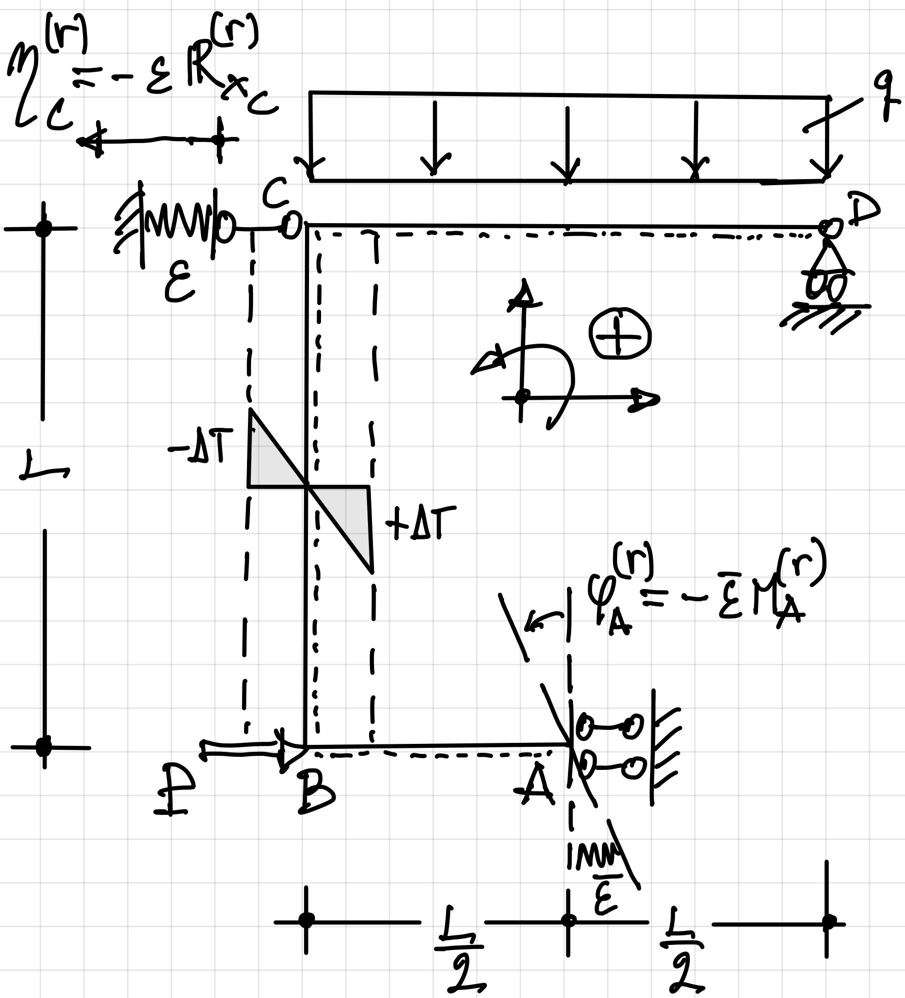
MECCANICA delle STRUTTURE – P. FUSCHI

TEST in ITINERE del 18 DIC. 2024

ES. #1 RISOLVERE LA STRUTTURA UNA VOLTA IPERSTATICA SEGUENTE DETERMINANDO IL DIAGRAMMA DEI MOMENTI_



POSIZIONI:

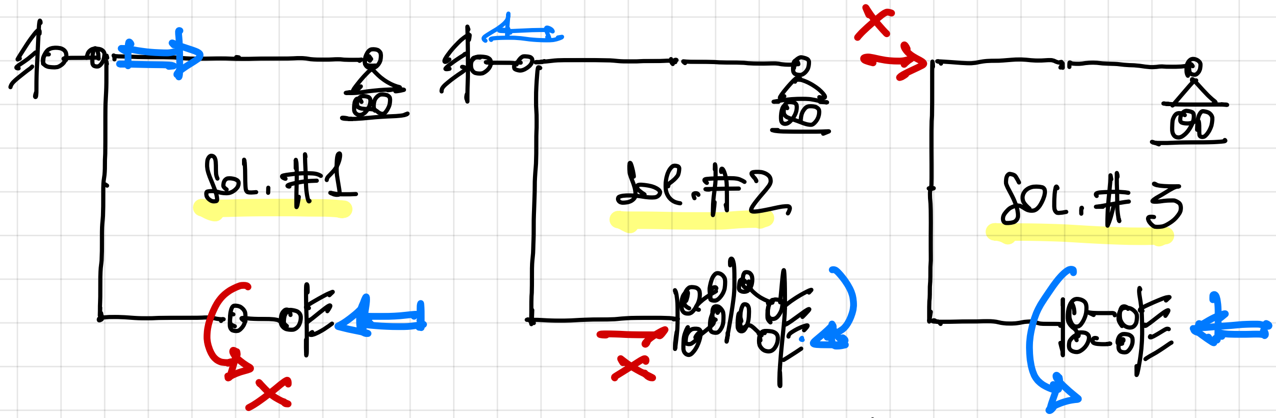
$|P| = qL$

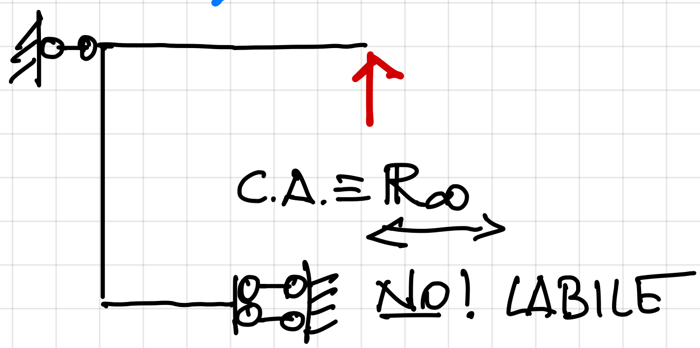
$|\varepsilon| = \frac{L^3}{12EI}$

$|\bar{\varepsilon}| = \frac{L}{12EI}$

$\left|\frac{\alpha \Delta T}{h}\right| = \frac{23}{12}\frac{qL^2}{EI}$

SOLUZIONI POSSIBILI



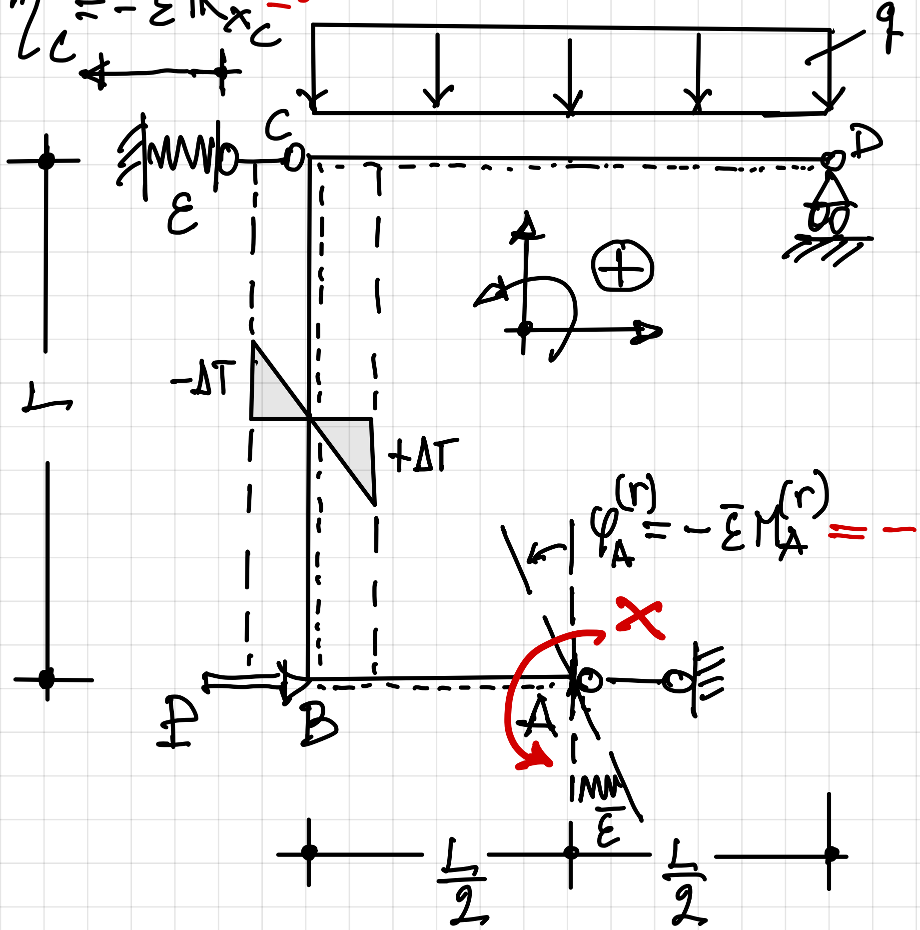


SOLUZIONE #1

⇨ SISTEMA PRINCIPALE ISOSTATICO

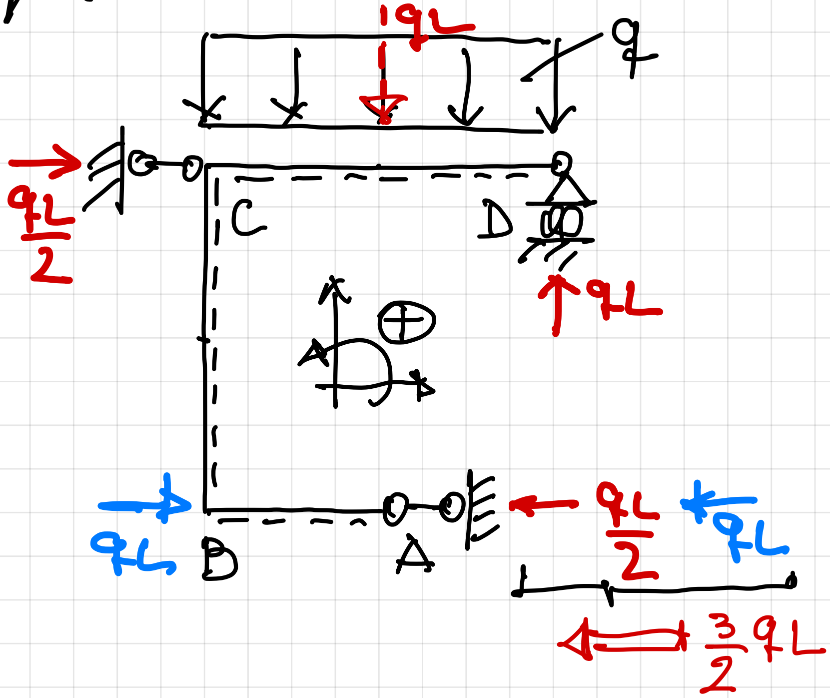
$$\eta_C^{(r)} = -\varepsilon R_{x_C}^{(r)} = -\varepsilon\left[R_{x_C}^{(0)} + X\, R_{x_C}^{(1)}\right]$$

$$\varphi_A^{(r)} = -\bar{\varepsilon} M_A^{(r)} = -\bar{\varepsilon} X$$



SOLUZIONI!

