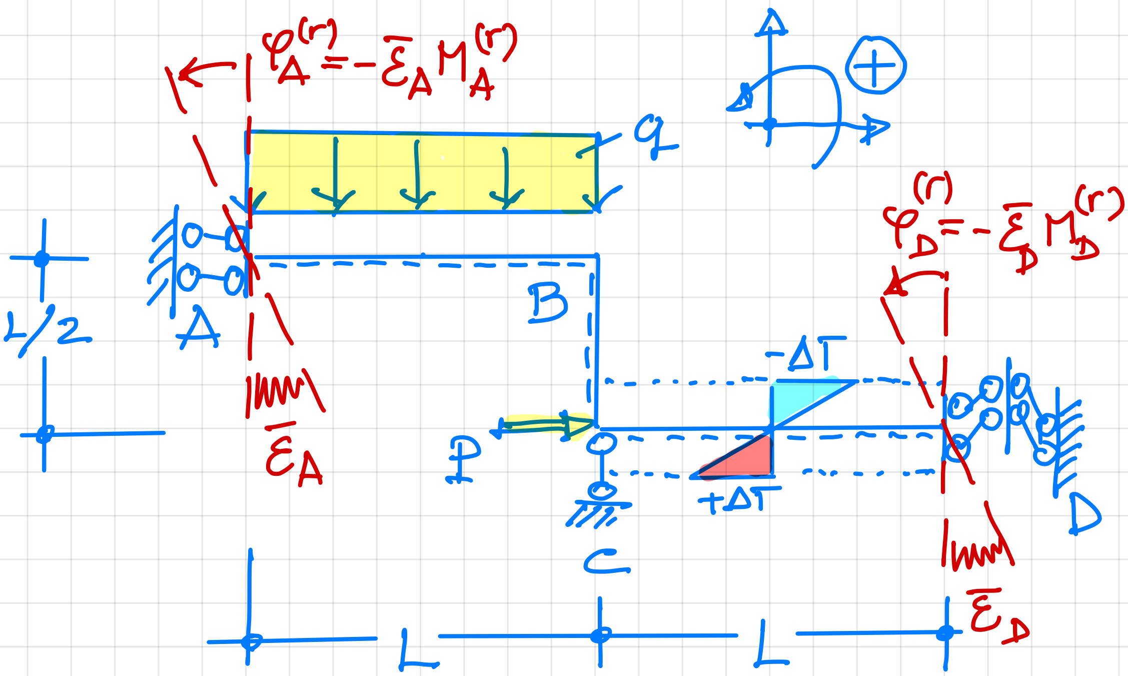
MECCANICA delle STRUTTURE - P. FUSCHI

PROVA SCRITTA del 9 APRILE 2025

TIPO 1

ES. #1

RISOLVERE LA STRUTTURA UNA VOLTA IPERSTATICA SEGUENTE DETERMINANDO IL DIAGRAMMA DEI MOMENTI



Posizioni

$|P| = qL$

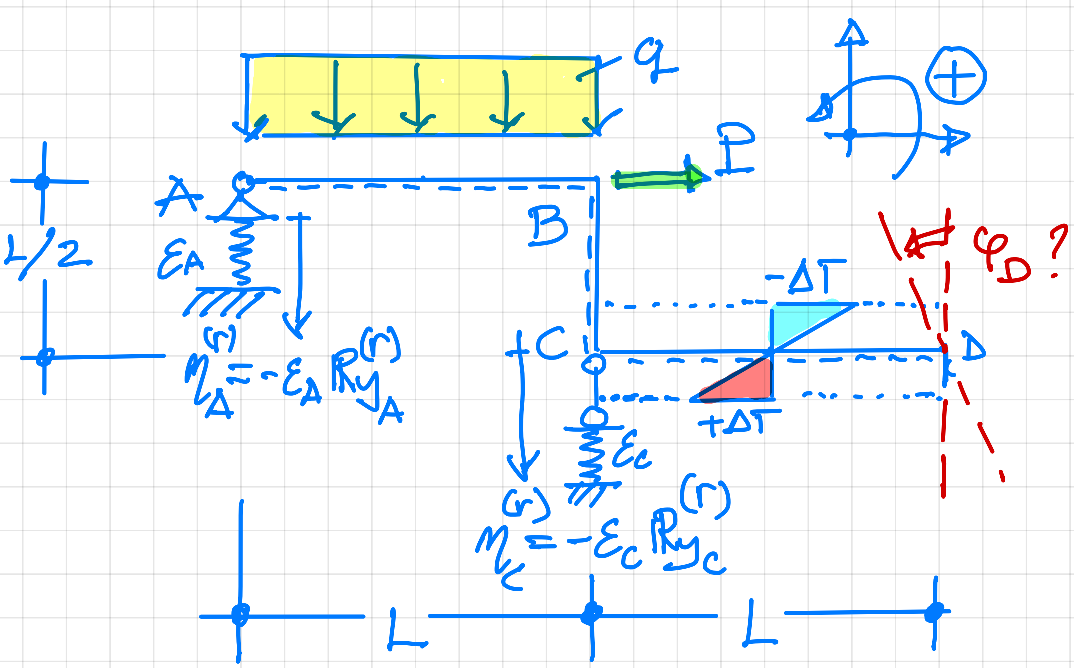
$|\bar{\varepsilon}_A| = \dfrac{L}{2EI}$

$|\bar{\varepsilon}_D| = \dfrac{L}{EI}$

$\left|\dfrac{\alpha \Delta T}{h}\right| = \dfrac{37}{24}\dfrac{qL^2}{EI}$

ES. #2

CALCOLARE LA ROTAZIONE DELLA SEZ. D DELLA STRUTTURA ISOSTATICA SEGUENTE CON IL METODO DELLA FORZA UNITARIA



Posizioni

$|P| = qL$

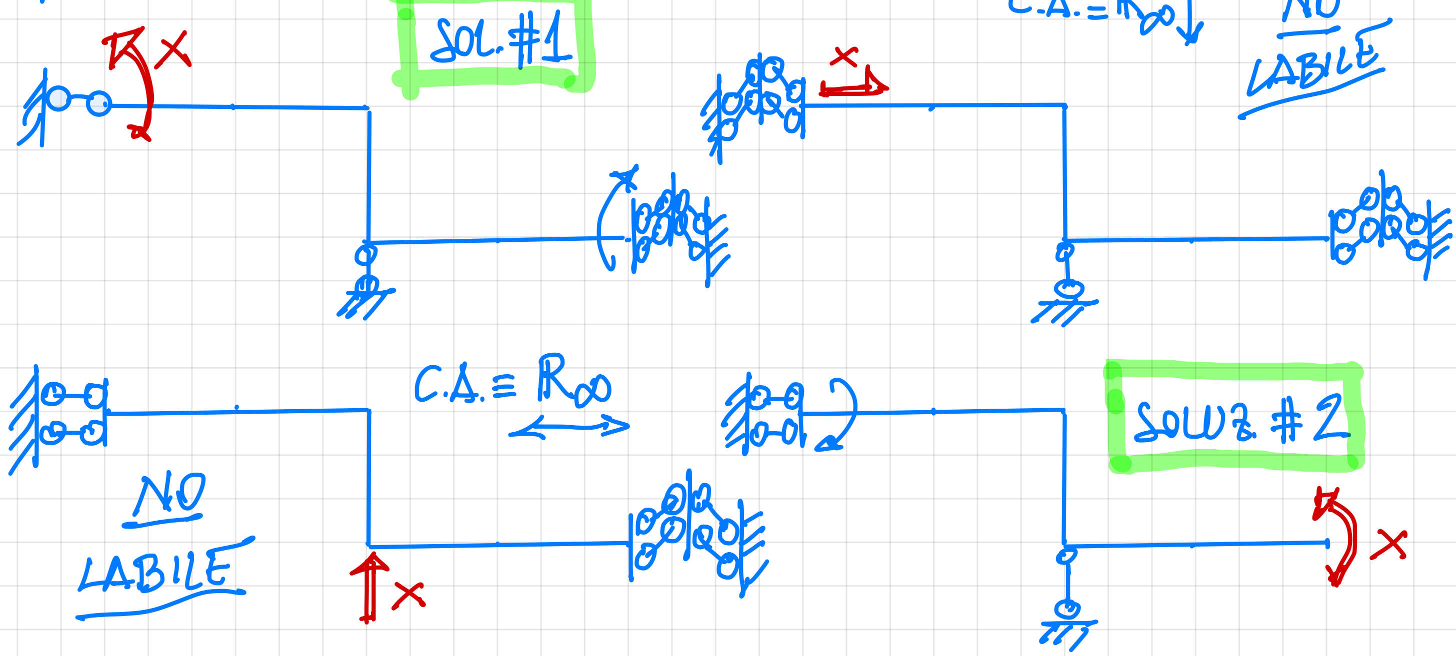
$|\varepsilon_A| = \dfrac{L^3}{EI}$

$|\varepsilon_C| = \dfrac{L^3}{12EI}$

$\left|\dfrac{\alpha \Delta T}{h}\right| = \dfrac{qL^2}{2EI}$

ES. #1 - Tipo 1 SOLUZIONI

SOLUZIONI POSSIBILI

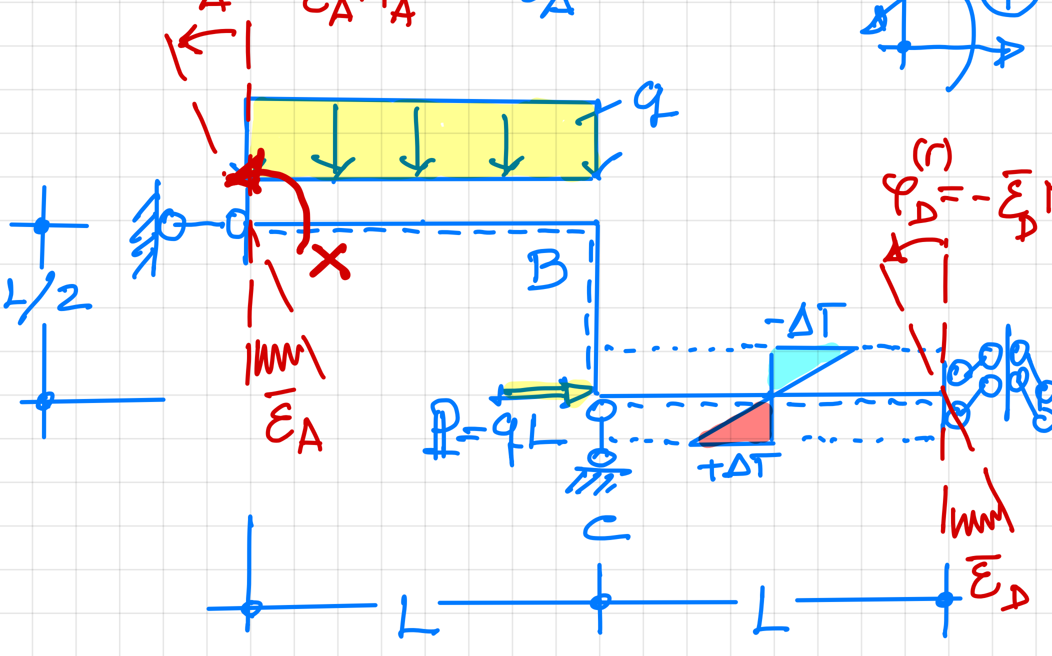


SOLUZIONE #1

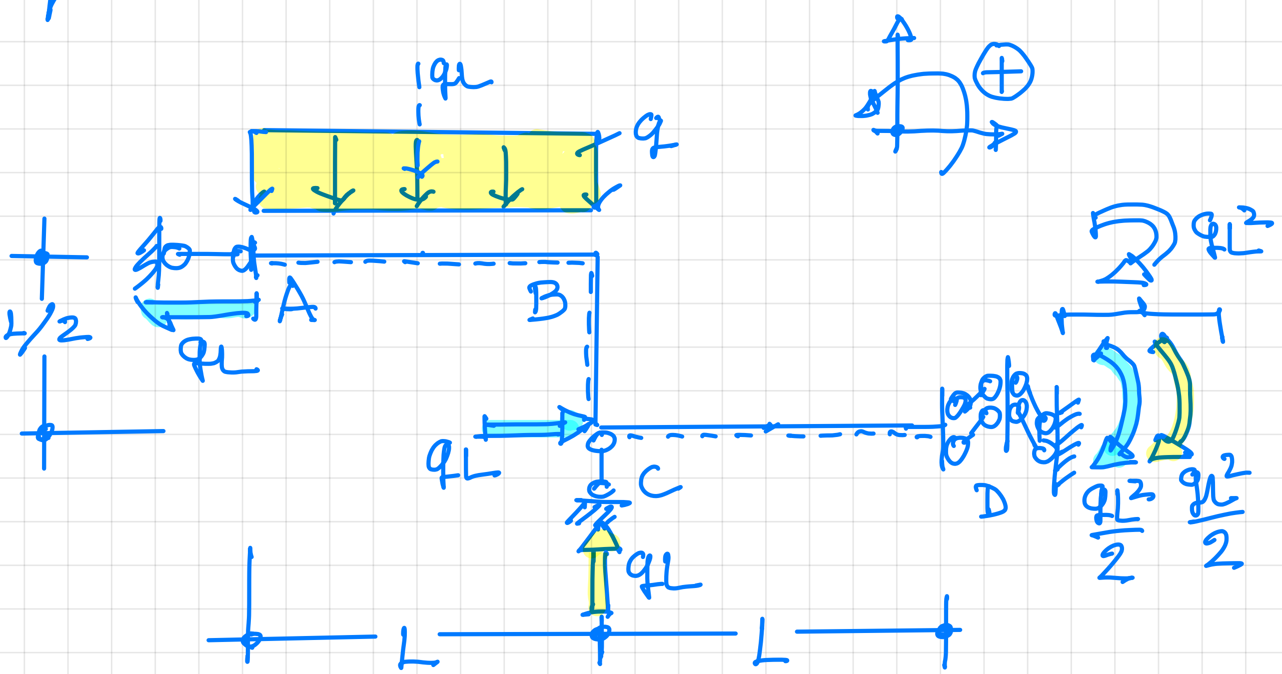
SISTEMA PRINCIPALE ISOSTATICO

$$\varphi_A^{(r)} = -\bar{\mathcal{E}}_A M_A^{(r)} = -\bar{\mathcal{E}}_A X$$

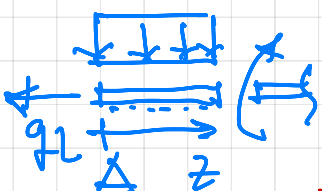
$$\varphi_D^{(r)} = -\bar{\mathcal{E}}_D M_D^{(r)} = -\bar{\mathcal{E}}_D \left[M_D^{(0)} + X M_D^{(1)} \right]$$



SCHEMA [0] – SOLO CARICHI ESTERNI



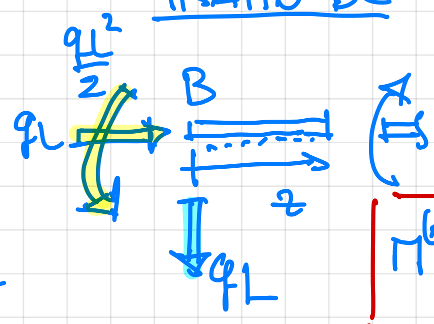
TRATTO AB $0 \le z \le L$



$$\boxed{M^{(0)}(z) = -\frac{q z^2}{2}}$$

$$\begin{cases} M_A = \phi \\ M_B = -\dfrac{qL^2}{2} \end{cases}$$

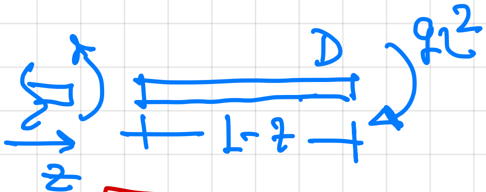
TRATTO BC $0 \le z \le \frac{L}{2}$



$$\boxed{M^{(0)}(z) = -\frac{qL^2}{2} - qL \cdot z}$$

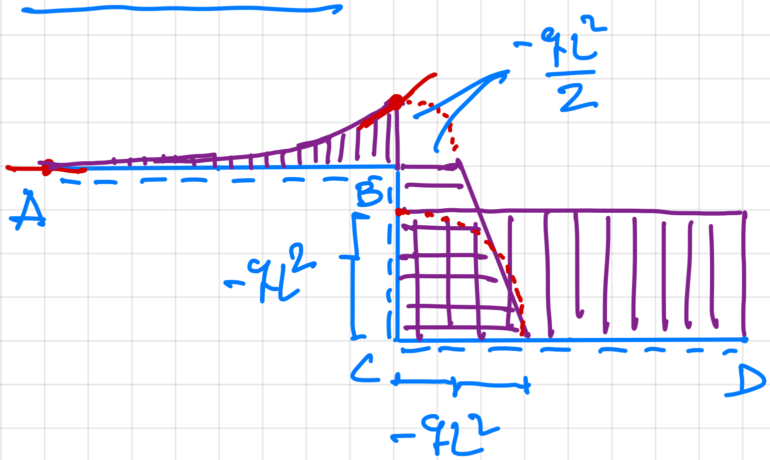
$$\begin{cases} M_B = -\dfrac{qL^2}{2} \\ M_C = -qL^2 \end{cases}$$