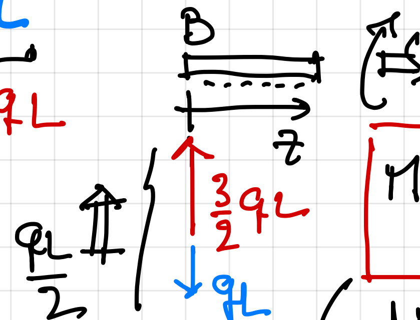
SCHEMA [0] SOLO CARICHI ESTERNI



TRATTO BA $0 \le z \le \frac{L}{2}$

$$\boxed{M^{(0)}(z) = \phi}$$

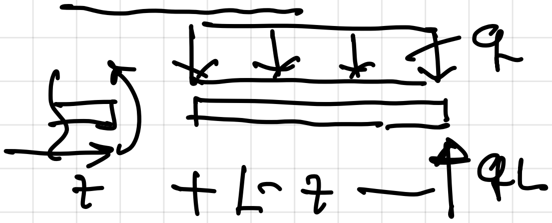
TRATTO BC $0 \le z \le L$



$$\boxed{M^{(0)}(z) = \frac{qL}{2} \cdot z}$$

$$\begin{cases} M_B = \phi \\ M_C = \dfrac{qL^2}{2} \end{cases}$$

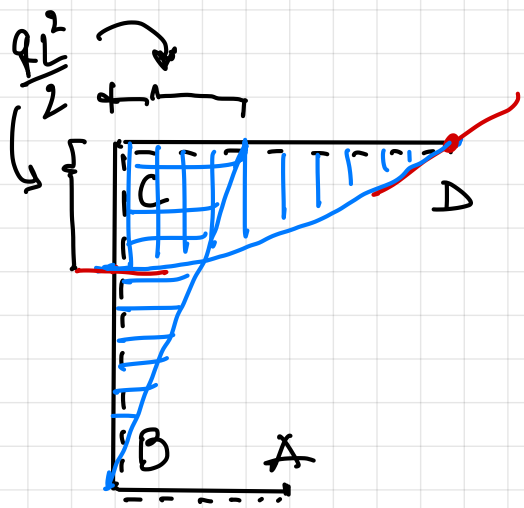
TRATTO CD $0 \le z \le L$



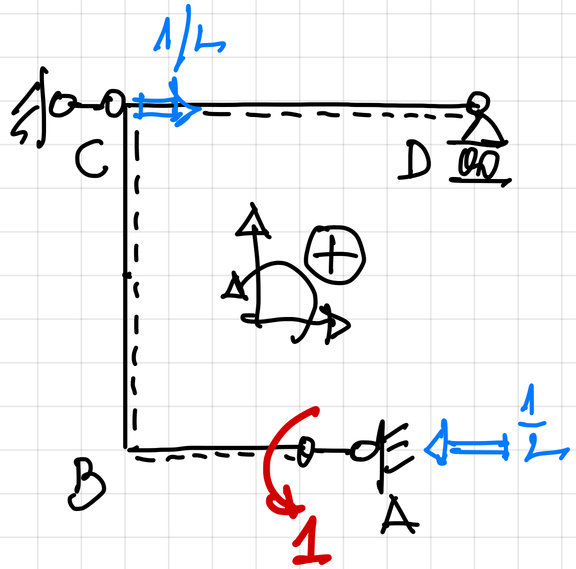
$$\boxed{M^{(0)}(z) = qL(L-z) - q\frac{(L-z)^2}{2}}$$

$$\begin{cases} M_C = \dfrac{qL^2}{2} \\ M_D = \phi \end{cases}$$

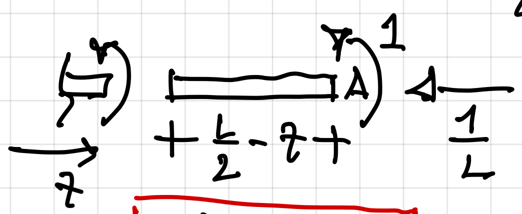
DIAGRAMMA $M^{(0)}(z)$



SCHEMA [1] solo $X=1$

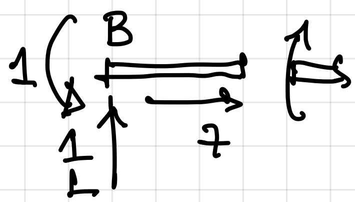


TRATTO BA $0 \le z \le \frac{L}{2}$



$$\boxed{M^{(1)}(z) = 1} \quad \text{cost.}$$

TRATTO BC $0 \le z \le L$

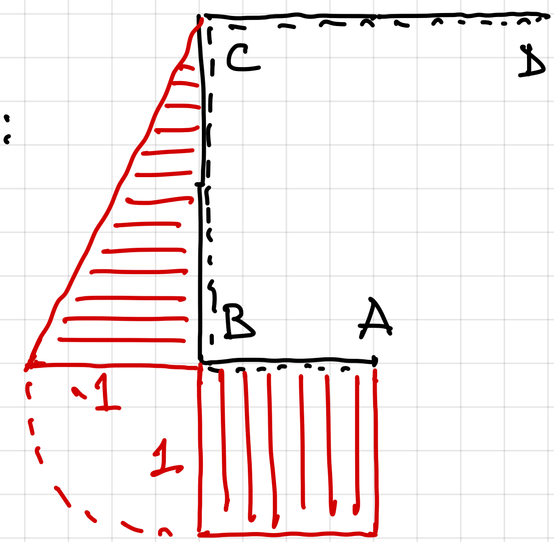


$$\boxed{M^{(1)}(z) = \frac{z}{L} - 1} \quad \begin{cases} M_B = -1 \\ M_C = \phi \end{cases}$$

TRATTO CD

$$\boxed{M^{(1)}(z) = \phi}$$

DIAGRAMMA $M^{(1)}(z)$:



$$\boxed{L_{ve}} = \underset{1}{X_i} \cdot \eta_i^{(r)} + \sum_j R_j^{(1)} \eta_j^{(r)} = 1 \cdot \varphi_A^{(r)} + R_{x_C}^{(1)} \cdot \overbrace{\eta_C^{(r)}}^{-\varepsilon\left[R_{x_C}^{(0)} + X R_{x_C}^{(1)}\right]} =$$

$$= -\bar{\varepsilon} X + \frac{1}{L}(-\varepsilon)\left[\frac{qL}{2} + X \cdot \frac{1}{L}\right] =$$

$$= \boxed{-\bar{\varepsilon} X - \frac{\varepsilon}{L}\left[\frac{qL}{2} + \frac{X}{L}\right]}$$

$$\boxed{L_{vi}} = \int_{str.} M^{(1)} \frac{\overbrace{M^{(r)}}^{\to M^{(0)}+M^{(1)}X}}{EI} ds_{tr} + \int_{str} M^{(1)} \frac{\alpha \Delta T}{h} ds_{tr} =$$

$$= \frac{1}{EI}\int_{str} M^{(1)} M^{(0)} ds_{tr} + \frac{X}{EI}\int_{str} [M^{(1)}]^2 ds_{tr} + \frac{\alpha\Delta T}{h}\int_{str} M^{(1)} ds_{tr} =$$

$$= \frac{1}{EI}\left[\int_0^L \underbrace{\left(\frac{z}{L}-1\right)\frac{qL}{2}\cdot z}_{\frac{q}{2}z^2 - \frac{qL}{2}z}\, dz\right] +$$

$$+ \frac{X}{EI}\left[\int_0^{\frac{L}{2}} 1 \cdot dz + \int_0^L \underbrace{\left(\frac{z}{L}-1\right)^2}_{\frac{z^2}{L^2}+1-\frac{2z}{L}} dz\right] + \frac{\alpha\Delta T}{h}\int_0^L \left(\frac{z}{L}-1\right) dz =$$

$$= \frac{1}{EI}\left[\frac{q}{2}\left[\frac{z^3}{3}\right]_0^L - \frac{qL}{2}\left[\frac{z^2}{2}\right]_0^L\right] +$$

$$+ \frac{X}{EI}\left[[z]_0^{\frac{L}{2}} + \frac{1}{L^2}\left[\frac{z^3}{3}\right]_0^L + [z]_0^L - \frac{2}{L}\left[\frac{z^2}{2}\right]_0^L\right] +$$

$$+ \frac{\alpha\Delta T}{h}\left\{\frac{1}{L}\left[\frac{z^2}{2}\right]_0^L - [z]_0^L\right\} =$$

$$= \frac{1}{EI}\left[\frac{qL^3}{6} - \frac{qL^3}{4}\right] + \frac{X}{EI}\left[\frac{L}{2} + \frac{L}{3} + \cancel{L} - \cancel{L}\right] +$$

$$+ \frac{\alpha\Delta T}{h}\left\{\frac{L}{2} - L\right\} =$$

$$= \boxed{-\frac{1}{12}\frac{qL^3}{EI} + \frac{5}{6}\frac{XL}{EI} - \frac{\alpha\Delta T}{h}\frac{L}{2}}$$

$\Rightarrow$ Lve = Lvi fornisce:

$$-\bar{\varepsilon}X - \frac{\varepsilon}{L}\left[\frac{qL}{2} + \frac{X}{L}\right] = -\frac{1}{12}\frac{qL^3}{EI} + \frac{5}{6}\frac{XL}{EI} - \frac{\alpha\Delta T}{h}\frac{L}{2}$$

$$X\left[-\bar{\varepsilon} - \frac{\varepsilon}{L^2} - \frac{5}{6}\frac{L}{EI}\right] = -\frac{1}{12}\frac{qL^3}{EI} - \frac{\alpha\Delta T}{h}\frac{L}{2} + \frac{\varepsilon q}{2}$$

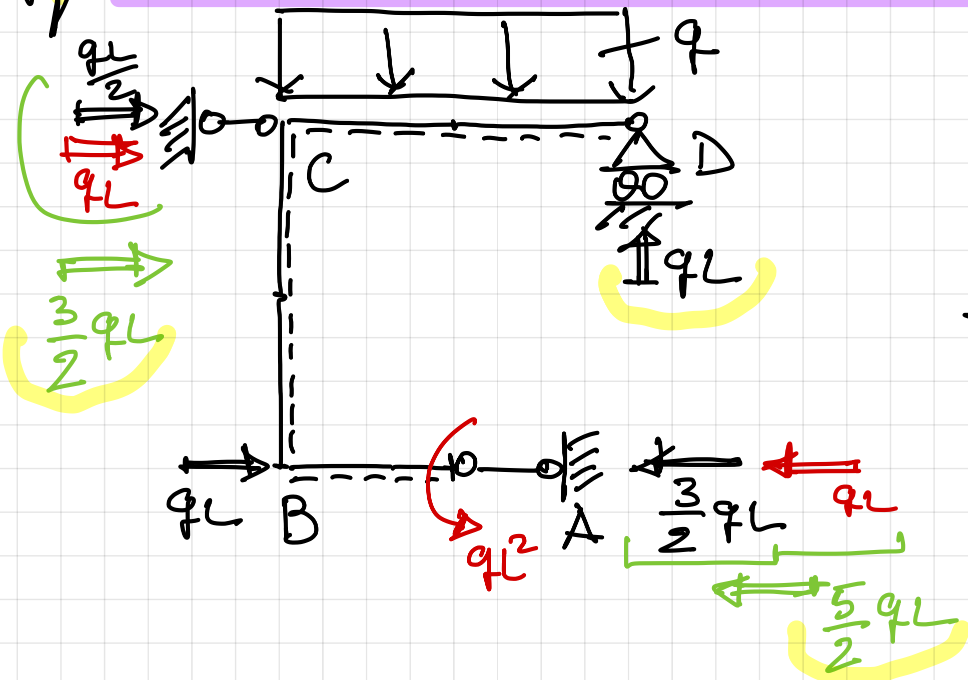
tenendo conto delle posizioni di pag. 1 si ha:

$$X\underbrace{\left[-\frac{L}{12EI} - \frac{L}{12EI} - \frac{5}{6}\frac{L}{EI}\right]}_{-\frac{L}{EI}\left[\frac{1}{12}+\frac{1}{12}+\frac{5}{6}\right] = \underbrace{\frac{2}{12}+\frac{10}{12}}_{1}} = \underbrace{-\frac{1}{12}\frac{qL^3}{EI} - \frac{23}{24}\frac{qL^3}{EI} + \frac{qL^3}{24EI}}_{\frac{qL^3}{EI}\left[\underbrace{-\frac{1}{12}}_{-\frac{2}{24}} \underbrace{- \frac{23}{24} + \frac{1}{24}}_{-\frac{22}{24}}\right],\quad -\frac{2}{24}-\frac{22}{24} = -1}$$

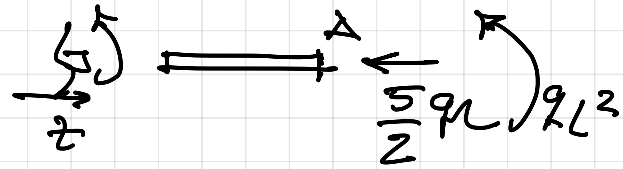
$$-\frac{\cancel{L}}{\cancel{EI}}X = -\frac{qL^{\cancel{3}\,2}}{\cancel{EI}}$$

da cui $\boxed{X = qL^2}$ $\Rightarrow$ POSITIVA! verso ipotizzato corretto!