TRATTO CD $0 \le z \le L$

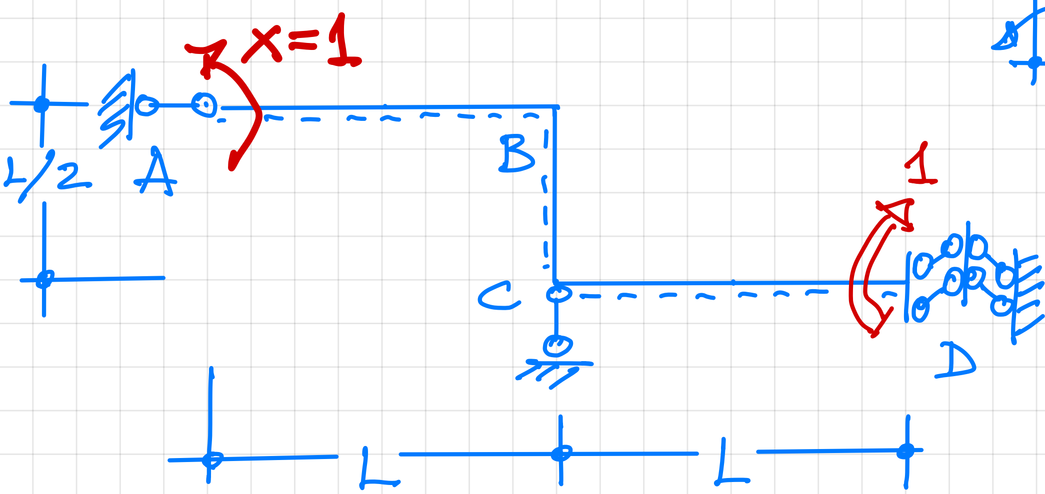


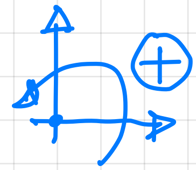
$$\boxed{M^{(0)}(z) = -qL^2 = \text{cost.}}$$

DIAGRAMMA $M^{(0)}(z)$

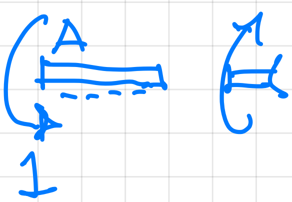


SCHEMA [1] – SOLO $X=1$



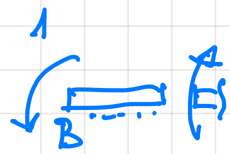


TRATTO AB $0 \le z \le L$



$\boxed{M^{(1)}(z) = -1}$ cost

TRATTO BC $0 \le z \le \frac{L}{2}$

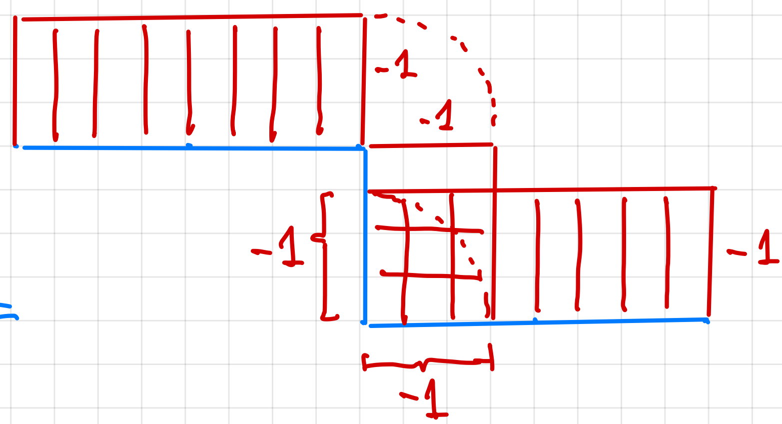


$\boxed{M^{(1)}(z) = -1}$ cost

TRATTO CD $0 \le z \le L$

$\boxed{M^{(1)}(z) = -1}$ cost.

DIAGRAMMA $M^{(1)}(z)$



$$L_{ve} = \underbrace{X_i}_{=1}\, \eta_i^{(r)} + \sum_j R_j^{(1)}\, \eta_j^{(r)} =$$

$$= 1 \cdot \varphi_A^{(r)} + \underbrace{M_D^{(1)}}_{=-1}\, \underbrace{\varphi_D^{(r)}}_{= -\bar{\varepsilon}_D\left[M_D^{(0)} + X M_D^{(1)}\right]} = -\bar{\varepsilon}_A X + \bar{\varepsilon}_D\left[-qL^2 + X(-1)\right] =$$

$$= \underbrace{-\bar{\varepsilon}_A X}\ \underbrace{- \bar{\varepsilon}_D\left[qL^2 + X\right]}$$

$$\Rightarrow L_{vi} = \int_{str} M^{(1)} \frac{\overbrace{M^{(r)}}^{M^{(0)} + X M^{(1)}}}{EI} \, dstr + \int_{str} M^{(1)} \frac{\alpha \Delta T}{h} \, dstr =$$

$$= \frac{1}{EI} \int_{str} M^{(1)} M^{(0)} \, dstr + \frac{X}{EI} \int_{str} \left[M^{(1)}\right]^2 dstr + \frac{\alpha \Delta T}{h} \int_{str} M^{(1)} \, dstr =$$

$$= \frac{1}{EI} \left\{ \int_0^L (-1)\left(-\frac{q z^2}{2}\right) dz + \int_0^{L/2} (-1)\left[-\frac{qL^2}{2} - qL \cdot z\right] dstr + \int_0^L (-1)\left[-qL^2\right] dstr \right\} +$$

$$+ \frac{X}{EI} \left\{ \int_0^L 1 \, dz + \int_0^{\frac{L}{2}} 1 \cdot dz + \int_0^L 1 \cdot dz \right\} + \frac{\alpha \Delta T}{h} \int_0^L (-1) \, dstr =$$

$$= \frac{1}{EI} \left\{ \frac{q}{2} \left[\frac{z^3}{3}\right]_0^L + \frac{qL^2}{2} \left[z\right]_0^{\frac{L}{2}} + qL \left[\frac{z^2}{2}\right]_0^{\frac{L}{2}} + qL^2 \left[z\right]_0^L \right\} +$$

$$+ \frac{X}{EI} \left\{ \left[z\right]_0^L + \left[z\right]_0^{\frac{L}{2}} + \left[z\right]_0^L \right\} - \frac{\alpha \Delta T}{h} \left[z\right]_0^L =$$

$$= \frac{1}{EI} \left\{ \frac{q}{6} \cdot L^3 + \frac{qL^2}{2} \cdot \frac{L}{2} + \frac{qL}{2} \cdot \frac{L^2}{4} + qL^2 \cdot L \right\} +$$

$$+ \frac{X}{EI} \left\{ L + \frac{L}{2} + L \right\} - \frac{\alpha \Delta T}{h} \cdot L =$$

$$= \frac{qL^3}{EI} \frac{37}{24} + \frac{XL}{EI} \cdot \frac{5}{2} - \frac{\alpha \Delta T}{h} \cdot L$$

$\Rightarrow$ $L_{ve} = L_{vi}$ fornisce

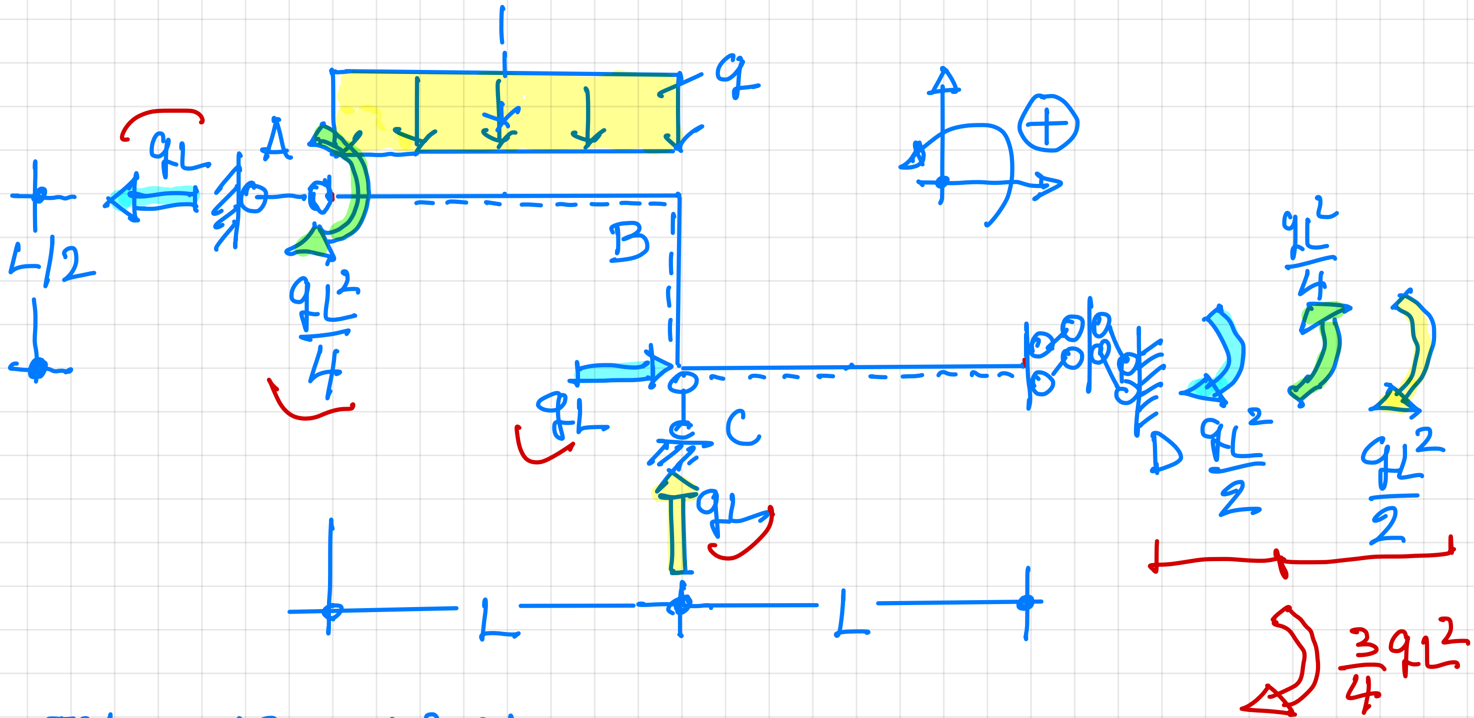
$$-\bar{\varepsilon}_A X - \bar{\varepsilon}_D \left[qL^2 + X \right] = \frac{qL^3}{EI}\frac{37}{24} + \frac{XL}{EI}\cdot\frac{5}{2} - \frac{\alpha\Delta T}{h}\cdot L$$

$$X\left[-\underbrace{\bar{\varepsilon}_A}_{\frac{L}{2EI}} - \underbrace{\bar{\varepsilon}_D}_{\frac{L}{EI}} - \frac{5}{2}\frac{L}{EI} \right] = \cancel{\frac{37}{24}\frac{qL^3}{EI}} + \underbrace{\bar{\varepsilon}_D}_{\frac{L}{EI}} qL^2 - \cancel{\underbrace{\frac{\alpha\Delta T}{h}\cdot L}_{\frac{37}{24}\frac{qL^2}{EI}}}$$

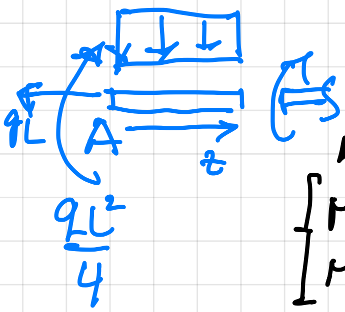
$$-X\frac{\cancel{L}}{\cancel{EI}}\underbrace{\left[\frac{1}{2} + 1 + \frac{5}{2} \right]}_{4} = \frac{qL^{\cancel{3}\,2}}{\cancel{EI}} \Rightarrow \boxed{X = -\frac{qL^2}{4}}$$

NEGATIVA! VERSO EFFETTIVO OPPOSTO A QUELLO IPOTIZZATO!

RISOLUZIONE SISTEMA PRINCIPALE ISOSTATICO & DIAGRAMMA DEL MOMENTO STRUTTURA IPERSTATICA



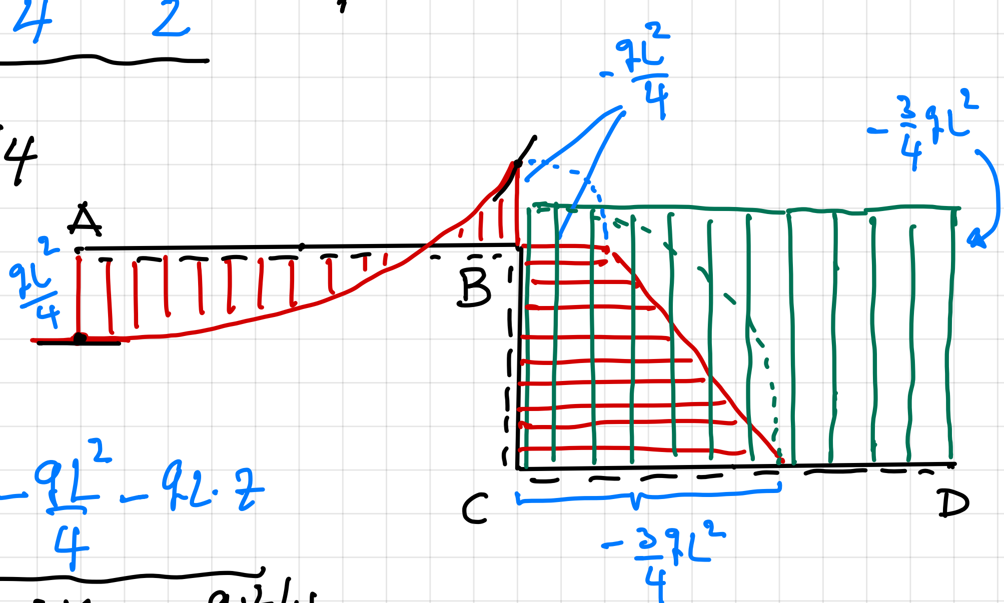
TRATTO AB $0 \le z \le L$



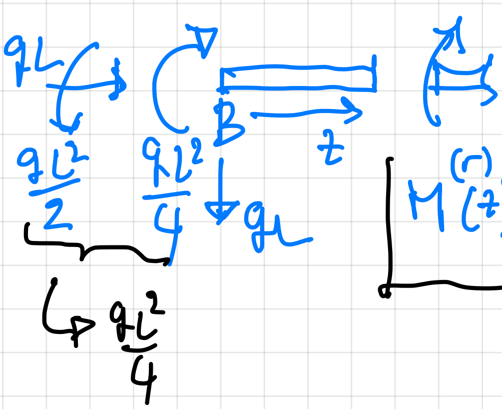
$$M^{(r)}(z) = \frac{qL^2}{4} - \frac{q \cdot z^2}{2}$$