SOLUZIONE SIST. PRINCIPALE ISOSTATICO

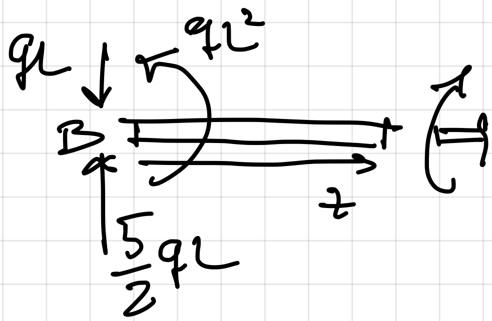


TRATTO BA $0 \le z \le \frac{L}{2}$



$$\boxed{M^{(r)}(z) = qL^2 \quad \text{cost.}}$$

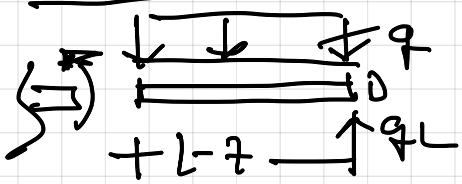
TRATTO BC $0 \le z \le L$



$$\boxed{M^{(r)}(z) = \frac{3}{2}qL \cdot z - qL^2}$$

$$\begin{cases} M_B = -qL^2 \\ M_C = \dfrac{qL^2}{2} \end{cases}$$

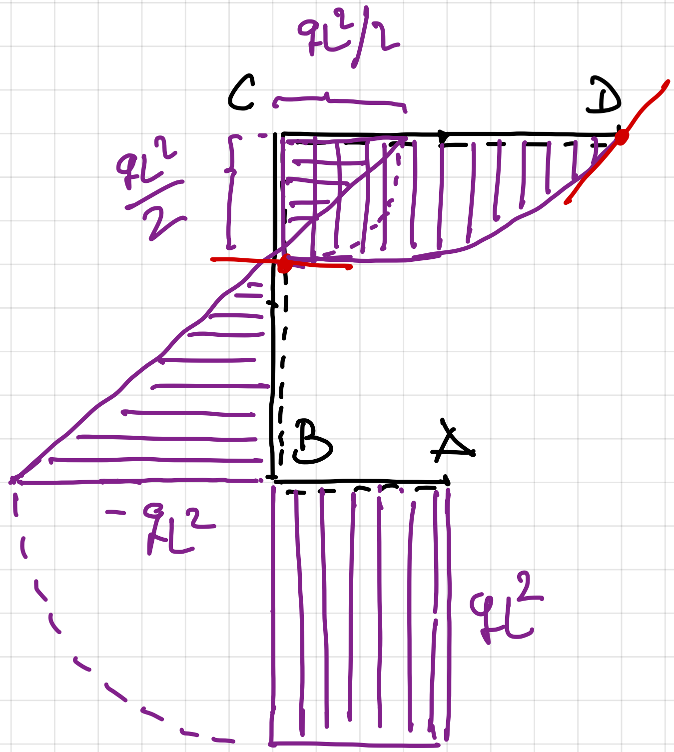
TRATTO CD $0 \le z \le L$



$$\boxed{M^{(r)}(z) = qL \cdot (L-z) - \frac{q(L-z)^2}{2}}$$

$$\hookrightarrow \begin{cases} M_C = \dfrac{qL^2}{2} \\ M_D = \emptyset \end{cases}$$

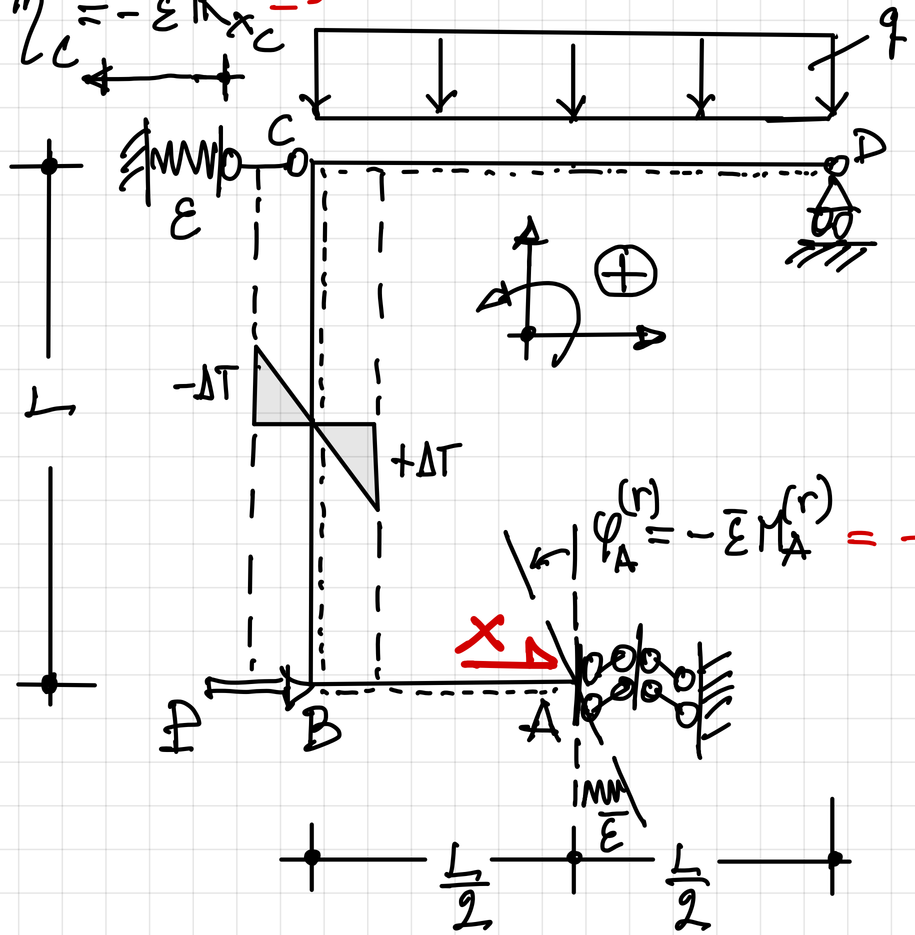
DIAGRAMMA DEI MOMENTI STRUTTURA IPERSTATICA



SOLUZIONE #2

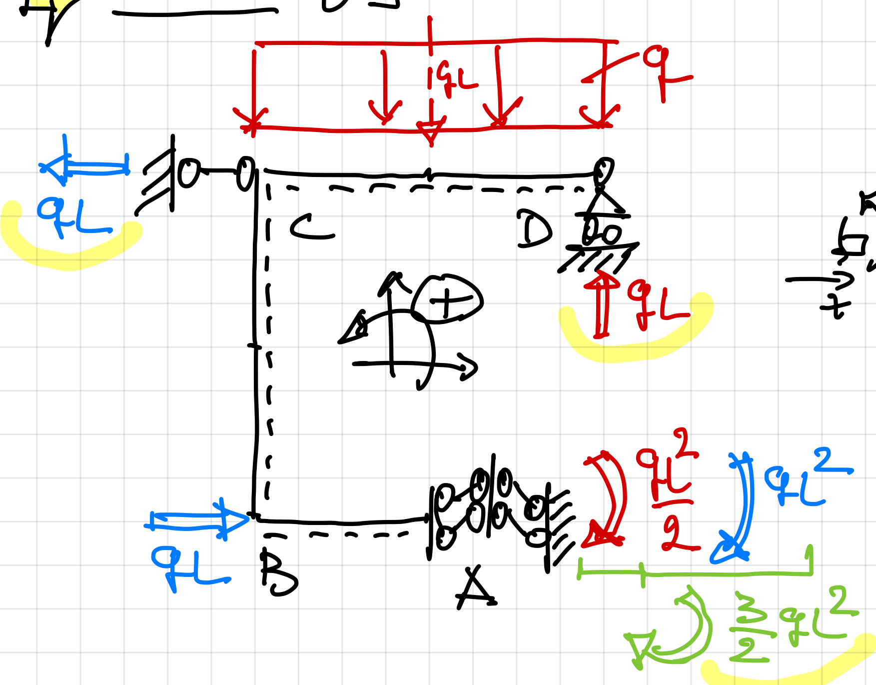
SISTEMA PRINCIPALE ISOSTATICO

$$\eta_C^{(r)} = -\varepsilon R_{x_C}^{(r)} = -\varepsilon\left[R_{x_C}^{(0)} + X R_{x_C}^{(1)}\right]$$

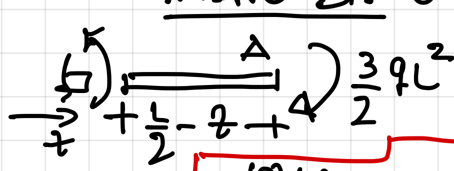


$$\varphi_A^{(r)} = -\bar{\varepsilon} M_A^{(r)} = -\bar{\varepsilon}\left[M_A^{(0)} + X M_A^{(1)}\right]$$