$$\begin{cases} M_A = qL^2/4 \\ M_B = -qL^2/4 \end{cases}$$

DIAGRAMMA MOMENTO STRUTTURA IPERSTATICA



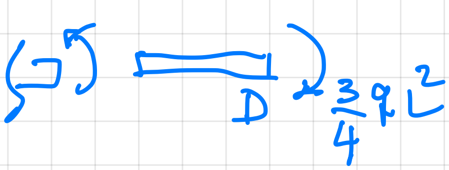
TRATTO BC $0 \le z \le \frac{L}{2}$



$$M^{(r)}(z) = -\frac{qL^2}{4} - qL \cdot z$$

$$\begin{cases} M_B = -qL^2/4 \\ M_C = -\frac{3}{4}qL^2 \end{cases}$$

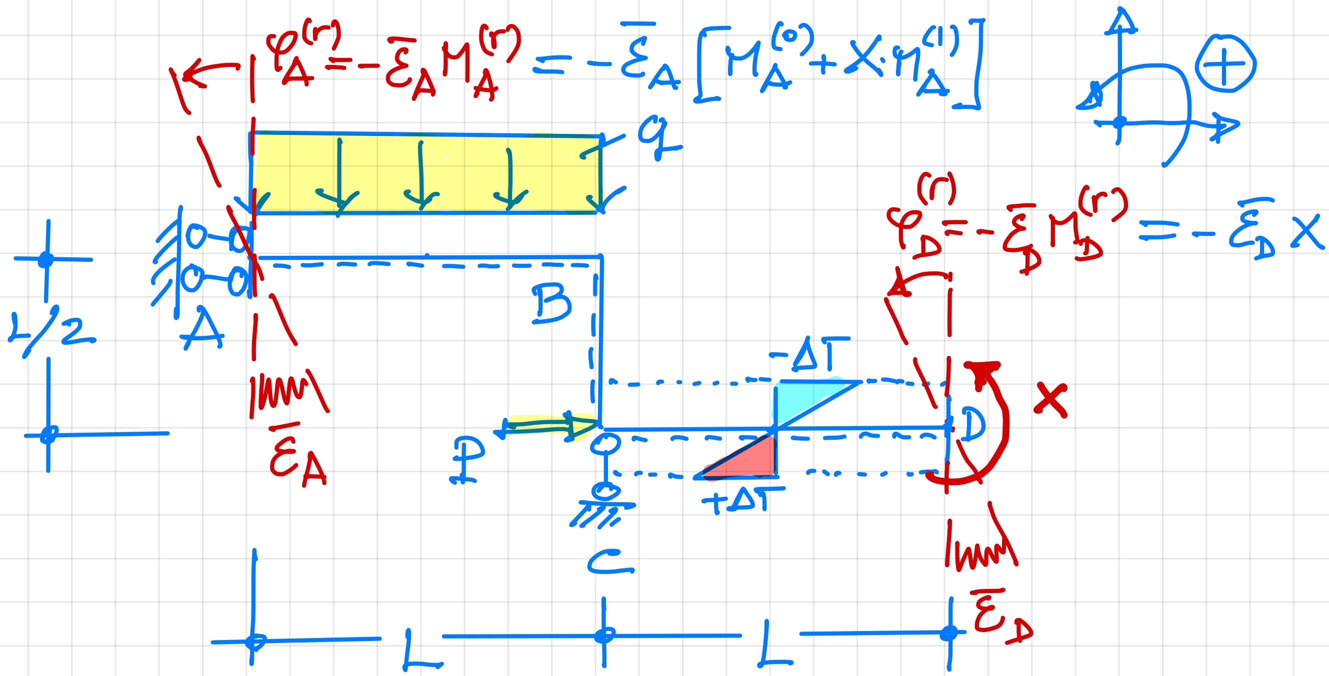
TRATTO CD $0 \le z \le L$



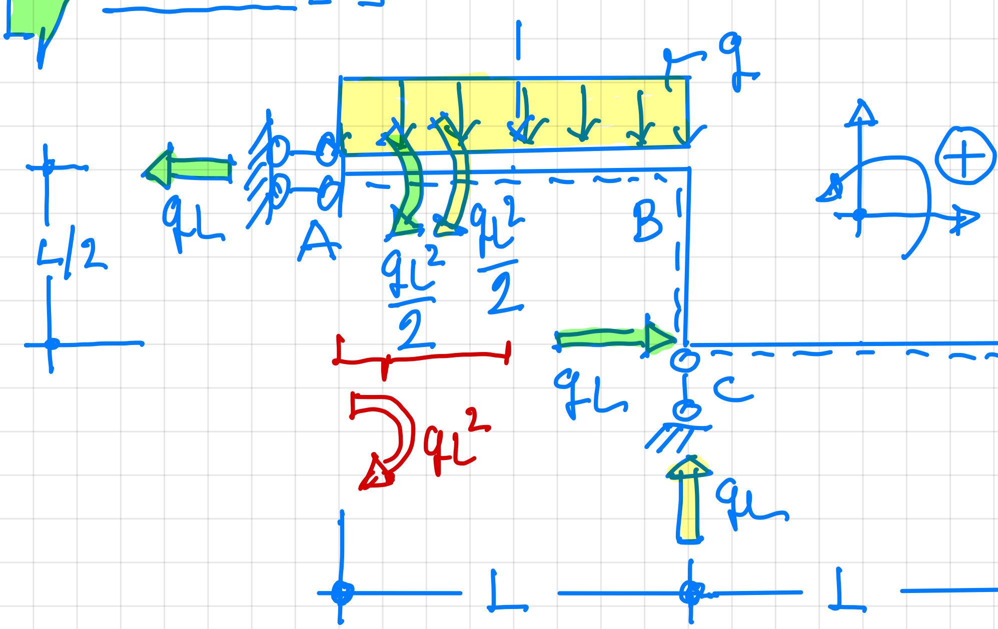
$$M^{(r)}(z) = -\frac{3}{4}qL^2$$

SOLUZIONE #2

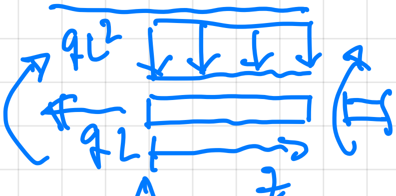
SISTEMA PRINCIPALE ISOSTATICO



SCHEMA [0] – SOLO CARICHI ESTERNI



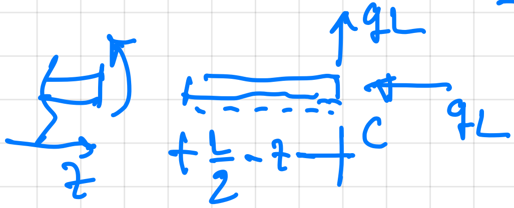
TRATTO AB $0 \le z \le L$



$$\boxed{M^{(0)}(z) = qL^2 - q\frac{z^2}{2}}$$

$$\begin{cases} M_A = qL^2 \\ M_B = \frac{qL^2}{2} \end{cases}$$

TRATTO BC $0 \le z \le \frac{L}{2}$



$$\boxed{M^{(0)}(z) = qL\left(\frac{L}{2} - z\right)}$$

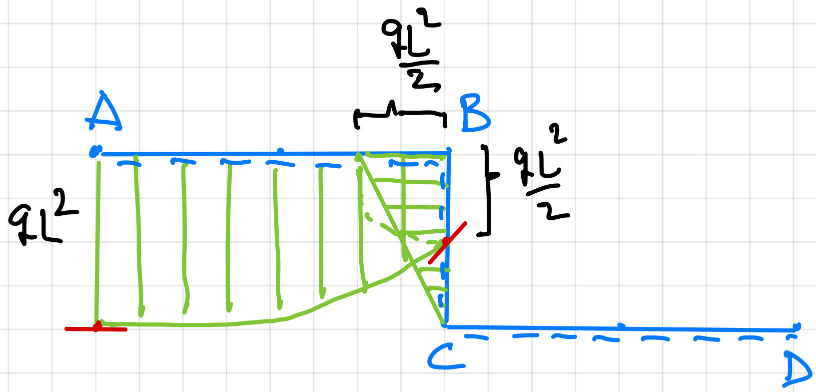
$$\hookrightarrow \begin{cases} M_B = \dfrac{qL^2}{2} \\ M_C = \phi \end{cases}$$

TRATTO CD

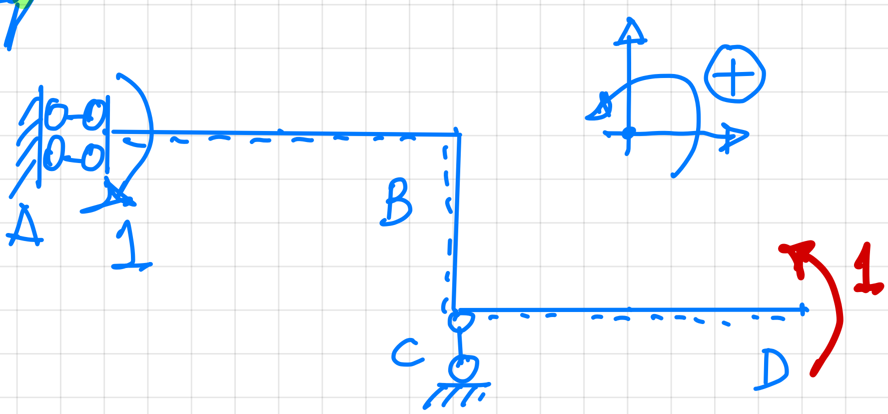
SCARICO!

$$\boxed{M^{(0)}(z) = \phi} \quad \text{cost.}$$

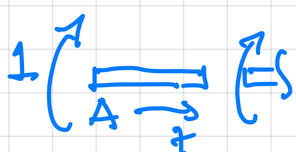
DIAGRAMMA $M^{(0)}(z)$



SCHEMA [1] - Solo $X = 1$



TRATTO AB $0 \le z \le L$



$$\boxed{M^{(1)}(z) = 1} \quad \text{cost.}$$

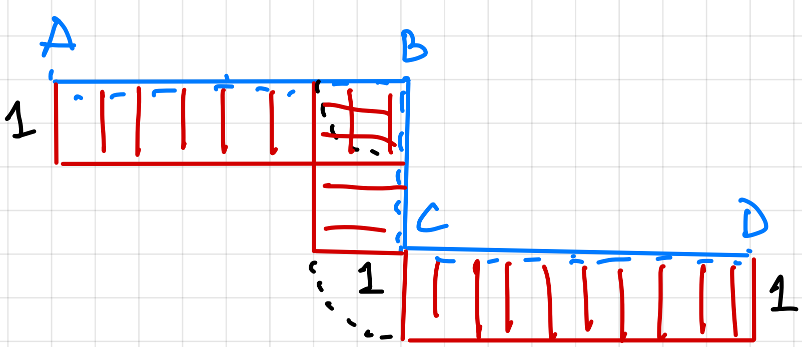
TRATTO BC $0 \le z \le \frac{L}{2}$

$$\boxed{M^{(1)}(z) = 1} \quad \text{cost}$$

TRATTO CD $0 \le z \le L$

$$\boxed{M^{(1)}(z) = 1} \quad \text{cost}$$

DIAGRAMMA $M^{(1)}(z)$



$$L_{ve} = \underset{\overset{\shortparallel}{1}}{X_i}\, \eta_i^{(r)} + \sum_j R_j^{(1)} \eta_j^{(r)} = 1 \cdot \underbrace{\varphi_D^{(r)}}_{-\bar{\varepsilon}_D \cdot X} + \underbrace{M_A^{(1)}}_{-1} \underbrace{\varphi_A^{(r)}}_{-\bar{\varepsilon}_A \left[\underbrace{M_A^{(0)}}_{-qL^2} + X \underbrace{M_A^{(1)}}_{-1} \right]} =$$

$$= \underbrace{-\bar{\varepsilon}_D \cdot X - \bar{\varepsilon}_A \left[qL^2 + X \right]}$$

$$L_{vi} = \int_{str} M^{(1)} \frac{\overbrace{M^{(r)}}^{M^{(0)} + X M^{(1)}}}{EI} \, ds_{tr} + \int_{str} M^{(1)} \frac{\alpha \Delta T}{h} \, ds_{tr} =$$

$$= \frac{1}{EI} \int_{str} M^{(1)} M^{(0)} \, ds_{tr} + \frac{X}{EI} \int_{str} \left[M^{(1)} \right]^2 ds_{tr} + \frac{\alpha \Delta T}{h} \int_{str} M^{(1)} \, ds_{tr} =$$

$$= \frac{1}{EI} \left[\int_0^L \left(qL^2 - \frac{qz^2}{2} \right) dz + \int_0^{\frac{L}{2}} qL \left(\frac{L}{2} - z \right) dz \right] +$$

$$+ \frac{X}{EI} \left\{ \int_0^L dz + \int_0^{\frac{L}{2}} dz + \int_0^L dz \right\} + \frac{\alpha \Delta T}{h} \int_0^L 1 \cdot dz =$$

$$= \frac{1}{EI} \left\{ qL^2 \left[z \right]_0^L - \frac{q}{2} \left[\frac{z^3}{3} \right]_0^L + \frac{qL^2}{2} \left[z \right]_0^{\frac{L}{2}} - qL \left[\frac{z^2}{2} \right]_0^{\frac{L}{2}} \right\} +$$

$$+ \frac{X}{EI} \left\{ \left[z \right]_0^L + \left[z \right]_0^{\frac{L}{2}} + \left[z \right]_0^L \right\} + \frac{\alpha \Delta T}{h} \left[z \right]_0^L =$$

$$= \frac{1}{EI} \left\{ qL^2 \cdot L - \frac{q}{6} L^3 + \frac{qL^2}{2} \cdot \frac{L}{2} - \frac{qL}{2} \cdot \frac{L^2}{4} \right\} +$$

$$+ \frac{X}{EI} \left\{ L + \frac{L}{2} + L \right\} + \frac{\alpha \Delta T}{h} \cdot L =$$

$$= \underbrace{\frac{qL^3}{EI} \cdot \frac{23}{24} + \frac{XL}{EI} \cdot \frac{5}{2} + \frac{\alpha \Delta T}{h} \cdot L}$$

$L_{ve} = L_{vi}$ fornisce

$$-\bar{\varepsilon}_D \cdot X - \bar{\varepsilon}_A \left[qL^2 + X\right] = \frac{qL^3}{EI} \cdot \frac{23}{24} + \frac{XL}{EI} \cdot \frac{5}{2} + \frac{\alpha \Delta T}{h} \cdot L$$

$$-X \left\{ \underbrace{\bar{\varepsilon}_D}_{\frac{L}{EI}} + \underbrace{\bar{\varepsilon}_A}_{\frac{L}{2EI}} + \frac{5}{2}\frac{L}{EI} \right\} = \frac{qL^3}{EI} \cdot \frac{23}{24} + \underbrace{\frac{\alpha \Delta T}{h}}_{= \frac{37}{24}\frac{qL^2}{EI}} \cdot L + \underbrace{\bar{\varepsilon}_A}_{= \frac{L}{2EI}} qL^2$$

$$-X \cancel{\frac{L}{EI}} \underbrace{\left\{1 + \frac{1}{2} + \frac{5}{2}\right\}}_{4} = \frac{qL^{\cancel{3}\,2}}{\cancel{EI}} \underbrace{\left\{\frac{23}{24} + \frac{37}{24} + \frac{1}{2}\right\}}_{\frac{72}{24}}$$

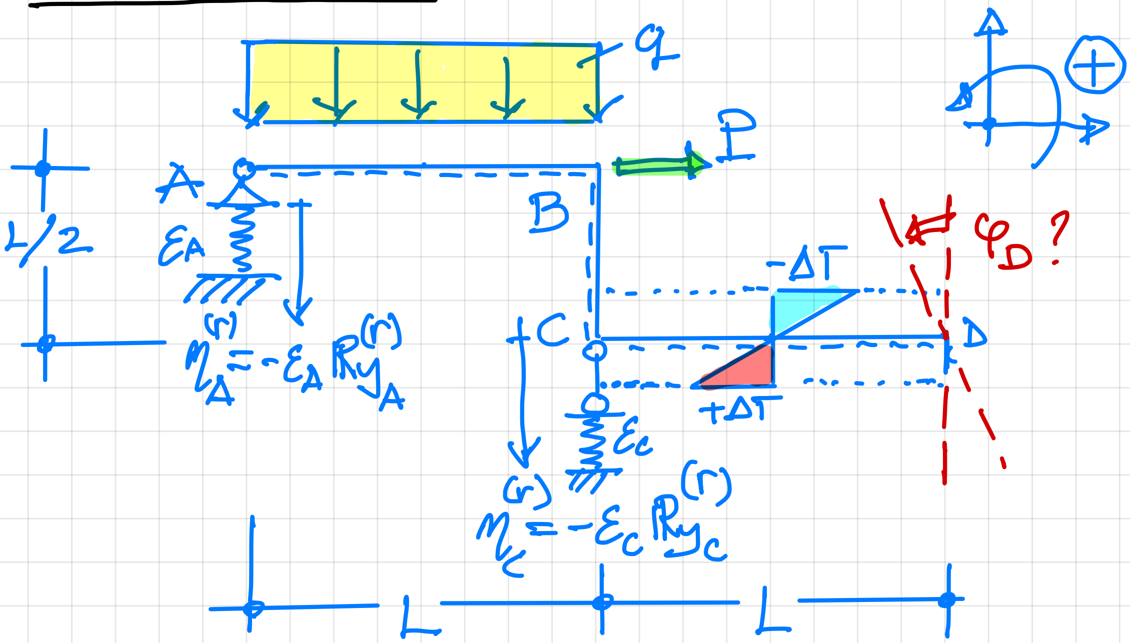
$$X = -\frac{72}{96} qL^2 = \boxed{-\frac{3}{4} qL^2}$$

NEGATIVA! VERSO OPPOSTO A QUELLO IPOTIZZATO!