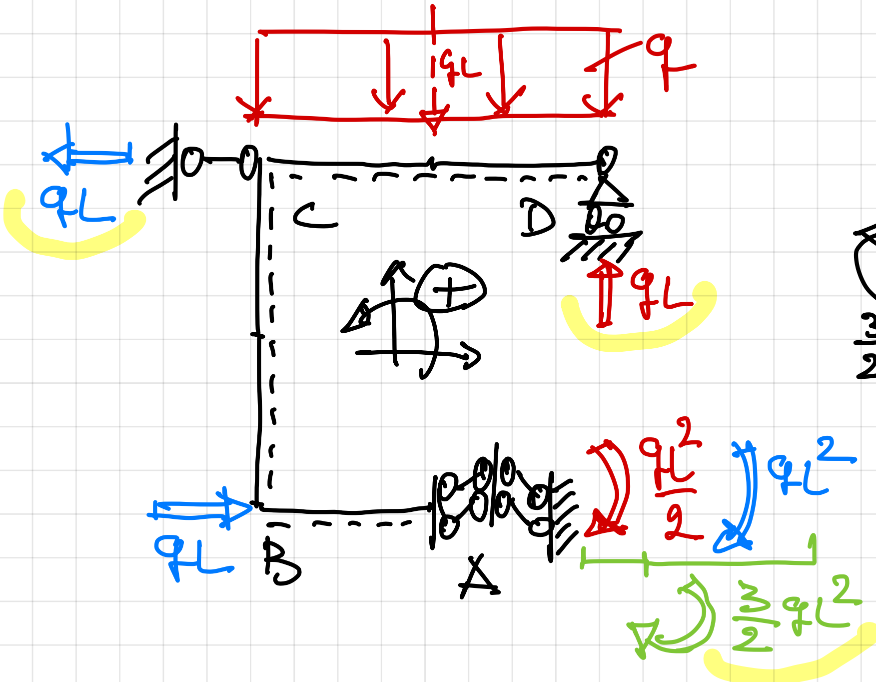
SCHEMA [0] SOLO CARICHI ESTERNI



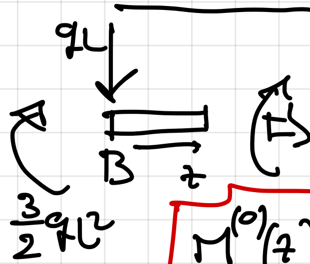
TRATTO BA $0 \le z \le \frac{L}{2}$



$$\boxed{M^{(0)}(z) = -\frac{3}{2} qL^2} \quad \text{cost.}$$



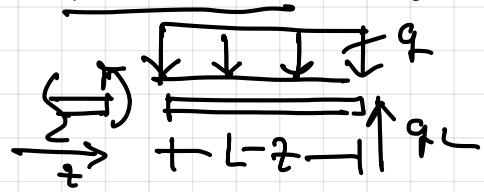
TRATTO BC $0 \le z \le L$



$$\boxed{M^{(0)}(z) = \frac{3}{2}qL^2 - qL\cdot z}$$

$\hookrightarrow \begin{cases} M_B = \frac{3}{2}qL^2 \\ M_C = \frac{qL^2}{2} \end{cases}$

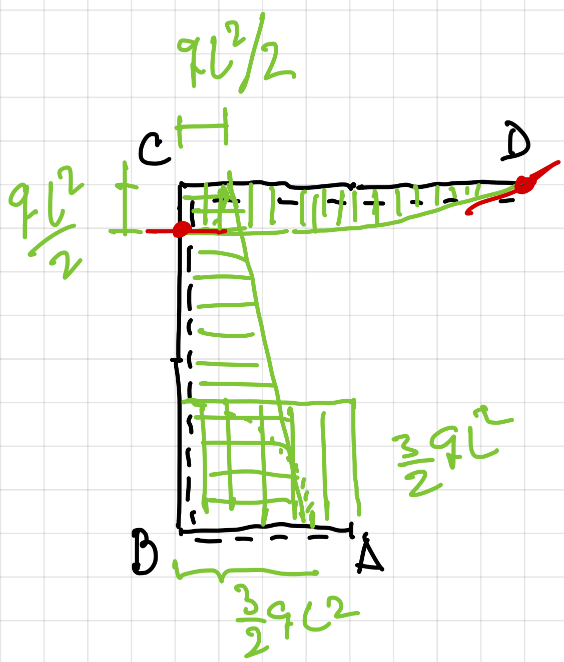
TRATTO CD $0 \le z \le L$



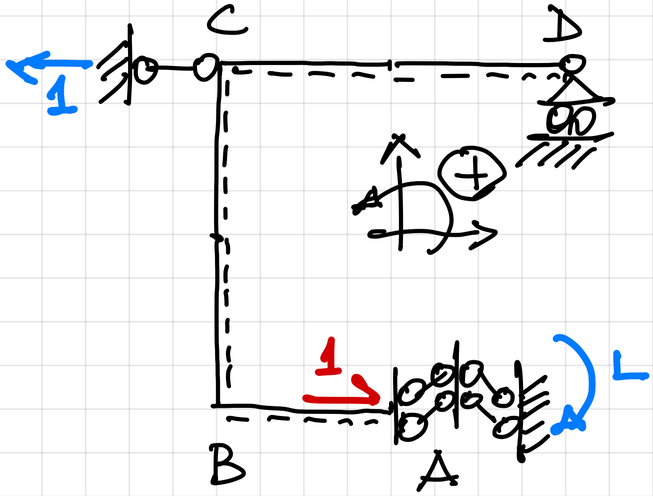
$$\boxed{M^{(0)}(z) = qL(L-z) - \frac{q(L-z)^2}{2}}$$

$\hookrightarrow \begin{cases} M_C = \frac{qL^2}{2} \\ M_D = \phi \end{cases}$

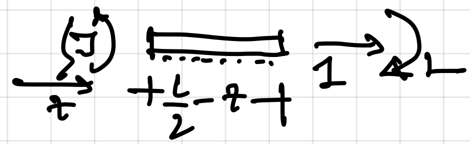
⇨ DIAGRAMMA $M^{(0)}(z)$



SCHEMA [1] solo $X = 1$

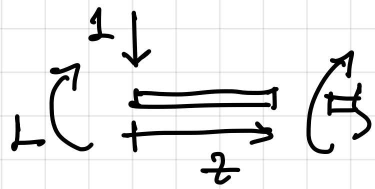


TRATTO BA $\quad 0 \le z \le \frac{L}{2}$



$$\boxed{M^{(1)}(z) = -L} \quad \text{cost.}$$

TRATTO BC $\quad 0 \le z \le L$



$$\boxed{M^{(1)}(z) = L - z}$$

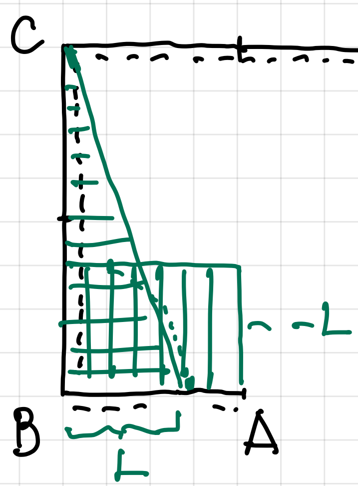
↳ $M_B = L$

$M_C = \phi$

TRATTO CD
scarico

$$\boxed{M^{(1)}(z) = \phi}$$

DIAGRAMMA $M^{(1)}(z)$



$$\varphi_A^{(r)} = -\varepsilon M_A^{(r)} = -\varepsilon\left[\overbrace{M_A^{(0)}}^{-\frac{3}{2}qL^2} + X \overbrace{M_A^{(1)}}^{-L}\right]$$

$$\boxed{L_{ve}} = \underset{\underset{1}{\parallel}}{X_i}\, \eta_i^{(r)} + \sum_j R_j^{(1)} \eta_j^{(r)} = 1 \cdot \phi + \underbrace{M_A^{(1)}}_{-L} \varphi_A^{(r)} + \overbrace{R_{x_C}^{(1)}}^{-1} \eta_C^{(r)} =$$

$$-\varepsilon R_{x_C}^{(r)} = -\varepsilon\left[\underbrace{R_{x_C}^{(0)}}_{-qL} + X \underbrace{R_{x_C}^{(1)}}_{-1}\right]$$

$$= +L\varepsilon\left[-\frac{3}{2}qL^2 - XL\right] + \varepsilon\left[-qL - X\right] =$$

$$= \boxed{-L\varepsilon\left[\frac{3}{2}qL^2 + XL\right] - \varepsilon\left[qL + X\right]}$$

$$L_{vi} = \int_{Str} M^{(1)} \frac{\overset{M^{(0)} + M^{(1)} X}{M^{(T)}}}{EI} \, dStr + \int_{Str} M^{(1)} \frac{\alpha \overline{\Delta T}}{h} \, dStr =$$

$$= \frac{1}{EI} \int_{Str} M^{(1)} M^{(0)} dStr + \frac{X}{EI} \int_{Str} [M^{(1)}]^2 dStr + \frac{\alpha \overline{\Delta T}}{h} \int_{Str} M^{(1)} dz =$$

$$= \frac{1}{EI} \left[\int_0^{\frac{L}{2}} (-L)\left(-\frac{3}{2} q L^2\right) dz + \int_0^L (L-z)\left(\frac{3}{2} q L^2 - q L z\right) dz \right] +$$

$$+ \frac{X}{EI} \left[\int_0^{\frac{L}{2}} L^2 dz + \int_0^L (L-z)^2 dz \right] + \frac{\alpha \overline{\Delta T}}{h} \int_0^L (L-z) dz =$$

$$= \frac{1}{EI} \left[\frac{3}{2} q L^3 \int_0^{\frac{L}{2}} dz + \int_0^L \left[\frac{3}{2} q L^3 - q L^2 z - \frac{3}{2} q L^2 z + q L z^2\right] dz \right]$$

$$+ \frac{X}{EI} \left[L^2 \int_0^{\frac{L}{2}} dz + \int_0^L (L^2 + z^2 - 2Lz) dz \right] + \frac{\alpha \overline{\Delta T}}{h} \int_0^L (L-z) dz =$$

$$= \frac{1}{EI} \left[\frac{3}{2} q L^3 [z]_0^{\frac{L}{2}} + \frac{3}{2} q L^3 [z]_0^L - q L^2 \left[\frac{z^2}{2}\right]_0^L - \frac{3}{2} q L^2 \left[\frac{z^2}{2}\right]_0^L + q L \left[\frac{z^3}{3}\right]_0^L \right]$$

$$+ \frac{X}{EI} \left[L^2 [z]_0^{\frac{L}{2}} + L^2 [z]_0^L + \left[\frac{z^3}{3}\right]_0^L - 2L \left[\frac{z^2}{2}\right]_0^L \right] +$$

$$+ \frac{\alpha \overline{\Delta T}}{h} \left[L [z]_0^L - \left[\frac{z^2}{2}\right]_0^L \right] =$$

$$= \frac{1}{EI} \left[\frac{3}{2} q L^3 \cdot \frac{L}{2} + \frac{3}{2} q L^3 \cdot L - q L^2 \cdot \frac{L^2}{2} - \frac{3}{4} q L^2 \cdot L^2 + \frac{qL}{3} \cdot L^3 \right] +$$

$$+ \frac{X}{EI} \left[L^2 \cdot \frac{L}{2} + L^2 \cdot L + \frac{L^3}{3} - L \cdot L^2 \right] +$$

$$+ \frac{\alpha \overline{\Delta T}}{h} \left[L^2 - \frac{L^2}{2} \right] = \text{continua} \rightarrow$$

$$\boxed{L_{vi}} = \frac{qL^4}{EI}\left[\cancel{\frac{3}{4}}+\frac{3}{2}-\frac{1}{2}-\cancel{\frac{3}{4}}+\frac{1}{3}\right]+$$

$$+\frac{XL^3}{EI}\left\{\frac{1}{2}+\cancel{1}+\frac{1}{3}-\cancel{1}\right]+\frac{\alpha\overline{\Delta T}}{h}\cdot\frac{L^2}{2}=$$

$$=\boxed{\frac{4}{3}\frac{qL^4}{EI}+\frac{5}{6}\frac{L^3}{EI}X+\frac{\alpha\overline{\Delta T}}{h}\cdot\frac{L^2}{2}}$$

$1+\frac{1}{3}$

$\frac{1}{2}+\frac{1}{3}$ $\quad \frac{3+2}{6}$

$\Rightarrow$ $L_{ve}=L_{vi}$ fornisce:

$$-L\bar{\varepsilon}\left[\frac{3}{2}qL^2+XL\right]-\varepsilon\left[qL+X\right]=$$

$$=\frac{4}{3}\frac{qL^4}{EI}+\frac{5}{6}\frac{L^3}{EI}X+\frac{\alpha\overline{\Delta T}}{h}\cdot\frac{L^2}{2}$$

$$X\left\{-L^2\bar{\varepsilon}-\varepsilon-\frac{5}{6}\frac{L^3}{EI}\right\}=\frac{4}{3}\frac{qL^4}{EI}+\frac{\alpha\overline{\Delta T}}{h}\cdot\frac{L^2}{2}+\frac{3}{2}qL^3\cdot\bar{\varepsilon}+qL\cdot\varepsilon$$

tenendo conto delle posizioni di pag 1 si ha:

$$X\left\{-L^2\cdot\frac{L}{12EI}-\frac{L^3}{12EI}-\frac{5}{6}\frac{L^3}{EI}\right\}=\frac{4}{3}\frac{qL^4}{EI}+\frac{L^2}{2}\frac{23}{12}\frac{qL^2}{EI}+\frac{3}{2}qL^3\cdot\frac{L}{12EI}+qL\cdot\frac{L^3}{12EI}$$

$$X\frac{\cancel{L^3}}{\cancel{EI}}\underbrace{\left\{-\frac{1}{12}-\frac{1}{12}-\frac{5}{6}\right\}}_{-1}=\frac{q\cancel{L^4}}{\cancel{EI}}\underbrace{\left\{\frac{4}{3}+\frac{23}{24}+\frac{3}{24}+\frac{1}{12}\right\}}_{}$$

$\cancel{\frac{60}{24}}\ \cancel{\frac{36}{12}}\ \frac{5}{2}$

da cui

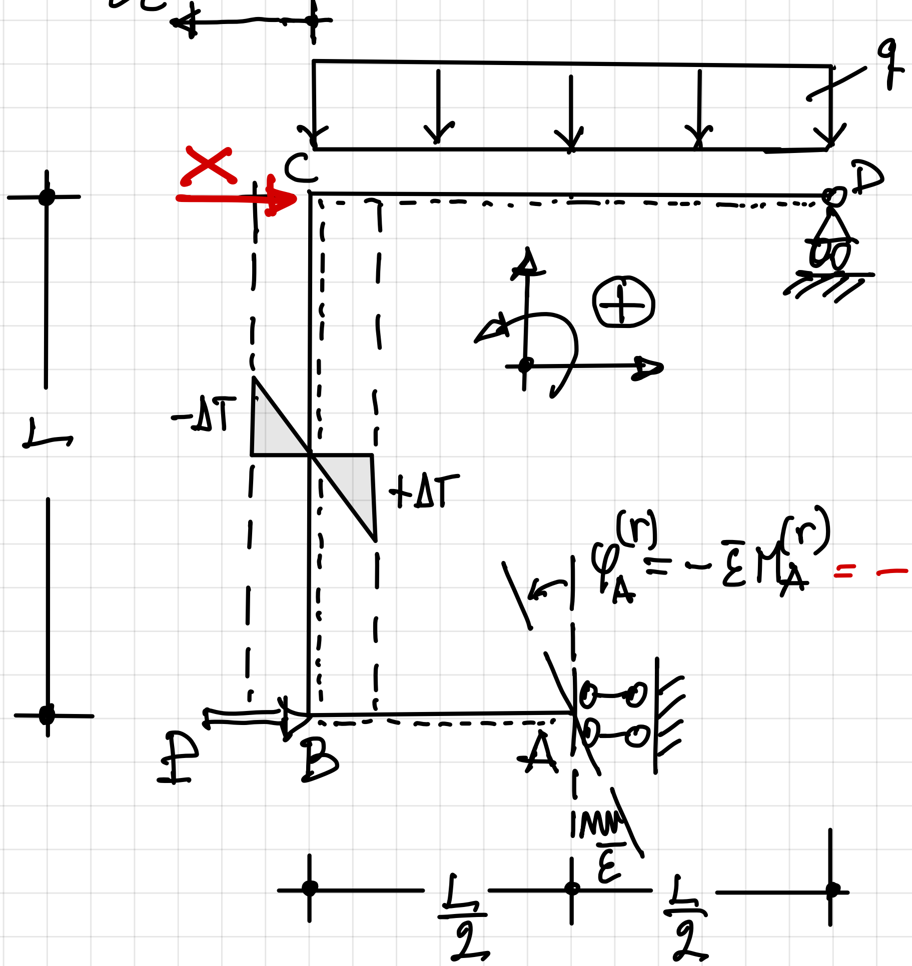
$$\boxed{X=-\frac{5}{2}qL}$$

VERSO OPPOSTO
A QUELLO IPOTIZZATO
VALORE OK! cfr. RV di pag. 7

SOLUZIONE # 3

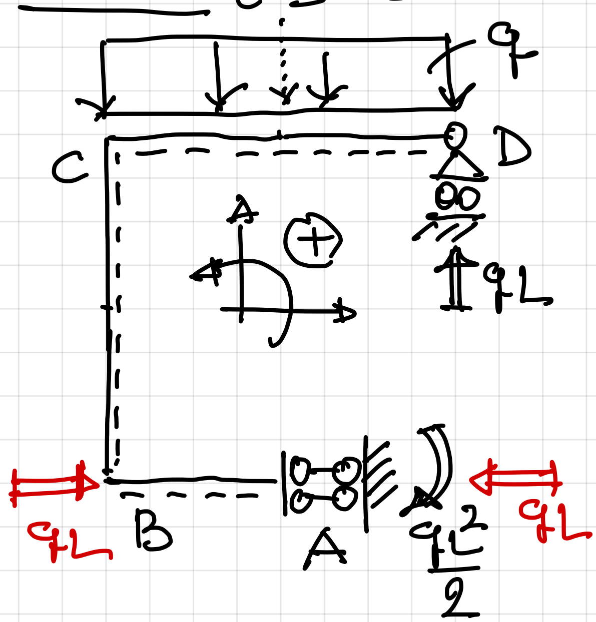
SISTEMA PRINCIPALE ISOSTATICO

$$\eta_C^{(r)} = -\varepsilon R_{x_C}^{(r)} = -\varepsilon X$$

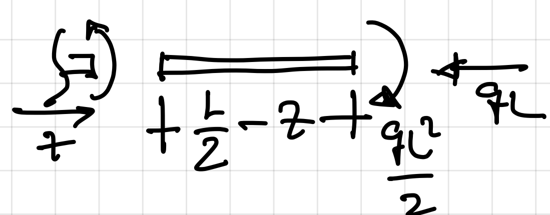


$$\varphi_A^{(r)} = -\bar{\varepsilon} M_A^{(r)} = -\bar{\varepsilon}\left[M_A^{(0)} + X M_A^{(1)}\right]$$

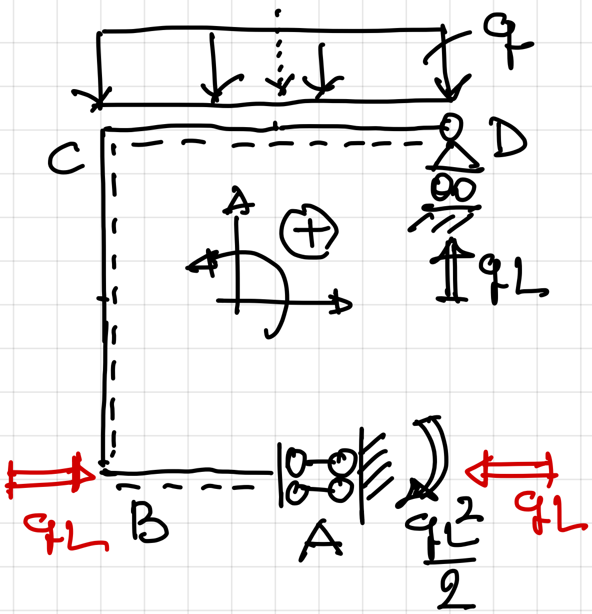
SCHEMA [0] SOLO CARICHI ESTERNI



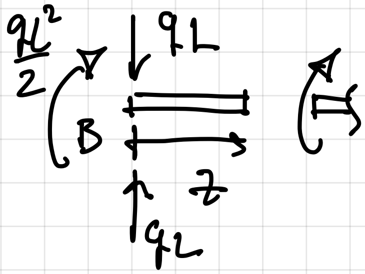
TRATTO BA $0 \le z \le \frac{L}{2}$



$$\boxed{M^{(0)}(z) = -\frac{qL^2}{2}} \quad \text{cost.}$$

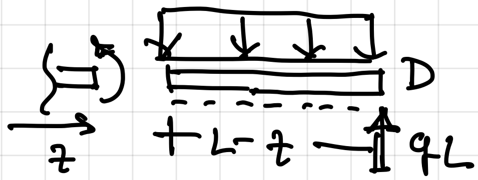


TRATTO BC $\quad 0 \le z \le L$



$$\boxed{M^{(0)}(z) = \frac{qL^2}{2}}\ \text{cost}$$

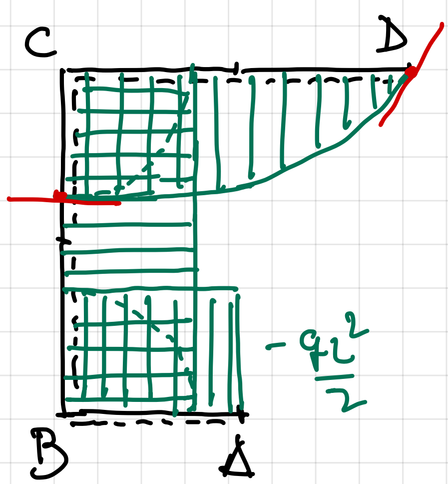
TRATTO CD $\quad 0 \le z \le L$



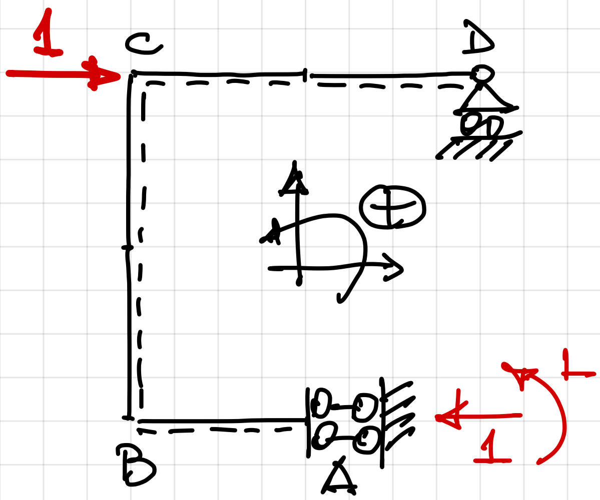
$$\boxed{M^{0}(z) = qL(L-z) - \frac{q(L-z)^2}{2}}$$