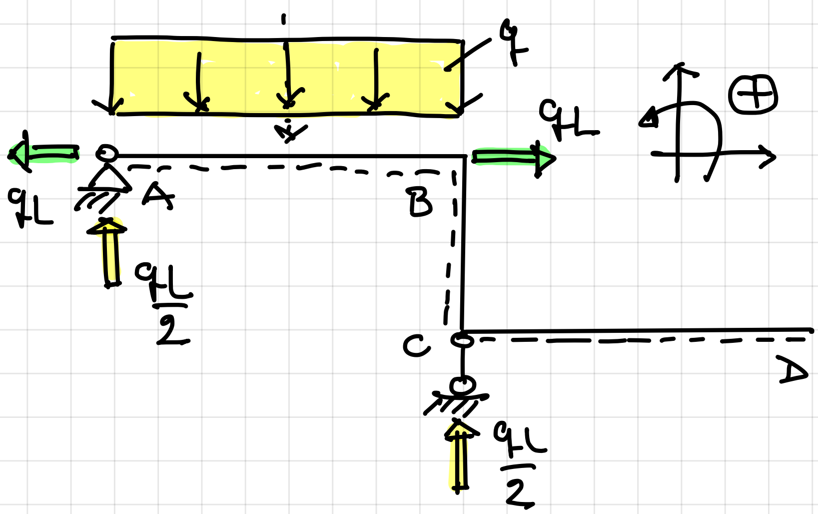
VALORE CORRETTO! cfr. RV di p. VI

ES#2 tipo 1 SOLUZIONE

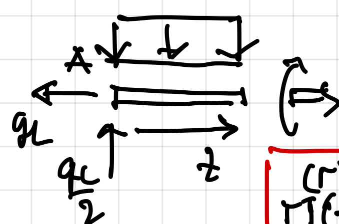
STRUTTURA REALE



CALCOLO RV e $M^{(r)}(z)$ NELLA STRUTTURA REALE



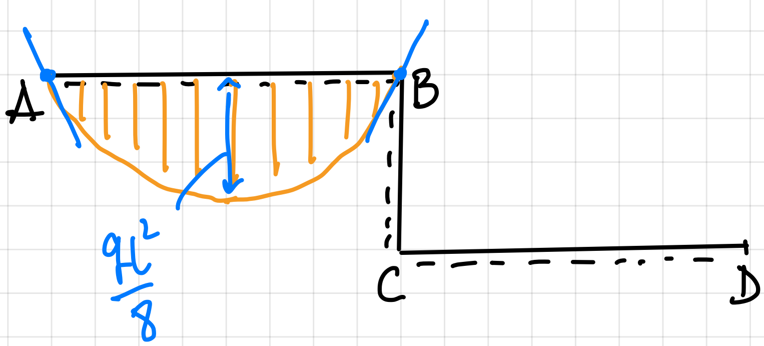
TRATTO AB $0 \le z \le L$



$$\boxed{M^{(r)}(z) = \frac{qL}{2} \cdot z - \frac{q \cdot z^2}{2}}$$

$$\begin{cases} M_A = 0 \\ M_B = 0 \\ M|_{\frac{L}{2}} = \frac{qL^2}{8} \end{cases}$$

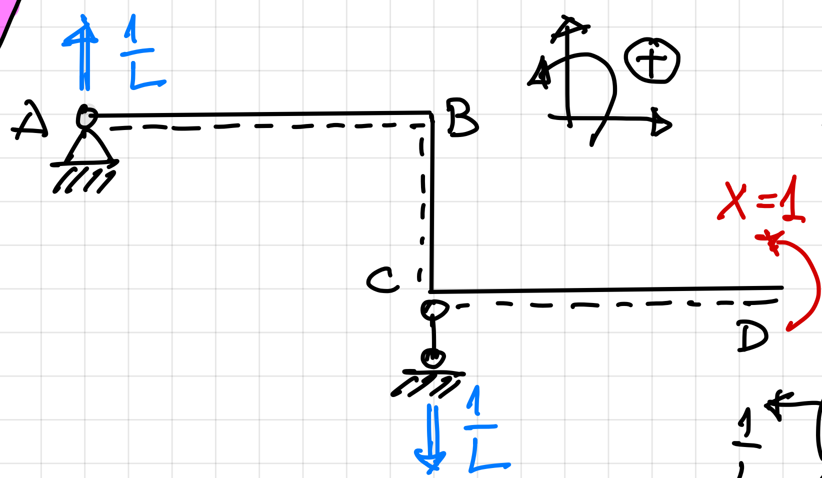
DIAGRAMMA $M^{(r)}(z)$



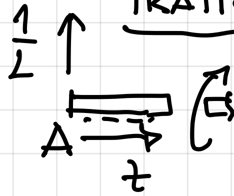
TRATTI BC e CD

$$\boxed{M^{(r)}(z) = 0}$$

STRUTTURA FITTIZIA PER IL CALCOLO DI φ_D

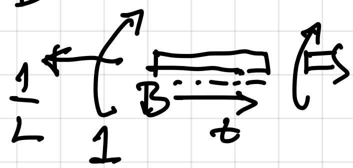


TRATTO AB $0 \le z \le L$



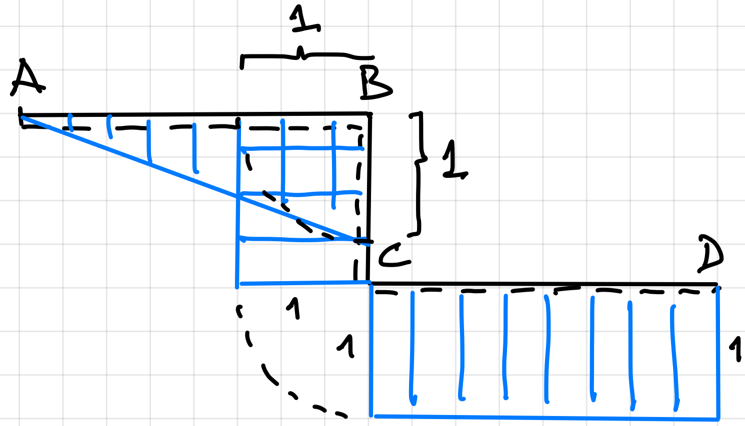
$$\boxed{M^{(f)}(z) = \frac{z}{L}} \qquad \begin{cases} M_A = \phi \\ M_B = 1 \end{cases}$$

TRATTO BC $0 \le z \le \frac{L}{2}$

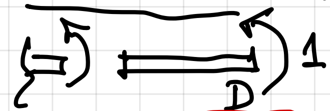


$$\boxed{M^{(f)}(z) = 1} \quad \text{cost.}$$

DIAGRAMMA $M^{(f)}(z)$



TRATTO CD $0 \le z \le L$



$$\boxed{M^{(f)}(z) = 1} \quad \text{cost.}$$

$$L_{ve} = 1 \cdot \eta_i^{(r)} + \sum_j R_j^{(f)} \eta_j^{(r)} = 1 \cdot \varphi_D^{(r)} + R_{y_A}^{(f)} \eta_A^{(r)} + R_{y_C}^{(f)} \eta_C^{(r)} =$$

$$= \varphi_D^{(r)} + \underbrace{R_{y_A}^{(f)}}_{\frac{1}{L}} (-\varepsilon_A) \underbrace{R_{y_A}^{(r)}}_{\frac{qL}{2}} + \underbrace{R_{y_C}^{(f)}}_{-\frac{1}{L}} (-\varepsilon_C) \underbrace{R_{y_C}^{(r)}}_{\frac{qL}{2}}$$

$$= \varphi_D^{(r)} - \frac{\varepsilon_A}{L}\left[\frac{qL}{2}\right] + \frac{\varepsilon_C}{L}\left[\frac{qL}{2}\right]$$

$$L_{vi} = \int_{str} M^{(f)} \frac{M^{(r)}}{EI} \, ds_{tr} + \int_{str} M^{(f)} \frac{\alpha \Delta T}{h} \, ds_{tr} =$$

$$= \frac{1}{EI}\left[\int_0^L \frac{z}{L}\left(\frac{qL}{2}\cdot z - \frac{qz^2}{2}\right)dz\right] + \frac{\alpha\Delta T}{h}\int_0^L dz =$$

$$= \frac{1}{EI}\left\{\frac{q}{2}\left[\frac{z^3}{3}\right]_0^L - \frac{q}{2L}\left[\frac{z^4}{4}\right]_0^L\right\} + \frac{\alpha\Delta T}{h}\left[z\right]_0^L =$$

$$= \frac{1}{EI}\left\{\frac{q}{6}\cdot L^3 - \frac{q}{8}L^3\right\} + \frac{\alpha\Delta T}{h}\cdot L = \underbrace{\frac{qL^3}{EI}\frac{1}{24} + \frac{\alpha\Delta T}{h}\cdot L}$$

$L_{ve} = L_{vi}$ fornisce :

$$\varphi_D^{(r)} - \frac{\varepsilon_A}{\cancel{L}}\left[\frac{q\cancel{L}}{2}\right] + \frac{\varepsilon_C}{\cancel{L}}\left[\frac{q\cancel{L}}{2}\right] = \frac{qL^3}{EI}\frac{1}{24} + \frac{\alpha\Delta T}{h}\cdot L$$

$$\underline{\varphi_D^{(r)}} = \frac{q}{2}\underbrace{\varepsilon_A}_{\frac{L^3}{EI}} - \cancel{\frac{q}{2}\underbrace{\varepsilon_C}_{\frac{L^3}{12EI}} + \frac{qL^3}{24EI}} + \underbrace{\frac{\alpha\Delta T}{h}}_{\frac{qL^2}{2EI}}\cdot L = \underline{\frac{qL^3}{EI}}$$

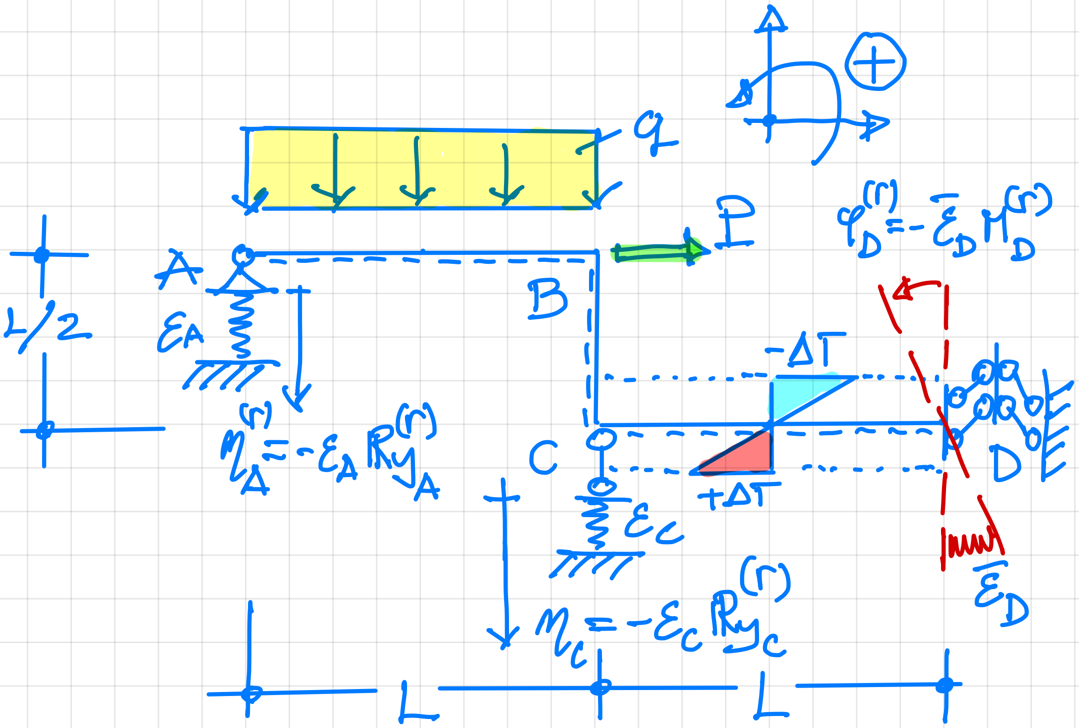
POSITIVA! ANTIORARIA

MECCANICA delle STRUTTURE – P. FUSCHI

PROVA SCRITTA del 9 APRILE 2025

TIPO 2

ES. #1 RISOLVERE LA STRUTTURA UNA VOLTA IPERSTATICA SEGUENTE DETERMINANDO IL DIAGRAMMA DEI MOMENTI



Posizioni

$|P| = qL$

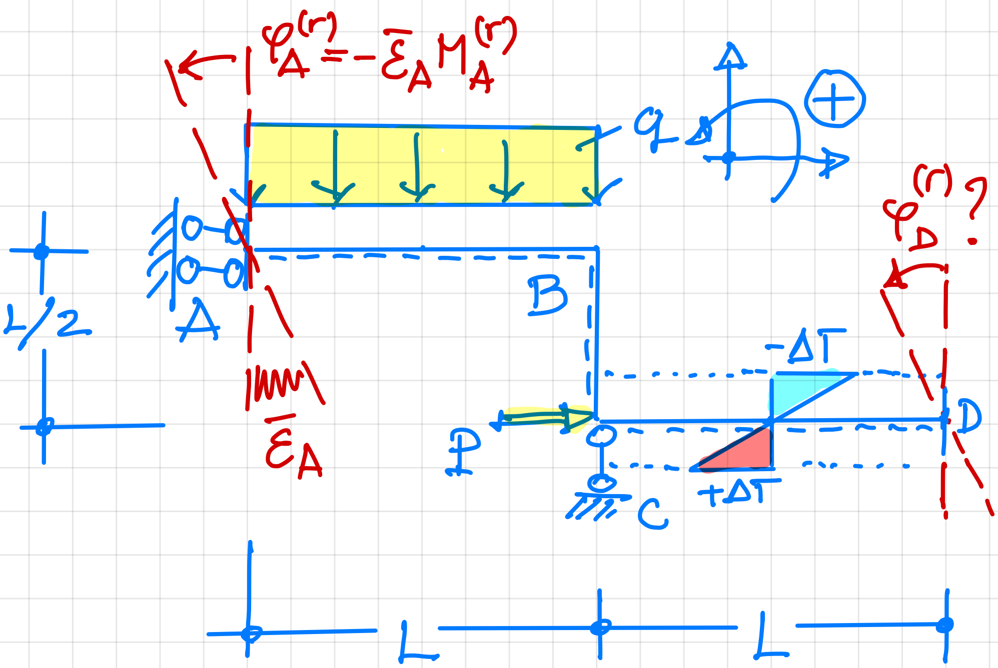
$|\varepsilon_A| = \frac{13}{12}\frac{L^3}{EI}$