$$\to \begin{cases} M_C = \dfrac{qL^2}{2} \\ M_D = \emptyset \end{cases}$$

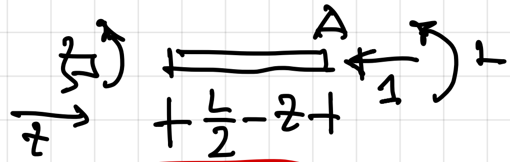
$\Rightarrow$ DIAGRAMMA $M^{(0)}(z)$



SCHEMA [1] solo $X=1$

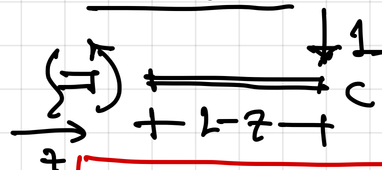


TRATTO BA $0 \le z \le L$



$$\boxed{M^{(1)}(z) = L} \quad \text{cost}$$

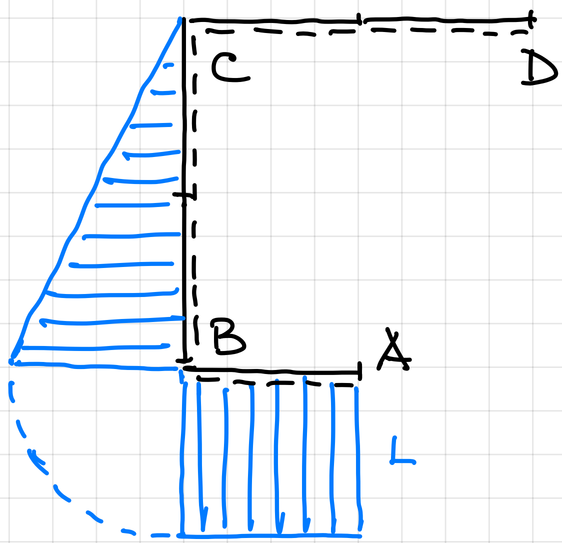
TRATTO BC $0 \le z \le L$



$$\boxed{M^{(1)}(z) = -(L-z)} \qquad \begin{cases} M_B = -L \\ M_C = 0 \end{cases}$$

TRATTO CD scarico $\boxed{M^{(1)}(z) = 0}$

DIAGRAMMA $M^{(1)}(z)$



$$\boxed{L_{ve}} = \underset{1}{\underset{\parallel}{X_i}}\,\eta_i^{(r)} + \sum_j R_j^{(1)}\eta_j^{(r)} = 1\cdot \overbrace{\eta_c^{(r)}}^{-\varepsilon X} + \underbrace{M_A^{(1)}}_{L}\,\overbrace{\varphi_A^{(r)}}^{-\varepsilon\left[\overbrace{M_A^{(0)}}^{-\frac{qL^2}{2}} + X\overbrace{M_A^{(1)}}^{L}\right]} =$$

$$= \boxed{-\varepsilon X - \varepsilon L\left[-\frac{qL^2}{2} + XL\right]}$$

$$\boxed{L_{vi}} = \int_{str} M^{(1)} \frac{\overbrace{M^{(r)}}^{M^{(0)}+XM^{(1)}}}{EI}\, ds_{tr} + \int_{str} M^{(1)} \frac{\alpha \overline{\Delta T}}{h}\, ds_{tr} =$$

$$= \frac{1}{EI}\int_{str} M^{(1)} M^{(0)}\, ds_{tr} + \frac{X}{EI}\int_{str} \left[M^{(1)}\right]^2 ds_{tr} + \frac{\alpha\overline{\Delta T}}{h}\int_{str} M^{(1)}\, ds_{tr} =$$

$$= \frac{1}{EI}\left\{\int_0^{\frac{L}{2}} L\cdot\left(-\frac{qL^2}{2}\right) dz + \int_0^L -(L-z)\frac{qL^2}{2}\, dz\right\} +$$

$$+ \frac{X}{EI}\left\{\int_0^{\frac{L}{2}} L^2\, dz + \int_0^L \overbrace{(L-z)^2}^{L^2+z^2-2Lz}\, dz\right\} + \frac{\alpha\overline{\Delta T}}{h}\int_0^L -(L-z)\, dz =$$

$$= \frac{1}{EI}\left\{-\frac{qL^3}{2}\cdot\frac{L}{2} - \frac{qL^3}{2}\cdot L + \frac{qL^2}{2}\cdot\frac{L^2}{2}\right\} +$$

$$+ \frac{X}{EI}\left\{L^2\cdot\frac{L}{2} + L^2\cdot L + \frac{L^3}{3} - \cancel{2}L\cdot\frac{L^2}{\cancel{2}}\right\} - \frac{\alpha\overline{\Delta T}}{h}\left[L\cdot L - \frac{L^2}{2}\right] =$$

$$= \frac{qL^4}{EI}\left\{\cancel{-\frac{1}{4}} - \frac{1}{2} \cancel{+\frac{1}{4}}\right\} + \frac{XL^3}{EI}\left[\frac{1}{2}\cancel{+1} + \frac{1}{3}\cancel{-1}\right] - \frac{\alpha\overline{\Delta T}}{h}\frac{L^2}{2} =$$

$$= \boxed{-\frac{qL^4}{2EI} + \frac{5}{6}\frac{XL^3}{EI} - \frac{\alpha\overline{\Delta T}}{h}\frac{L^2}{2}}$$

$L_{ve} = L_{vi}$ fornisce:

$$-\varepsilon X - \varepsilon L\left[-\frac{qL^2}{2} + XL\right] = -\frac{qL^4}{2EI} + \frac{5}{6}\frac{XL^3}{EI} - \frac{\alpha\overline{\Delta T}}{h}\frac{L^2}{2}$$

$$-\varepsilon X - \varepsilon L\left[-\frac{qL^2}{2} + XL\right] = -\frac{qL^4}{2EI} + \frac{5}{6}\frac{XL^3}{EI} - \frac{\alpha\Delta T}{h}\frac{L^2}{2}$$

$$X\left[-\underset{\uparrow\ \frac{L^3}{12EI}}{\varepsilon} - \underset{\uparrow\ \frac{L}{12EI}}{\varepsilon} L^2 - \frac{5}{6}\frac{L^3}{EI}\right] = -\frac{qL^4}{2EI} - \underbrace{\frac{\alpha\Delta T}{h}}_{=\frac{23}{12}\frac{qL^2}{EI}}\cdot\frac{L^2}{2} - \frac{\underset{\uparrow\ \frac{L}{12EI}}{\varepsilon}\, qL^3}{2}$$

$$X\frac{\cancel{L^3}}{\cancel{EI}}\underbrace{\left[-\frac{1}{12}-\frac{1}{12}-\frac{5}{6}\right]}_{\frac{-1-1-10}{12} = -1} = \frac{qL^{\cancel{4}}}{\cancel{EI}}\left[-\frac{1}{2}\underbrace{-\frac{23}{24}-\frac{1}{24}}\right]$$

$$-\frac{1}{2} - 1 = -\frac{3}{2}$$

$$\boxed{X = \frac{3}{2}qL}$$

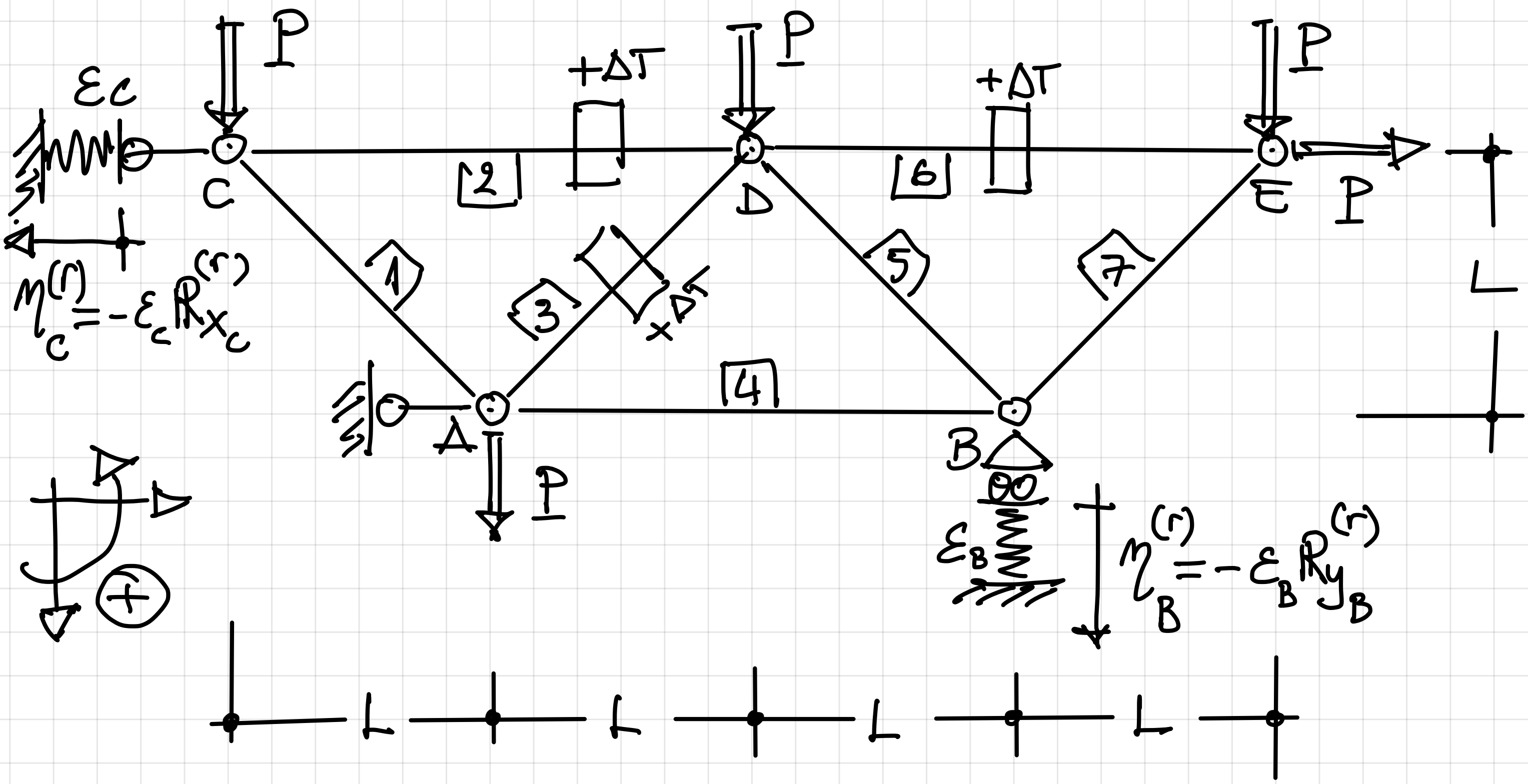
POSITIVO!
VERSO IPOTIZZATO CORRETTO

VALORE CORRETTO
CONFRONTA RV di pag. 7

MECCANICA delle STRUTTURE – P. FUSCHI

TEST in ITINERE del 18 DIC. 2024

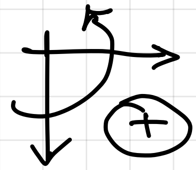
ES. #2 | CALCOLARE LO SPOSTAMENTO VERTICALE DEL NODO E DELLA TRAVATURA RETICOLARE SEGUENTE CON IL METODO DELLA FORZA UNITARIA