$|\varepsilon_C| = \frac{11}{12}\frac{L^3}{EI}$

$|\bar{\varepsilon}_D| = \frac{L}{6EI}$

$\left|\frac{\alpha \Delta T}{h}\right| = \frac{7}{8}\frac{qL^2}{EI}$

ES. #2 CALCOLARE LA ROTAZIONE DELLA SEZ. D DELLA STRUTTURA ISOSTATICA SEGUENTE CON IL METODO DELLA FORZA UNITARIA



Posizioni

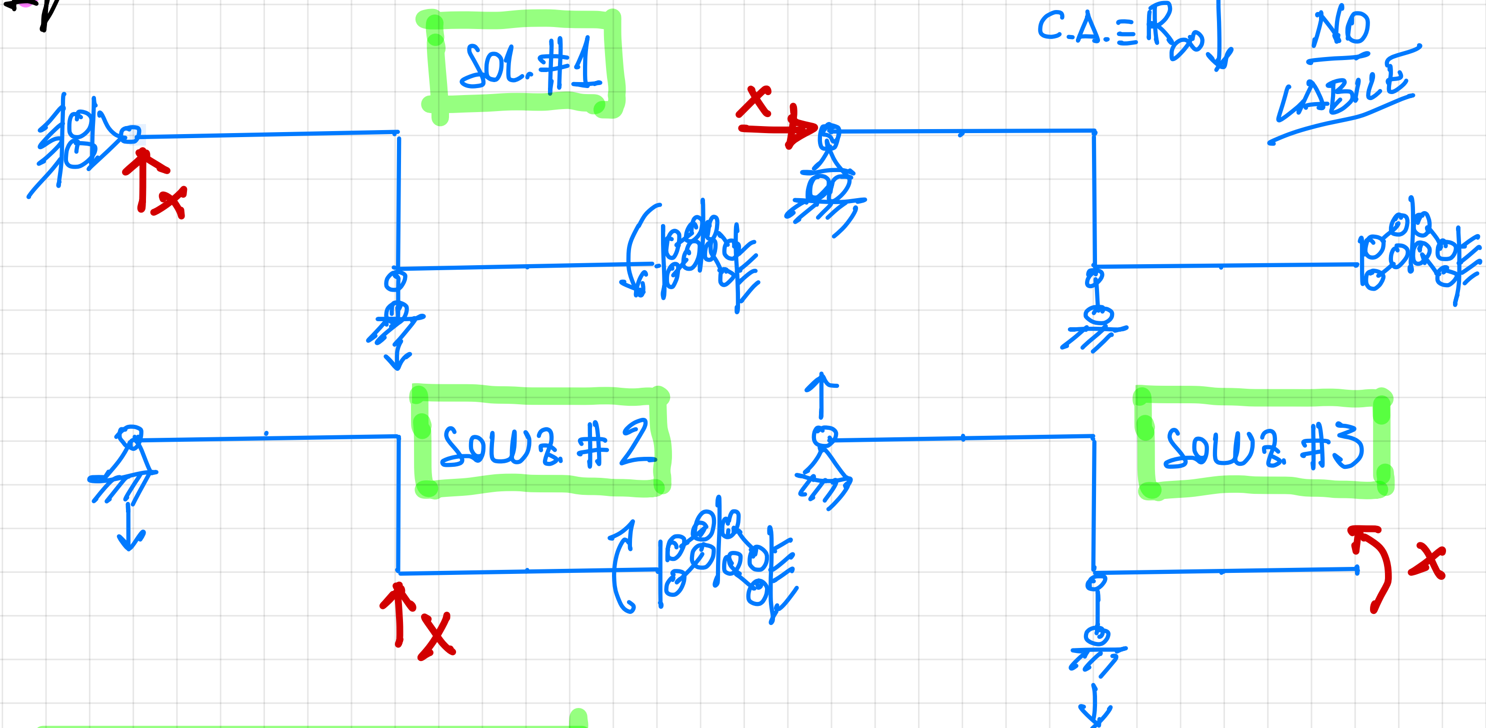
$|P| = qL$

$|\bar{\varepsilon}_A| = \frac{L}{EI}$

$\left|\frac{\alpha \Delta T}{h}\right| = \frac{qL^2}{24EI}$

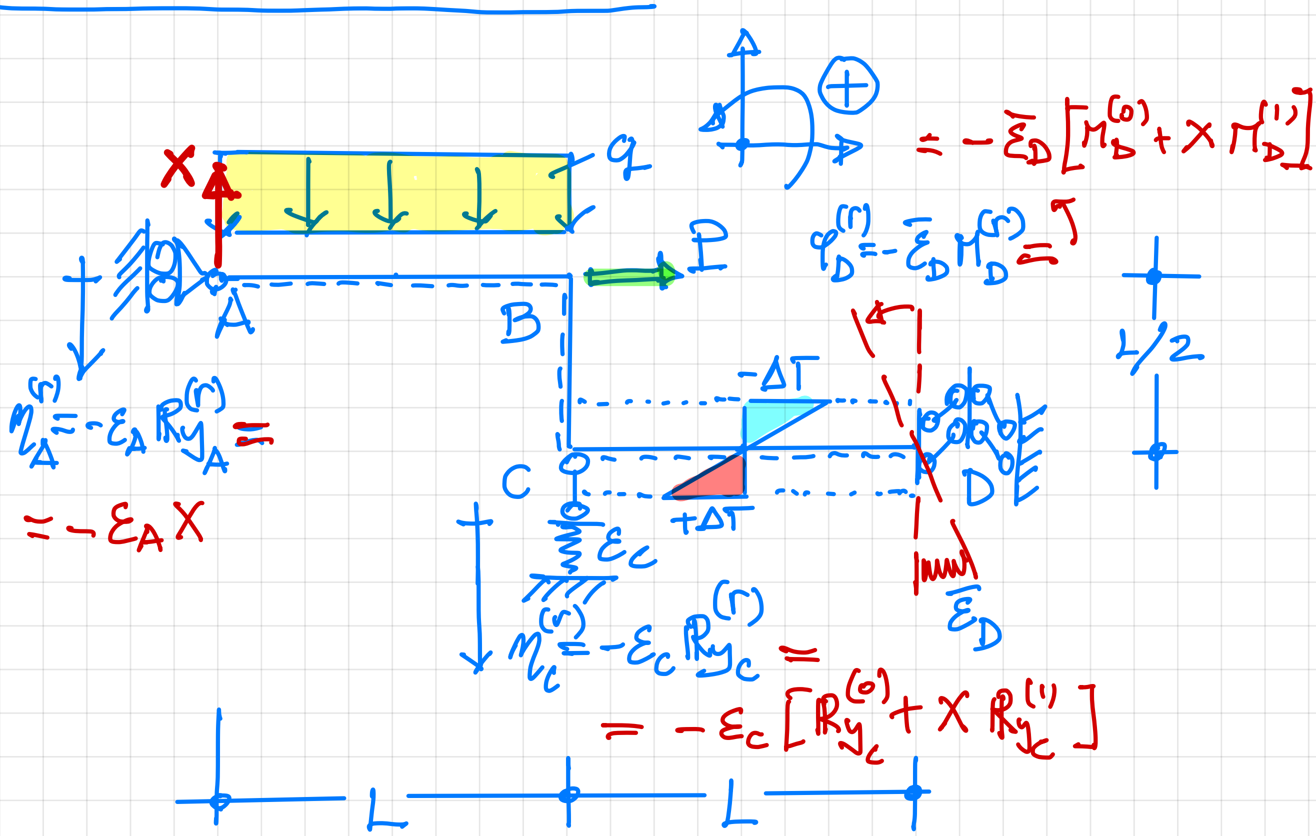
ES. #1 - Tipo 2 SOLUZIONI

SOLUZIONI POSSIBILI

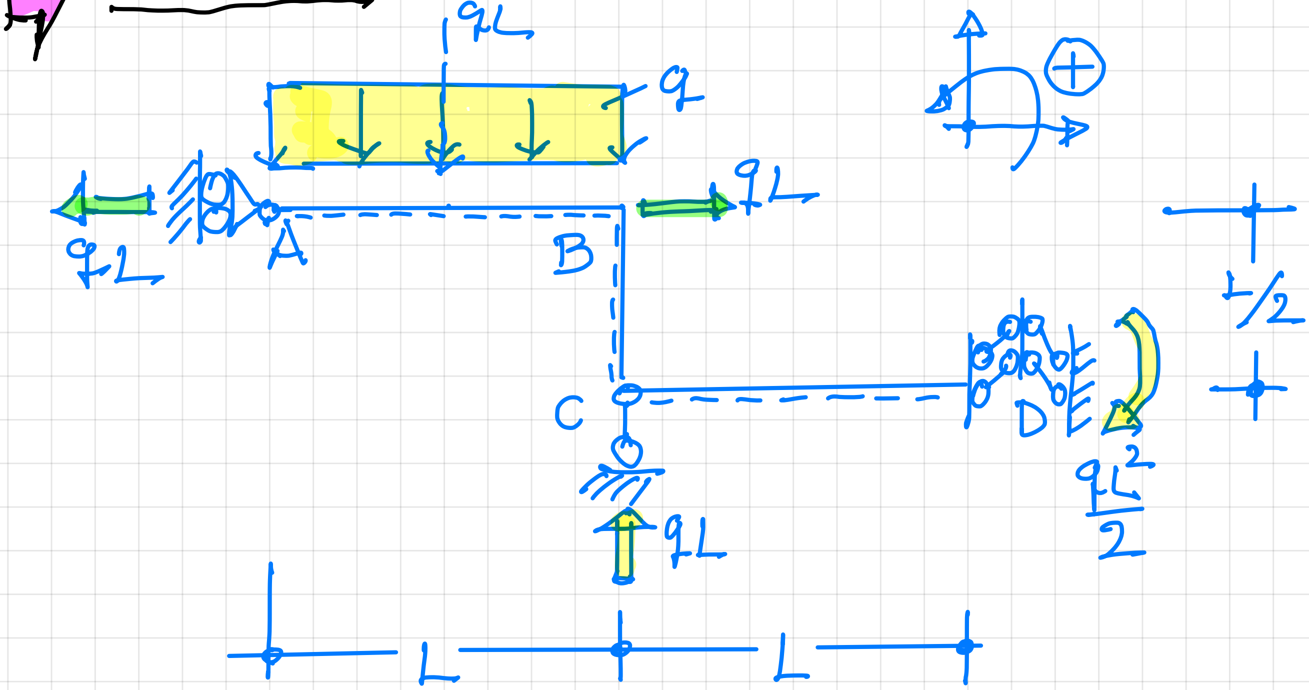


SOLUZIONE #1

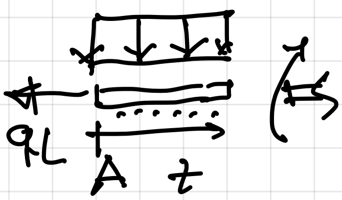
SISTEMA PRINCIPALE ISOSTATICO



SCHEMA [0] – SOLO CARICHI ESTERNI



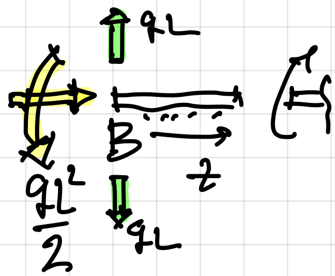
TRATTO AB $0 \le z \le L$



$$\boxed{M^{(0)}(z) = -\frac{q z^2}{2}}$$

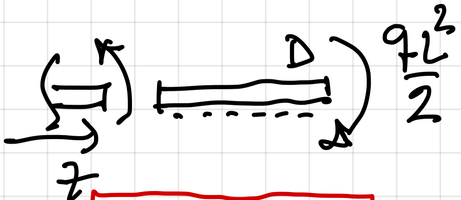
$$\begin{cases} M_A = 0 \\ M_B = -\dfrac{qL^2}{2} \end{cases}$$

TRATTO BC $0 \le z \le \frac{L}{2}$



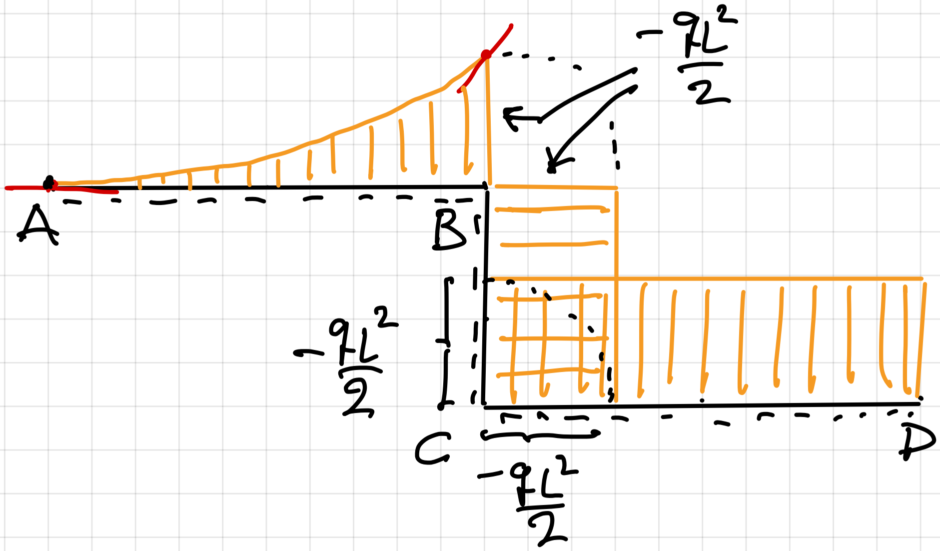
$$\boxed{M^{(0)}(z) = -\frac{qL^2}{2}} \quad \text{cost.}$$

TRATTO CD $0 \le z \le L$

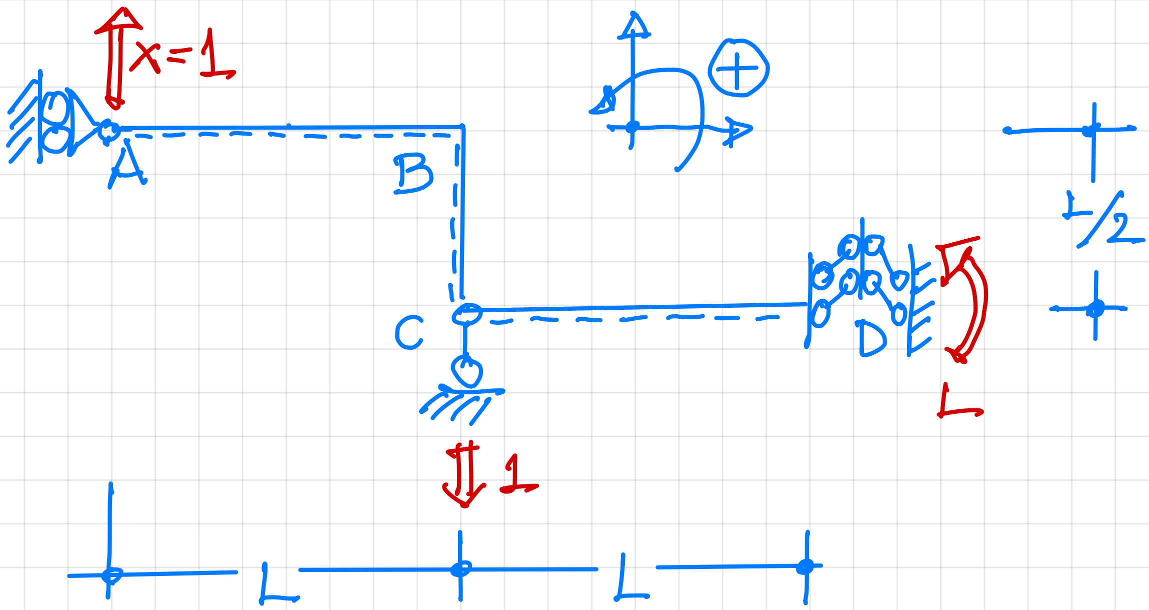


$$\boxed{M^{(0)}(z) = -\frac{qL^2}{2}} \quad \text{cost.}$$

DIAGRAMMA $M^{(0)}(z)$



SCHEMA [1] – Solo $X=1$



TRATTO AB $0 \le z \le L$

$$\boxed{M^{(1)}(z) = z} \qquad \begin{cases} M_A = \phi \\ M_B = L \end{cases}$$

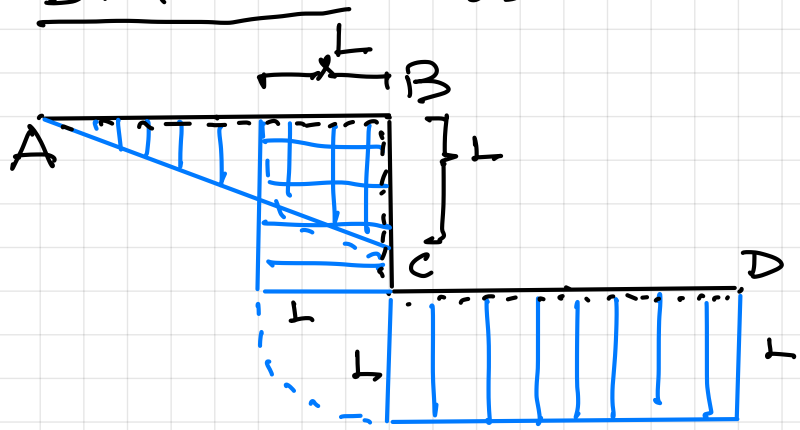
TRATTO BC $0 \le z \le \frac{L}{2}$

$$\boxed{M^{(1)}(z) = L} \quad \text{cost.}$$

TRATTO CD $0 \le z \le L$

$$\boxed{M^{(1)}(z) = L} \quad \text{cost.}$$

DIAGRAMMA $M^{(1)}(z)$



$$L_{ve} = \underset{\overset{\shortparallel}{1}}{X_i}\, \eta_i^{(r)} + \sum_j R_j^{(1)} \eta_j^{(r)} =$$

$$= 1 \cdot \underbrace{\eta_A^{(r)}}_{-\varepsilon_A X} + \underbrace{R_{y_c}^{(1)}}_{-1} \cdot \underbrace{\eta_c^{(r)}}_{-\varepsilon_c \left[\underbrace{R_{y_c}^{(0)}}_{qL} + X \underbrace{R_{y_c}^{(1)}}_{-1} \right]} + \underbrace{M_D^{(1)}}_{L} \underbrace{\varphi_D^{(r)}}_{-\varepsilon_D \left[\underbrace{M_D^{(0)}}_{-\frac{qL^2}{2}} + X \underbrace{M_D^{(1)}}_{L} \right]} =$$

$$= -\mathcal{E}_A X + \mathcal{E}_C \left[qL - X \right] - \mathcal{E}_D L \left[-\frac{qL^2}{2} + XL \right]$$

$$L_{vi} = \int_{str} M^{(1)} \frac{M^{(r)}}{EI} dstr + \int_{str} M^{(1)} \frac{\alpha \overline{\Delta T}}{h} dstr =$$

$M^{(r)} = M^{(0)} + X M^{(1)}$

$$= \frac{1}{EI} \int_{str} M^{(1)} M^{(0)} dstr + \frac{X}{EI} \int_{str} \left[M^{(1)} \right]^2 dstr + \frac{\alpha \overline{\Delta T}}{h} \int_{str} M^{(1)} dstr =$$

$$= \frac{1}{EI} \left\{ \int_0^L z \left(-\frac{qz^2}{2} \right) dz + \int_0^{L/2} L \left[-\frac{qL^2}{2} \right] dstr + \int_0^L L \left[-\frac{qL^2}{2} \right] dstr \right\} +$$

$$+ \frac{X}{EI} \left\{ \int_0^L z^2 dz + \int_0^{L/2} L^2 dz + \int_0^L L^2 \right\} + \frac{\alpha \overline{\Delta T}}{h} \int_0^L L\, dz =$$

$$= \frac{1}{EI} \left\{ -\frac{q}{2} \left[\frac{z^4}{4} \right]_0^L - \frac{qL^3}{2} \left[z \right]_0^{L/2} - \frac{qL^3}{2} \left[z \right]_0^L \right\} +$$

$$+ \frac{X}{EI} \left\{ \left[\frac{z^3}{3} \right]_0^L + L^2 \left[z \right]_0^{L/2} + L^2 \left[z \right]_0^L \right\} + \frac{\alpha \overline{\Delta T}}{h} \cdot L \left[z \right]_0^L =$$

$$= \frac{1}{EI} \left\{ -\frac{q}{8} L^4 - \frac{qL^3}{2} \cdot \frac{L}{2} - \frac{qL^3}{2} \cdot L \right\} +$$

$$+ \frac{X}{EI} \left\{ \frac{L^3}{3} + \frac{L^3}{2} + L^3 \right\} + \frac{\alpha \overline{\Delta T}}{h} \cdot L^2 =$$

$$= -\frac{qL^4}{EI} \cdot \frac{7}{8} + \frac{11}{6} \frac{XL^3}{EI} + \frac{\alpha \overline{\Delta T}}{h} L^2$$

$\Rightarrow$ $L_{ve} = L_{vi}$ fornisce :

$$-\bar{\varepsilon}_A X + \bar{\varepsilon}_C \left[qL - X\right] - \bar{\varepsilon}_D L \left[-\frac{qL^2}{2} + XL\right] =$$

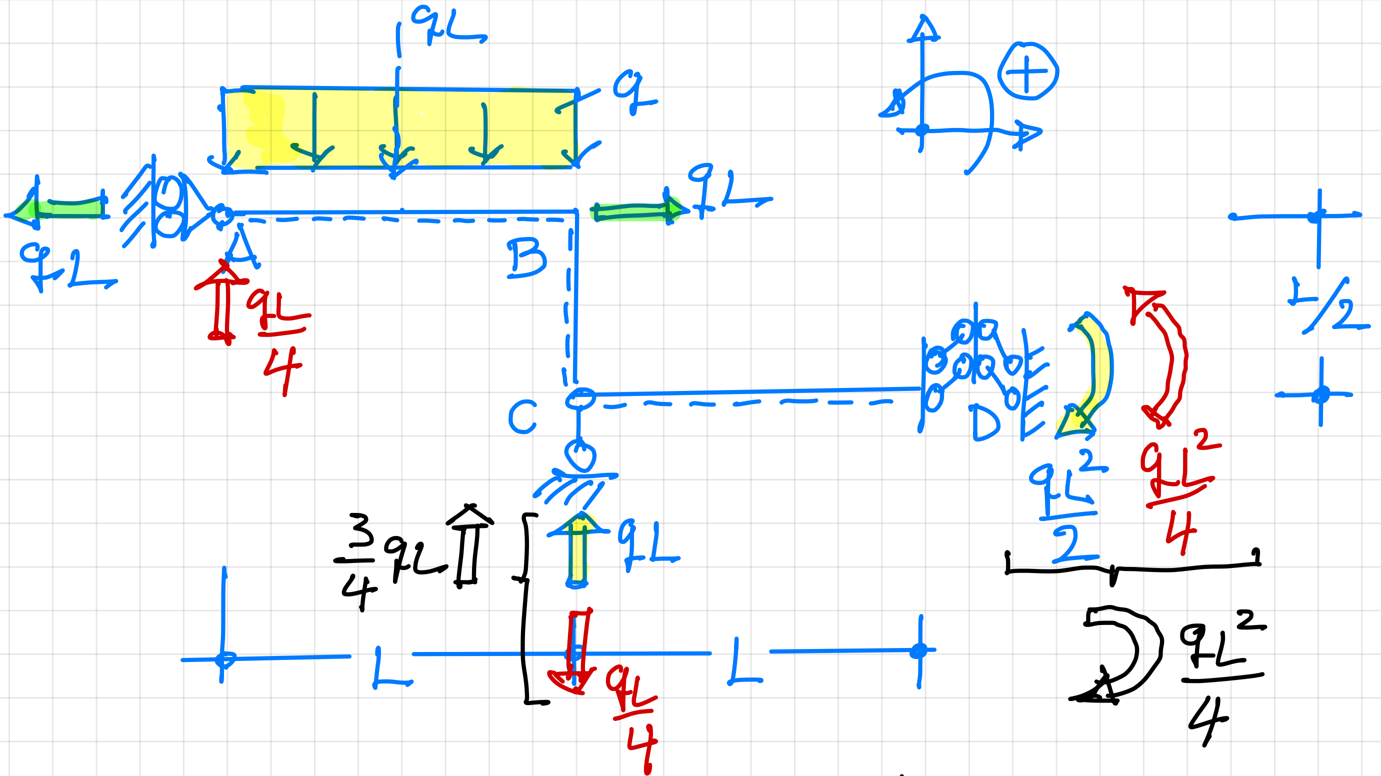
$$= -\frac{qL^4}{EI}\cdot\frac{7}{8} + \frac{11}{6}\frac{XL^3}{EI} + \frac{\alpha\Delta T}{h} L^2$$

$$X\Big[-\underbrace{\bar{\varepsilon}_A}_{\frac{13}{12}\frac{L^3}{EI}} - \underbrace{\bar{\varepsilon}_C}_{\frac{11}{12}\frac{L^3}{EI}} - \underbrace{\bar{\varepsilon}_D}_{\frac{L}{6EI}} L^2 - \frac{11}{6}\frac{L^3}{EI}\Big] = \underbrace{\cancel{-\frac{7}{8}\frac{qL^4}{EI}} + \cancel{\frac{\alpha\Delta T}{h} L^2}}_{\frac{7}{8}\frac{qL^2}{EI}} - \underbrace{\bar{\varepsilon}_C}_{\frac{11}{12}\frac{L^3}{EI}} qL - \frac{qL^3}{2}\underbrace{\bar{\varepsilon}_D}_{\frac{L}{6EI}}$$

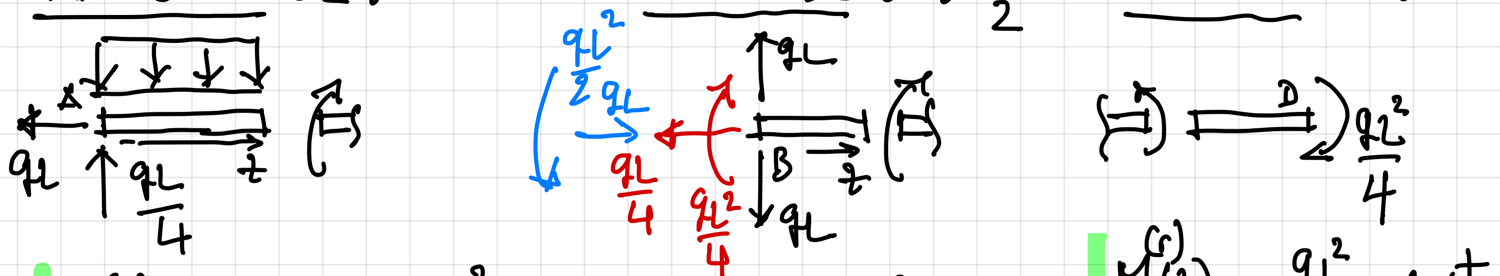
$$-X\cancel{\frac{L^3}{EI}}\Big[\underbrace{\frac{13}{12} + \frac{11}{12} + \frac{1}{6} + \frac{11}{6}}_{\frac{48}{12} = 4}\Big] = -\frac{qL^{\cancel{4}}}{\cancel{EI}}\Big[\underbrace{\frac{11}{12} + \frac{1}{12}}_{1}\Big]$$

da cui $\boxed{X = \frac{qL}{4}}$ POSITIVA! VERSO IPOTIZZATO CORRETTO !!

RISOLUZIONE SISTEMA PRINCIPALE ISOSTATICO E DIAGRAMMA DEI MOMENTI STRUTTURA IPERSTATICA



TRATTO AB $0 \le z \le L$ | **TRATTO BC** $0 \le z \le \frac{L}{2}$ | **TRATTO CD** $0 \le z \le L$



$$M^{(r)}(z) = \frac{qL}{4} \cdot z - \frac{qz^2}{2}$$

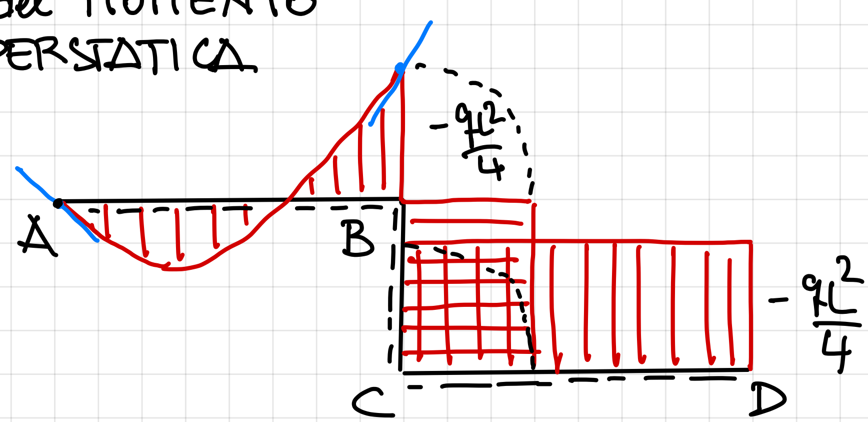
$M_A = \phi$; $M_B = -\frac{qL^2}{4}$

$$M^{(r)}(z) = \frac{qL^2}{4} - \frac{qL^2}{2} = -\frac{qL^2}{4}$$

cost.

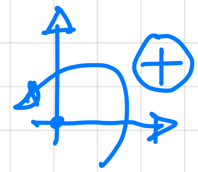
$$M^{(r)}(z) = -\frac{qL^2}{4} \quad \text{cost}$$

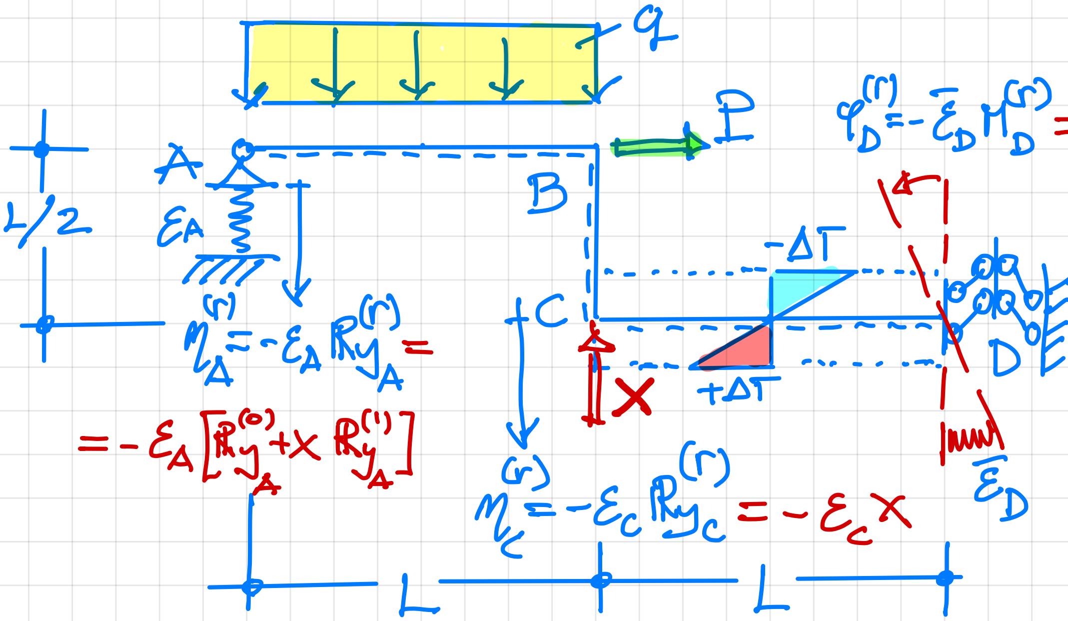
DIAGRAMMA del MOMENTO STRUTTURA IPERSTATICA



SOLUZIONE #2

SISTEMA PRINCIPALE ISOSTATICO



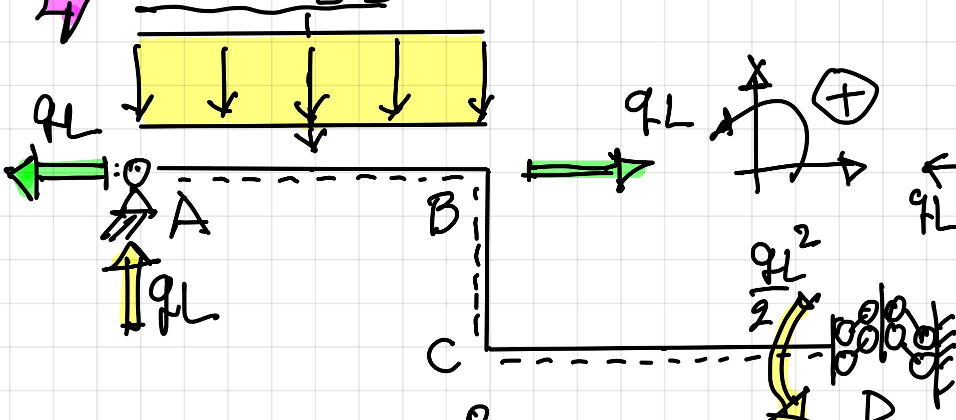


$\varphi_D^{(r)} = -\bar{\varepsilon}_D M_D^{(r)} = -\bar{\varepsilon}_D\left[M_D^{(0)} + X\, M_D^{(1)}\right]$

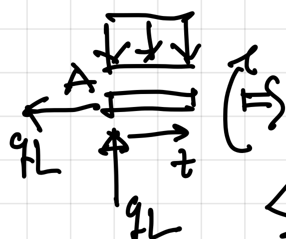
$M_A^{(r)} = -\varepsilon_A R_{y_A}^{(r)} = -\varepsilon_A\left[R_{y_A}^{(0)} + X\, R_{y_A}^{(1)}\right]$

$M_C^{(r)} = -\varepsilon_C R_{y_C}^{(r)} = -\varepsilon_C X$

SCHEMA [0] – SOLO CARICHI ESTERNI



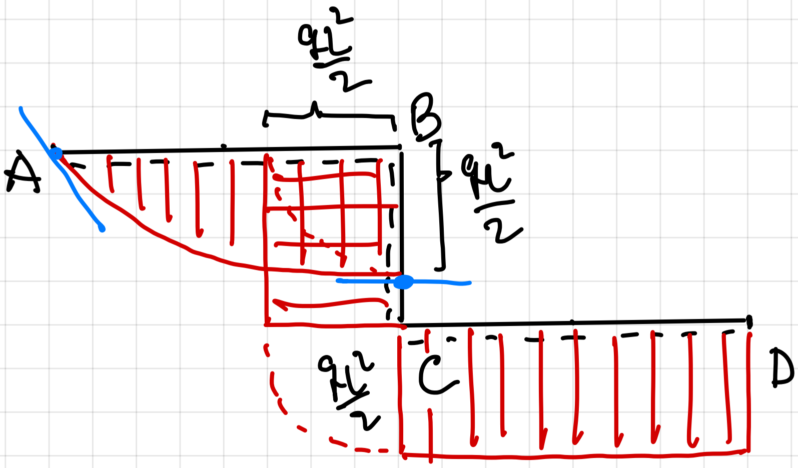
TRATTO AB $0 \le z \le L$



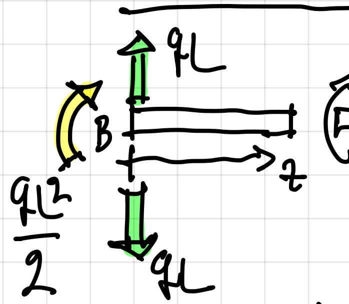
$\boxed{M^{(0)}(z) = qL\cdot z - q\frac{z^2}{2}}$

$\begin{cases} M_A = \phi \\ M_B = \frac{qL^2}{2} \end{cases}$

DIAGRAMMA $M^{(0)}(z)$

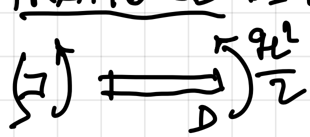


TRATTO BC $0 \le z \le \frac{L}{2}$



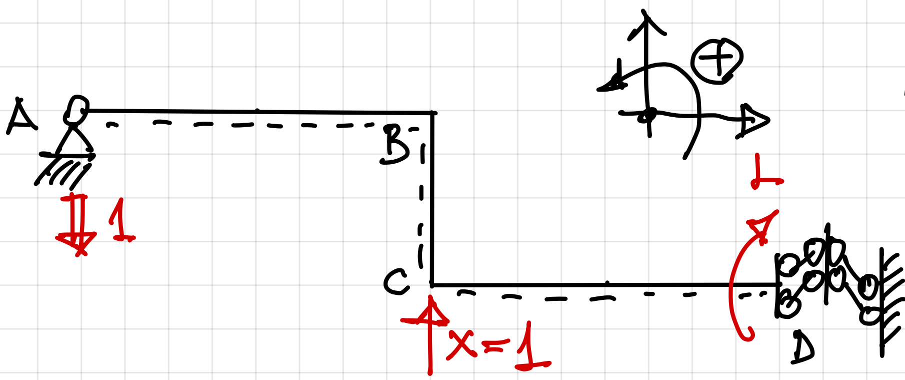
$\boxed{M^{(0)}(z) = \frac{qL^2}{2}}$ cost.