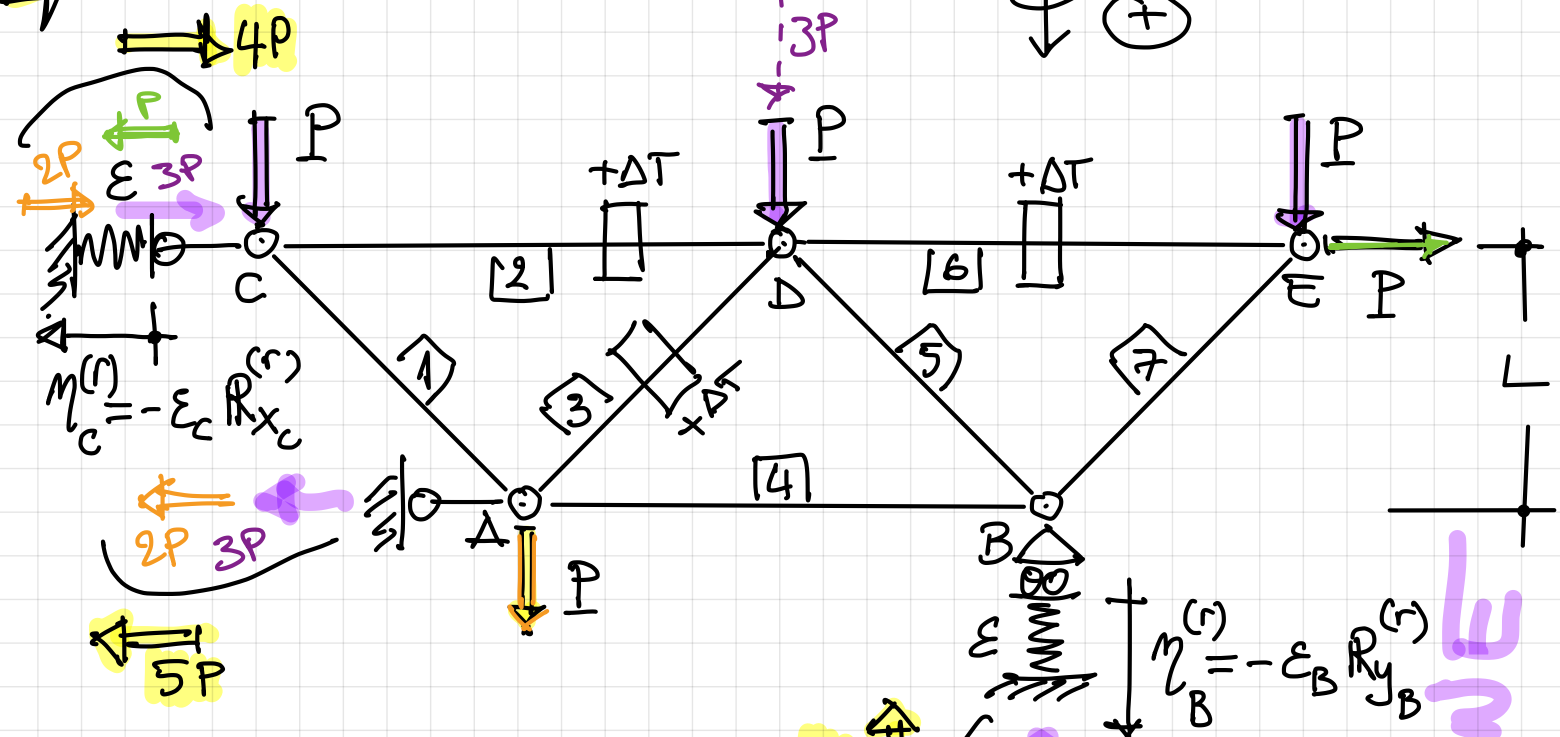


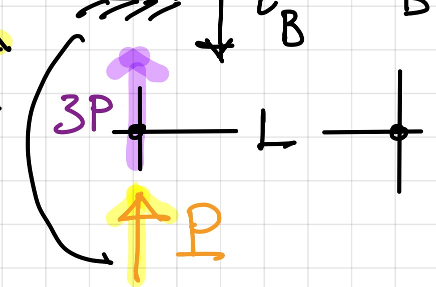
Posizioni : $|\mathcal{E}_B| = \dfrac{2L}{EA}$; $|\mathcal{E}_C| = \dfrac{\sqrt{2}\,L}{EA}$

$$|\alpha \Delta T| = \frac{P\sqrt{2}}{2EA}$$

STRUTTURA REALE





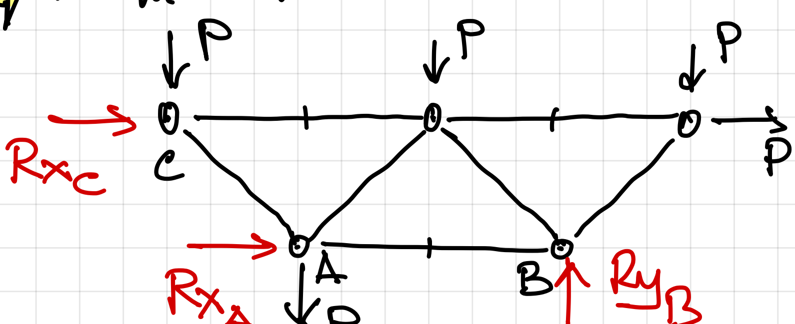


SOLUZIONE

RV metodo grafico:

vedi colori e applica il princ. di sovrapp. degli effetti ... RV in giallo!

RV metodo analitico:



$\Sigma F_x = 0 \quad R_{x_C} + R_{x_A} + P = 0$

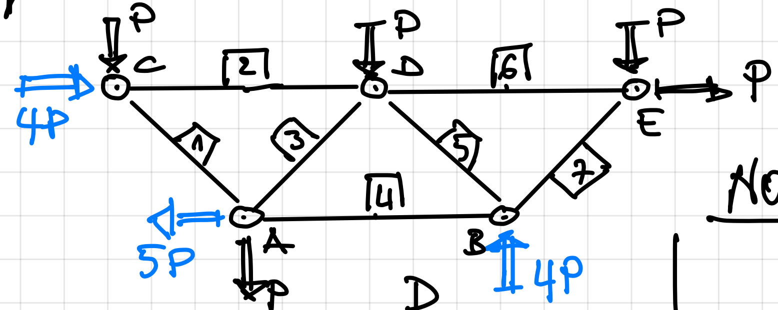
$\Sigma F_y = 0 \quad R_{y_B} - 4P = 0$

$\boxed{R_{y_B} = 4P}$

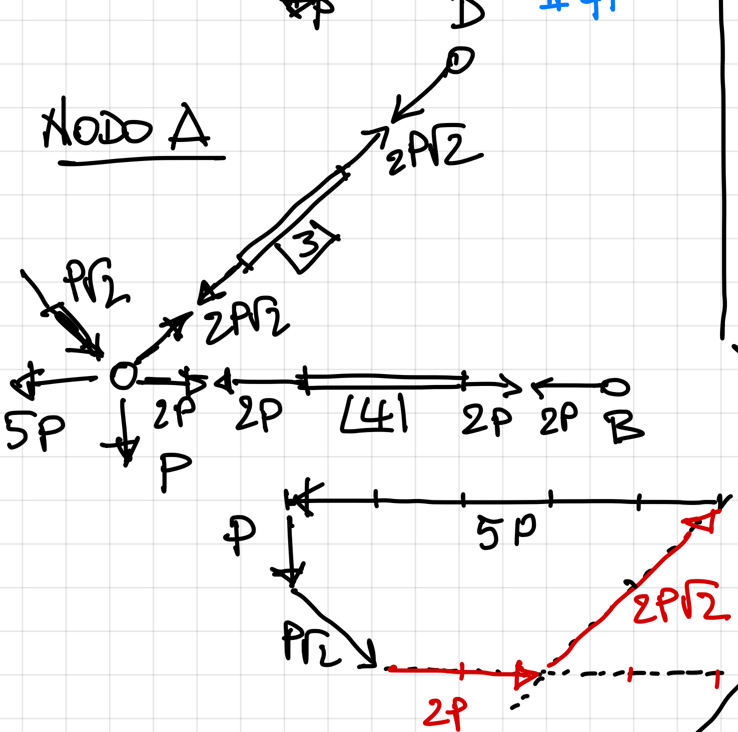
$\Sigma M_B = 0 \quad - R_{x_C} L + P3L + \cancel{PL} - PL + P2L - \cancel{PL} = 0$

$R_{x_C} = 4P$ dalla prima $R_{x_A} = -P - R_{x_C} = -5P$

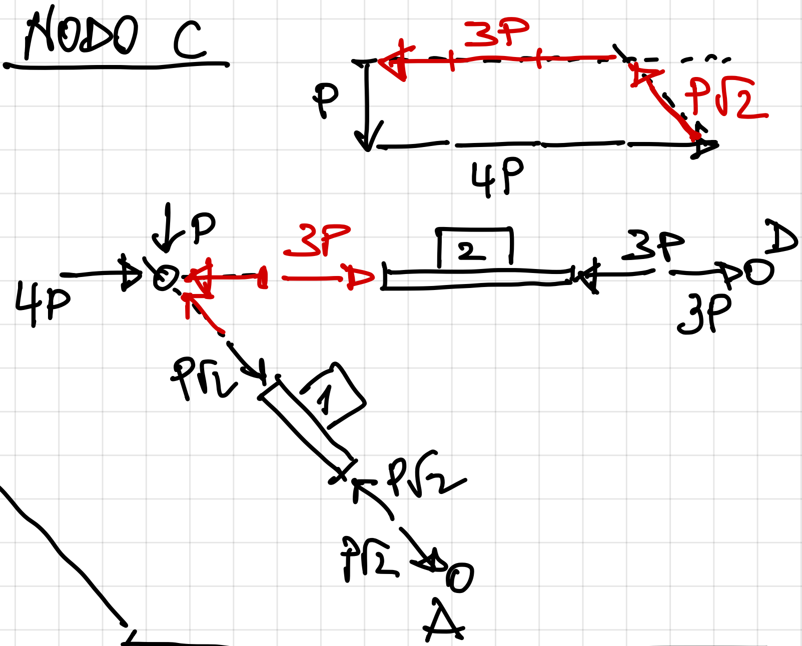
CALCOLO SFORZI NORMALI NELLA STRUTT. REALE