TRATTO CD $0 \le z \le L$

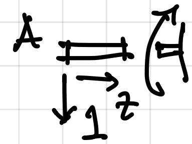


$\boxed{M^{(0)}(z) = \frac{qL^2}{2}}$ cost

SCHEMA [1] – Solo $X=1$

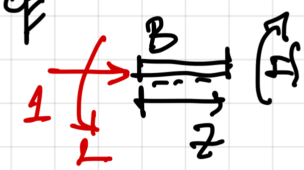


TRATTO AB $0 \le z \le L$



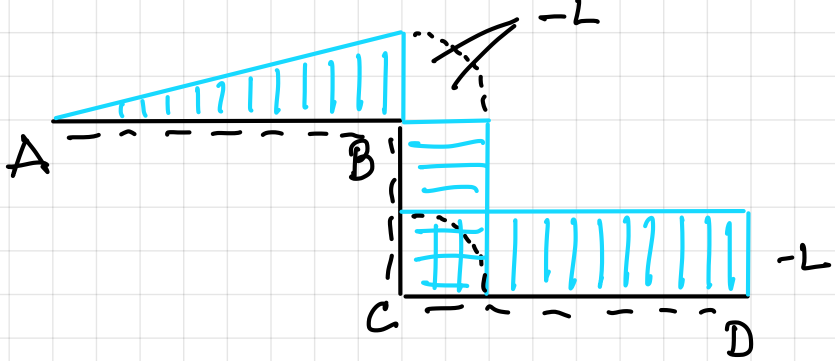
$$\boxed{M^{(1)}(z) = -z} \quad \begin{cases} M_A = 0 \\ M_B = -L \end{cases}$$

TRATTO BC $0 \le z \le \frac{L}{2}$

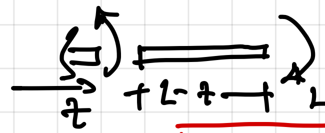


$$\boxed{M^{(1)}(z) = -L} \quad \text{cost.}$$

DIAGRAMMA $M^{(1)}(z)$



TRATTO CD $0 \le z \le L$



$$\boxed{M^{(1)}(z) = -L} \quad \text{cost.}$$

$$L_{ve} = \underbrace{X_i}_{1} \cdot \eta_i^{(r)} + \sum_j R_j^{(1)} \eta_j^{(r)} =$$

$$= 1 \cdot \underbrace{\eta_C^{(r)}}_{-\varepsilon_C X} + \underbrace{R_{y_A}^{(1)}}_{-1} \underbrace{\eta_A^{(r)}}_{-\varepsilon_A\left[\underbrace{R_{y_A}^{(0)}}_{qL} + X \underbrace{R_{y_A}^{(1)}}_{-1}\right]} + \underbrace{M_D^{(1)}}_{-L} \underbrace{\varphi_D^{(r)}}_{-\bar{\varepsilon}_D\left[\underbrace{M_D^{(0)}}_{\frac{qL^2}{2}} + X \underbrace{M_D^{(1)}}_{-L}\right]} =$$

$$= -\varepsilon_C X + \varepsilon_A\left[qL - X\right] + \bar{\varepsilon}_D L \left[\frac{qL^2}{2} - XL\right]$$

$$L_{vi} = \int_{str} M^{(1)} \frac{\overbrace{M^{(r)}}^{M^{(0)} + M^{(1)}X}}{EI} \, ds_{tr} + \int_{str} M^{(1)} \frac{d\Delta T}{h} \, ds_{tr} =$$

$$= \frac{1}{EI}\int_{str} M^{(1)}M^{(0)}\,ds_{tr} + \frac{X}{EI}\int_{str}\left[M^{(1)}\right]^2 ds_{tr} + \frac{\alpha\Delta T}{h}\int_{str} M^{(1)}\,ds_{tr} =$$

$$= \frac{1}{EI}\left\{\int_0^L (-z)\left[qLz - q\frac{z^2}{2}\right]dz + \int_0^{\frac{L}{2}}(-L)\left(\frac{qL^2}{2}\right)dz + \int_0^L (-L)\left(\frac{qL^2}{2}\right)dz\right\} +$$

$$+ \frac{X}{EI}\left\{\int_0^L z^2\,dz + \int_0^{\frac{L}{2}} L^2\,dz + \int_0^L L^2\,dz\right\} + \frac{\alpha\Delta T}{h}\int_0^L -L\,dz =$$

$$= \frac{1}{EI}\left\{-qL\left[\frac{z^3}{3}\right]_0^L + \frac{q}{2}\left[\frac{z^4}{4}\right]_0^L - \frac{qL^3}{2}\left[z\right]_0^{\frac{L}{2}} - \frac{qL^3}{2}\left[z\right]_0^L\right\} +$$

$$+ \frac{X}{EI}\left\{\left[\frac{z^3}{3}\right]_0^L + L^2\left[z\right]_0^{L/2} + L^2\left[z\right]_0^L\right\} - \frac{\alpha\Delta T}{h}\cdot L\left[z\right]_0^L =$$

$$= \frac{qL^4}{EI}\left\{-\frac{1}{3} + \frac{1}{8} - \frac{1}{4} - \frac{1}{2}\right\} + \frac{XL^3}{EI}\left\{\frac{1}{3} + \frac{1}{2} + 1\right\} - \frac{\alpha\Delta T}{h}\cdot L^2 =$$

$$= \underbrace{-\frac{23}{24}\frac{qL^4}{EI}}\underbrace{+\frac{11}{6}\frac{XL^3}{EI} - \frac{\alpha\Delta T}{h}\cdot L^2}$$

$L_{ve} = L_{vi}$ fornisce

$$-\underbrace{\varepsilon_C}_{\frac{11}{12}\frac{L^3}{EI}} X + \underbrace{\varepsilon_A}_{\frac{13}{12}\frac{L^3}{EI}}\left[qL - X\right] + \overbrace{\varepsilon_D}^{\frac{L}{6EI}} L\left[\frac{qL^2}{2} - XL\right] = \overbrace{}^{\frac{7}{8}\frac{qL^2}{EI}}$$

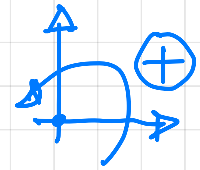
$$= -\frac{23}{24}\frac{qL^4}{EI} + \frac{11}{6}\frac{XL^3}{EI} - \frac{\alpha\Delta T}{h}\cdot L^2$$

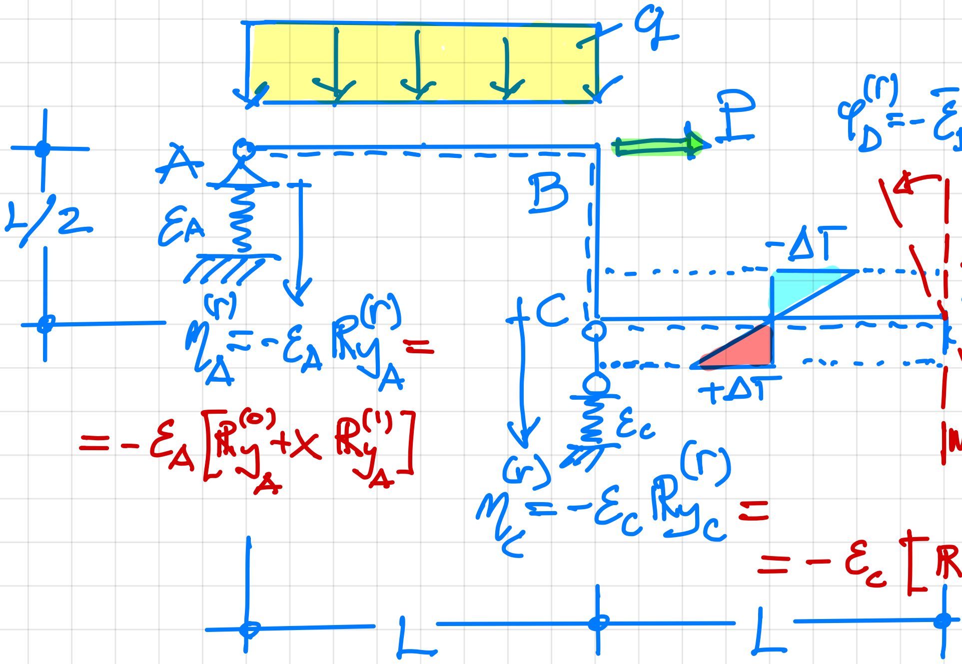
$$X\frac{\cancel{L^3}}{\cancel{EI}}\left\{\overbrace{-\frac{11}{12} - \frac{13}{12} - \frac{1}{6}}^{\frac{48}{12}=4} - \frac{11}{6}\right\} = \frac{qL^{\cancel{4}}}{\cancel{EI}}\left\{\underbrace{-\frac{23}{24} - \frac{7}{8} - \frac{13}{12} - \frac{1}{12}}_{\frac{72}{24}=3}\right\}$$

$$\boxed{X = \frac{3}{4}qL}$$ Ok! cfr. RV ↓ pag. 6

SOLUZIONE #3

SISTEMA PRINCIPALE ISOSTATICO



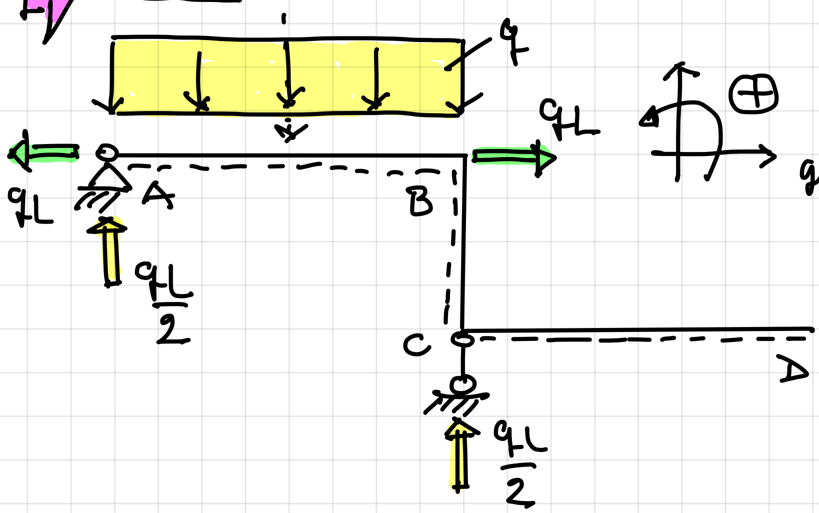


$\varphi_D^{(r)} = -\bar{\varepsilon}_D M_D^{(r)} = -\bar{\varepsilon}_D X$

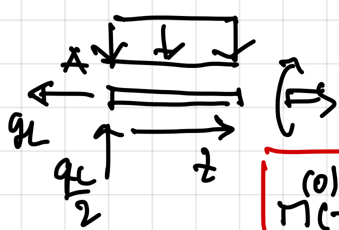
$M_A^{(r)} = -\varepsilon_A R_{y_A}^{(r)} = -\varepsilon_A \left[R_{y_A}^{(0)} + X R_{y_A}^{(1)}\right]$

$M_C^{(r)} = -\varepsilon_C R_{y_C}^{(r)} = -\varepsilon_C \left[R_{y_C}^{(0)} + X R_{y_C}^{(1)}\right]$

SCHEMA [0] – SOLO CARICHI ESTERNI



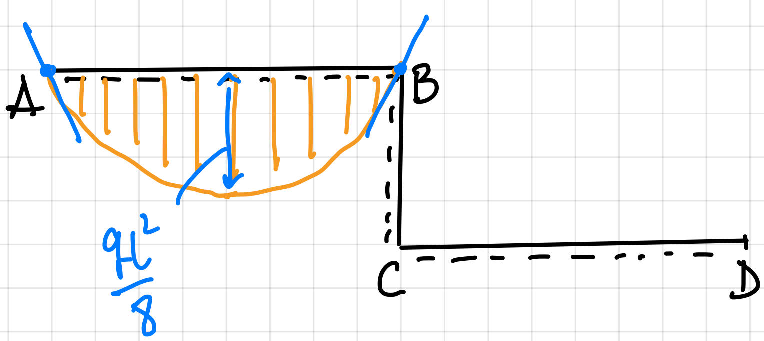
TRATTO AB $0 \le z \le L$



$$\boxed{M^{(0)}(z) = \frac{qL}{2} z - \frac{q z^2}{2}}$$

$$\begin{cases} M_A = 0 \\ M_B = 0 \\ M|_{\frac{L}{2}} = \frac{qL^2}{8} \end{cases}$$

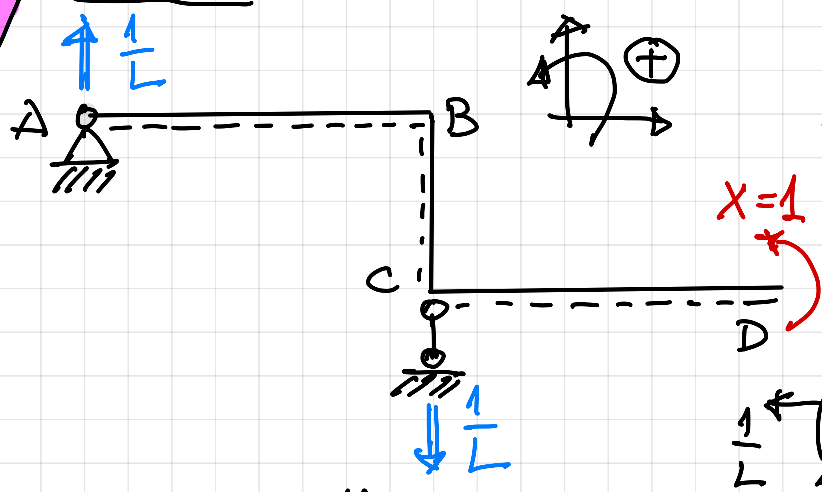
DIAGRAMMA $M^{(0)}(z)$



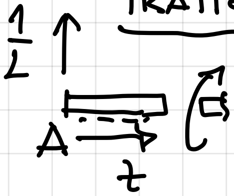
TRATTI BC e CD

$$\boxed{M^{(0)}(z) = 0}$$

SCHEMA [1] – solo X=1

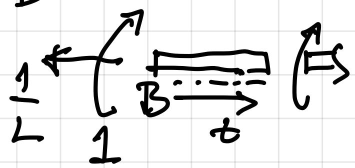


TRATTO AB $0 \le z \le L$



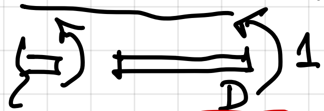
$$\boxed{M^{(1)}(z) = \frac{z}{L}} \qquad \begin{cases} M_A = 0 \\ M_B = 1 \end{cases}$$

TRATTO BC $0 \le z \le \frac{L}{2}$



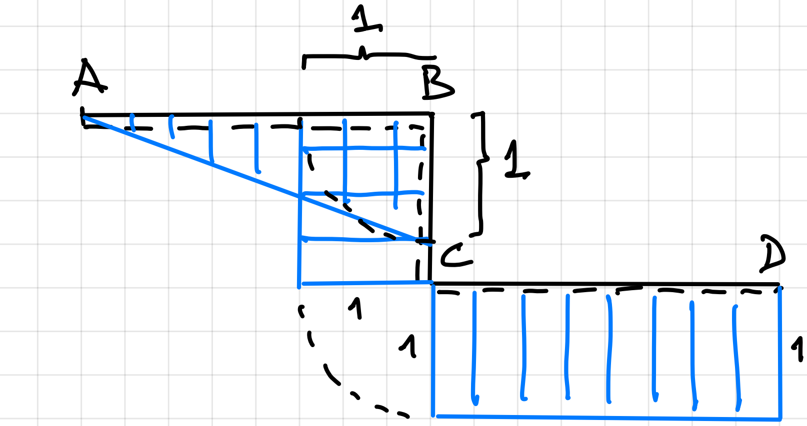
$$\boxed{M^{(1)}(z) = 1} \quad \text{cost.}$$