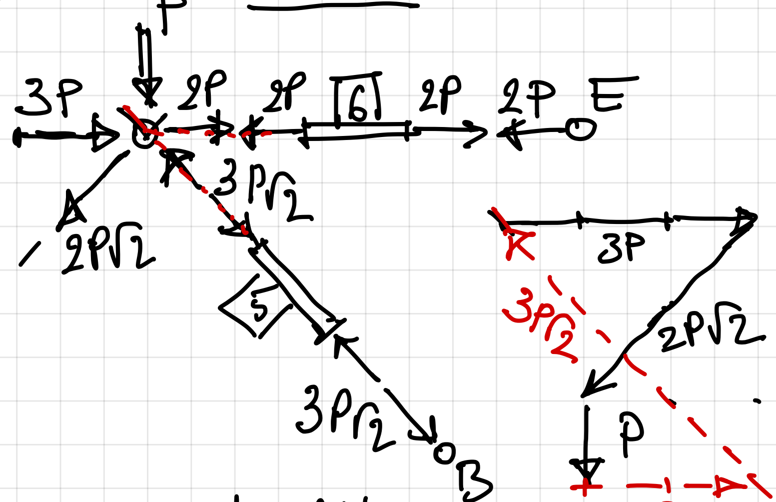
NODO A



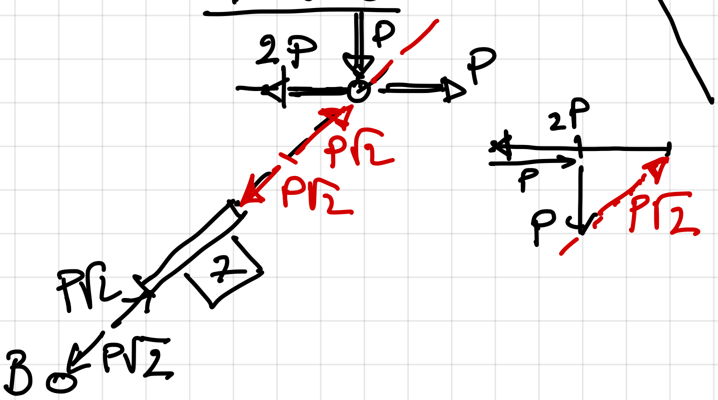
NODO C



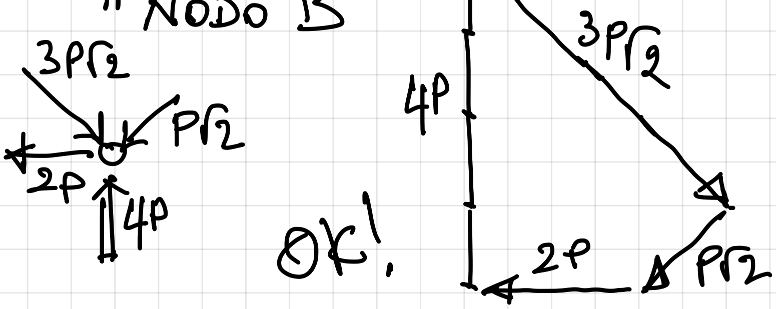
NODO D



NODO E



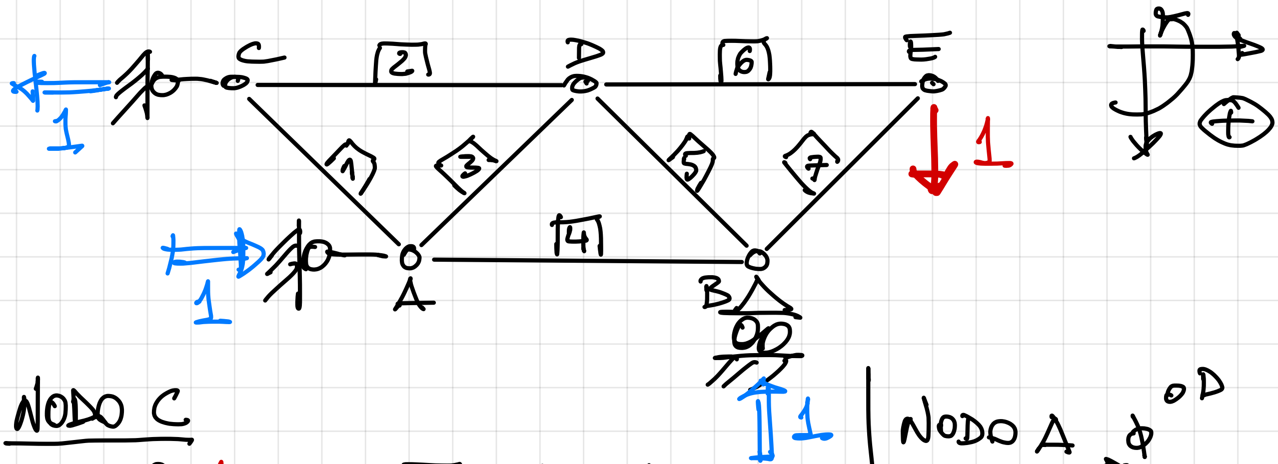
VERIFICA AL NODO B



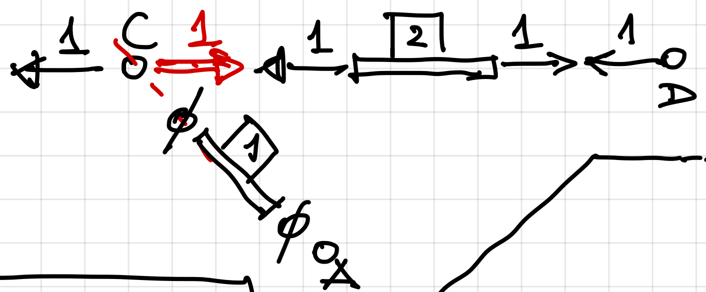
OK!

ASTA	$N^{(r)}$	Comp. mecc
1	$-P\sqrt{2}$	puntone
2	$-3P$	" "
3	$2P\sqrt{2}$	tirante
4	$2P$	tirante
5	$-3P\sqrt{2}$	puntone
6	$2P$	tirante
7	$-P\sqrt{2}$	puntone

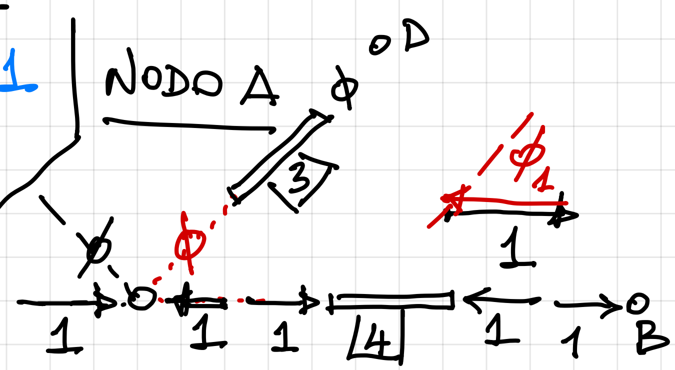
STRUTTURA FITTIZIA PER IL CALCOLO DELLO SPOSTAMENTO VERTICALE DEL NODO E



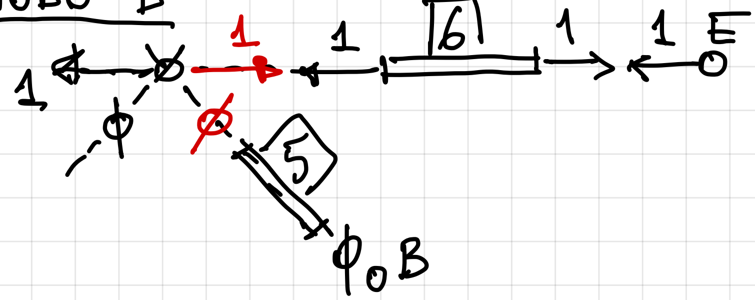
NODO C



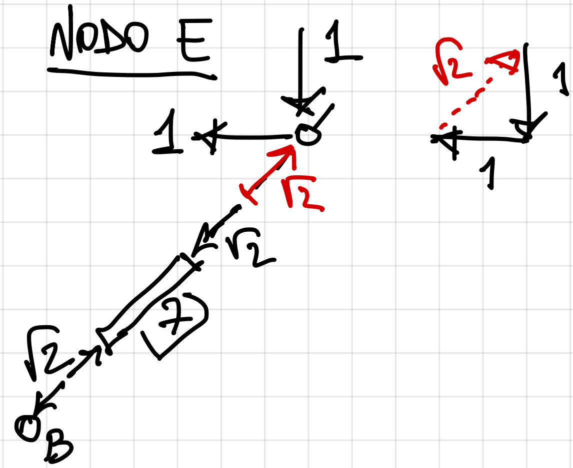
NODO A



NODO D

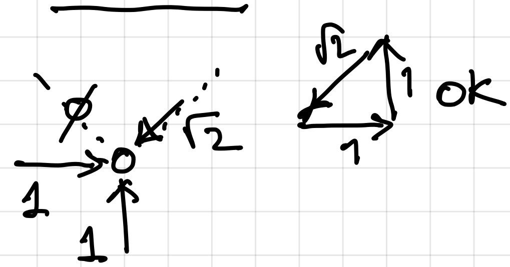


NODO E



ASTA	$N^{(f)}$	Comp. mecc.
1	∅	—
2	1	tirante
3	∅	—
4	-1	puntone
5	∅	—
6	1	tirante
7	$-\sqrt{2}$	puntone

VERIFICA AL NODO B



$$\boxed{L_{ve}} = 1 \cdot \eta_E + \sum_j R_j^{(f)} \eta_j^{(r)} =$$

$$= \eta_E + \underbrace{R_{y_B}^{(f)}}_{-1} \underbrace{\eta_B^{(r)}}_{-\varepsilon_B \underbrace{R_{y_B}^{(r)}}_{-4P}} + \underbrace{R_{x_C}^{(f)}}_{-1} \underbrace{\eta_C^{(r)}}_{-\varepsilon_C \underbrace{R_{x_C}^{(r)}}_{4P}} =$$

$$= \boxed{\eta_E - 4\varepsilon_B P + 4\varepsilon_C P}$$

$$\boxed{L_{vi}} = \underbrace{\sum_i N_i^{(f)} \frac{N_i^{(r)}}{EA} L_i}_{\text{solo aste } 2,4,6,7} + \underbrace{\sum_j N_j^{(f)} \alpha L_j \Delta T}_{\text{solo aste } 2,6} =$$

$$= (1)\left(\frac{-3P}{EA}\right) \cdot 2L + (-1)\left(\frac{2P}{EA}\right) \cdot 2L +$$

$$+ (1)\left(\frac{2P}{EA}\right) 2L + (-\sqrt{2})\left(\frac{-P\sqrt{2}}{EA}\right) L\sqrt{2} +$$

$$+ (1)\alpha\Delta T \cdot 2L + (1)\alpha\Delta T \cdot 2L =$$

$$= -6\frac{PL}{EA} - \cancel{4\frac{PL}{EA}} + \cancel{4\frac{PL}{EA}} + \frac{2P\sqrt{2}L}{EA} + 4\alpha\Delta T L =$$

$$= \boxed{-6\frac{PL}{EA} + \frac{2P\sqrt{2}L}{EA} + 4\alpha\Delta T L}$$

⇨ $L_{ve} = L_{vi}$ fornisce

$$\eta_E - 4\varepsilon_B P + 4\varepsilon_c P = -6\frac{PL}{EA} + \frac{2P\sqrt{2}L}{EA} + 4\alpha\Delta T L$$

da cui:

$$\eta_E = 4\underbrace{\varepsilon_B}_{\frac{2L}{EA}} P - 4\underbrace{\varepsilon_c}_{\frac{\sqrt{2}L}{EA}} P - 6\frac{PL}{EA} + \frac{2P\sqrt{2}L}{EA} + 4\underbrace{\alpha\Delta T}_{\frac{P\sqrt{2}}{2EA}} L =$$

$$\frac{8PL}{EA} \qquad -\frac{4\sqrt{2}PL}{EA} \qquad \frac{2P\sqrt{2}L}{EA}$$

$$= \frac{8PL}{EA} - \cancel{4\frac{\sqrt{2}PL}{EA}} - 6\frac{PL}{EA} + \cancel{\frac{2P\sqrt{2}L}{EA}} + \cancel{\frac{2P\sqrt{2}L}{EA}} =$$

$$= \boxed{\frac{2PL}{EA}}$$

POSITIVO ⇨ VERSO IL BASSO! COME LA FORZA UNITARIA CONSIDERATA IN E CONCORDE ALLO SPOST. IPOTIZZATO