TRATTO CD $0 \le z \le L$



$$\boxed{M^{(1)}(z) = 1} \quad \text{cost.}$$

DIAGRAMMA $M^{(1)}(z)$



$$L_{ve} = \underset{=1}{X_i}\,\eta_i^{(r)} + \sum_j R_j^{(1)}\eta_j^{(r)} = 1\cdot\varphi_D^{(r)} + R_{y_A}^{(1)}\eta_A^{(r)} + R_{y_C}^{(1)}\eta_C^{(r)} =$$

$$= -\bar{\varepsilon}_D X + \underbrace{R_{y_A}^{(1)}}_{\frac{1}{L}}(-\varepsilon_A)\Big[\underbrace{R_{y_A}^{(0)}}_{\frac{qL}{2}} + X\underbrace{R_{y_A}^{(1)}}_{\frac{1}{L}}\Big] + \underbrace{R_{y_C}^{(1)}}_{-\frac{1}{L}}(-\varepsilon_C)\Big[\underbrace{R_{y_C}^{(0)}}_{\frac{qL}{2}} + X\underbrace{R_{y_C}^{(1)}}_{-\frac{1}{L}}\Big] =$$

$$= -\bar{\varepsilon}_D X - \frac{\varepsilon_A}{L}\left[\frac{qL}{2} + \frac{X}{L}\right] + \frac{\varepsilon_C}{L}\left[\frac{qL}{2} - \frac{X}{L}\right]$$

$$L_{vi} = \int_{str} M^{(1)}\frac{M^{(r)}}{EI}\,ds_{tr} + \int_{str} M^{(1)}\frac{\alpha\Delta T}{h}\,ds_{tr} =$$

$$M^{(r)} \rightarrow M^{(0)} + M^{(1)}X$$

$$= \frac{1}{EI}\int_{str} M^{(1)} M^{(0)} ds_{tr} + \frac{X}{EI}\int_{str} \left[M^{(1)}\right]^2 ds_{tr} + \frac{\alpha \Delta T}{h}\int_{str} M^{(1)} ds_{tr} =$$

$$= \frac{1}{EI}\left\{\int_0^L \frac{z}{L}\left(\frac{qL}{2}\cdot z - \frac{qz^2}{2}\right) dz\right\} +$$

$$+ \frac{X}{EI}\left\{\int_0^L \frac{z^2}{L^2} dz + \int_0^{\frac{L}{2}} dz + \int_0^L dz\right\} + \frac{\alpha \Delta T}{h}\int_0^L dz =$$

$$= \frac{1}{EI}\left\{\frac{q}{2}\left[\frac{z^3}{3}\right]_0^L - \frac{q}{2L}\left[\frac{z^4}{4}\right]_0^L\right\} +$$

$$+ \frac{X}{EI}\left\{\frac{1}{L^2}\left[\frac{z^3}{3}\right]_0^L + \left[z\right]_0^{\frac{L}{2}} + \left[z\right]_0^L\right\} + \frac{\alpha \Delta T}{h}\left[z\right]_0^L =$$

$$= \frac{1}{EI}\left\{\frac{q}{6}\cdot L^3 - \frac{q}{8} L^3\right\} + \frac{X}{EI}\left\{\frac{L}{3} + \frac{L}{2} + L\right\} + \frac{\alpha \Delta T}{h}\cdot L =$$

$$= \underbrace{\frac{qL^3}{EI}\frac{1}{24} + \frac{X}{EI}\frac{11\cdot L}{6} + \frac{\alpha \Delta T}{h}\cdot L}$$

$L_{ve} = L_{ri}$ fornisce :

$$-\underbrace{\bar{\varepsilon}_D}_{\frac{L}{6EI}} X - \underbrace{\frac{\varepsilon_A}{L}}_{\frac{13}{12}\frac{L^3}{EI}}\left[\frac{qL}{2} + \frac{X}{L}\right] + \overbrace{\frac{\varepsilon_C}{L}}^{\frac{11}{12}\frac{L^3}{EI}}\left[\frac{qL}{2} - \frac{X}{L}\right] = \frac{qL^3}{EI}\frac{1}{24} + \frac{X}{EI}\frac{11\cdot L}{6} + \overbrace{\frac{\alpha \Delta T}{h}\cdot L}^{\frac{7}{8}\frac{qL^2}{EI}}$$

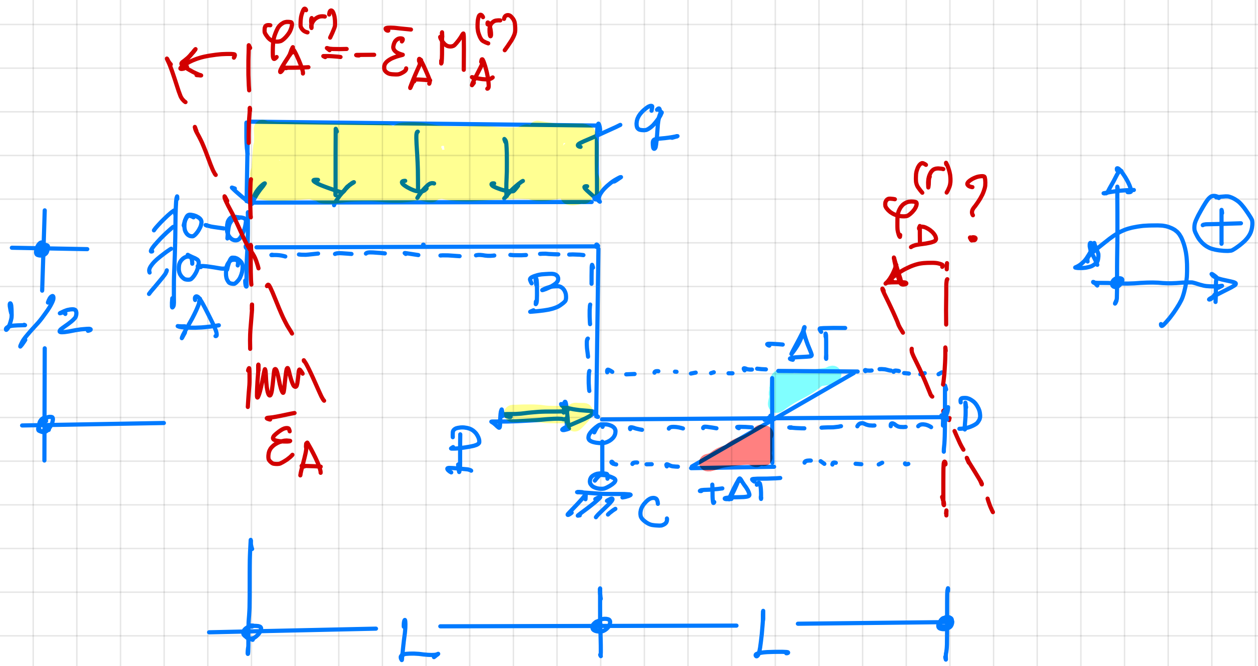
$$X\frac{\cancel{L}}{\cancel{EI}}\underbrace{\left\{-\frac{1}{6} - \frac{13}{12} - \frac{11}{12} - \frac{11}{6}\right\}}_{-\cancel{\frac{48}{12}}\ -4} = \frac{qL^{\cancel{3}\,2}}{\cancel{EI}}\underbrace{\left[\frac{1}{24} + \frac{7}{8} + \frac{13}{24} - \frac{11}{24}\right]}_{\frac{24}{24} = 1}$$

$$\boxed{X = -\frac{qL^2}{4}}$$

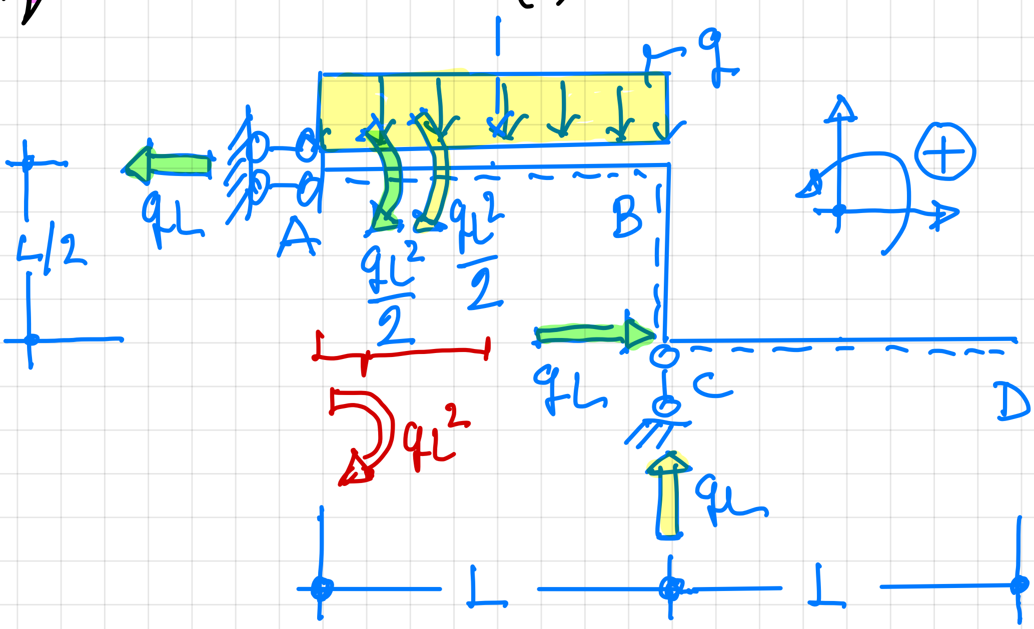
NEGATIVA → ORARIA OK! cfr. RV di p.6

ES #2 tipo 2 SOLUZIONE

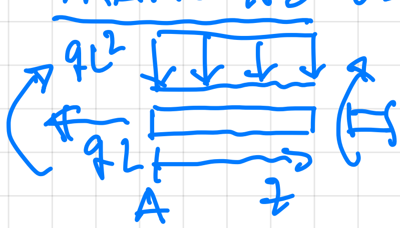
STRUTTURA REALE



CALCOLO RV e $M^{(r)}(z)$ NELLA STRUTTURA REALE



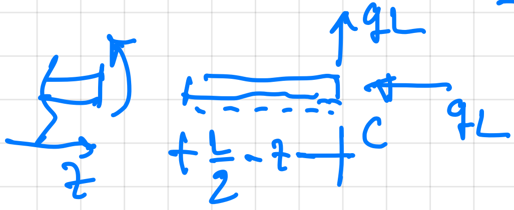
TRATTO AB $0 \le z \le L$



$$\boxed{M^{(r)}(z) = qL^2 - q\frac{z^2}{2}}$$

$$\begin{cases} M_A = qL^2 \\ M_B = \dfrac{qL^2}{2} \end{cases}$$

TRATTO BC $0 \le z \le \frac{L}{2}$



$$\boxed{M^{(r)}(z) = qL\left(\frac{L}{2} - z\right)}$$

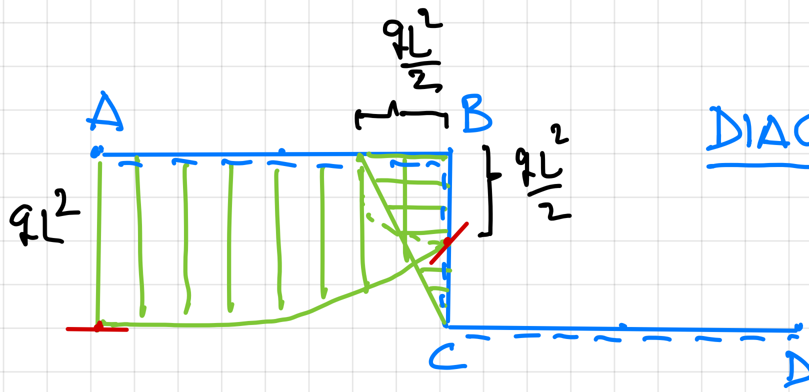
$$\hookrightarrow \begin{cases} M_B = \dfrac{qL^2}{2} \\ M_C = \phi \end{cases}$$

TRATTO CD

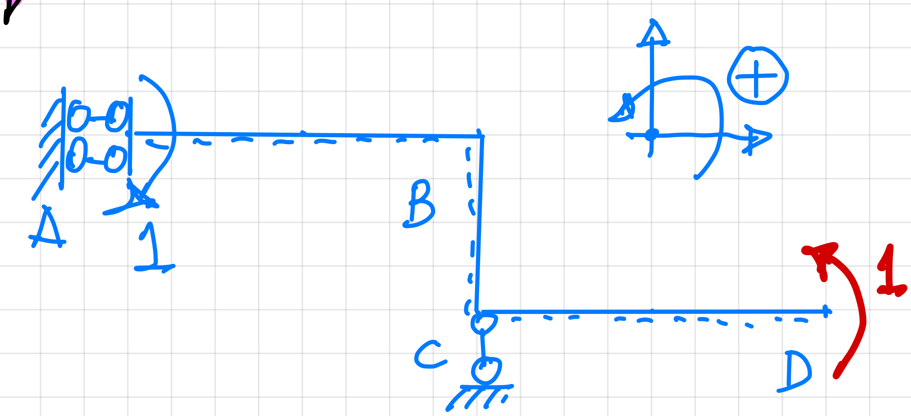
SCARICO!

$$\boxed{M^{(r)}(z) = \phi} \quad \text{cost.}$$

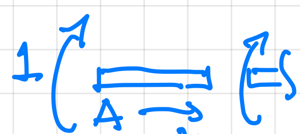
DIAGRAMMA $M^{(r)}(z)$



⇒ STRUTTURA FITTIZIA PER IL CALCOLO DI φ_D

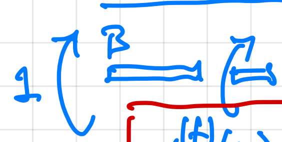


TRATTO AB $0 \le z \le L$



$$\boxed{M^{(f)}(z) = 1} \quad \text{cost.}$$

TRATTO BC $0 \le z \le \frac{L}{2}$

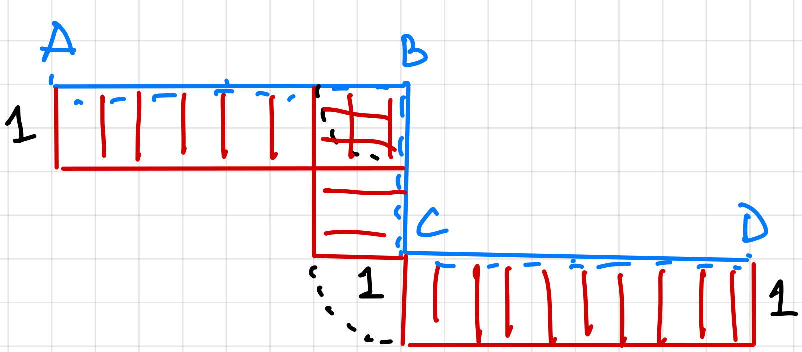


$$\boxed{M^{(f)}(z) = 1} \quad \text{cost}$$

TRATTO CD $0 \le z \le L$

$$\boxed{M^{(f)}(z) = 1} \quad \text{cost}$$

DIAGRAMMA $M^{(f)}(z)$



$$\Rightarrow\ L_{ve} = 1\cdot \eta_i^{(r)} + \sum_j R_j^{(f)} \eta_j^{(r)} = 1\cdot \varphi_D^{(r)} + \underbrace{M_A^{(f)}}_{-1}\ \underbrace{\varphi_A^{(r)}}_{-\bar{\varepsilon}_A \left[\underbrace{M_A^{(r)}}_{-qL^2}\right]} =$$

$$= \varphi_D^{(r)} - \bar{\varepsilon}_A qL^2$$

$$\Rightarrow\ L_{vi} = \int_{str} M^{(f)} \frac{M^{(r)}}{EI}\, ds_{tr} + \int_{str} M^{(f)} \frac{\alpha \Delta T}{h}\, ds_{tr} =$$

$$= \frac{1}{EI}\left\{ \int_0^L \left(qL^2 - \frac{qz^2}{2}\right) dz + \int_0^{\frac{L}{2}} qL\left(\frac{L}{2} - z\right) dz \right\} + \frac{\alpha \Delta T}{h} \int_0^L 1\cdot dz =$$

$$= \frac{1}{EI}\left\{ qL^2 \left[z\right]_0^L - \frac{q}{2}\left[\frac{z^3}{3}\right]_0^L + \frac{qL^2}{2}\left[z\right]_0^{\frac{L}{2}} - qL\left[\frac{z^2}{2}\right]_0^{\frac{L}{2}} \right\} + \frac{\alpha \Delta T}{h}\left[z\right]_0^L =$$

$$= \frac{1}{EI}\left\{ qL^2 \cdot L - \frac{q}{6}L^3 + \frac{qL^2}{2}\cdot\frac{L}{2} - \frac{qL}{2}\cdot\frac{L^2}{4} \right\} + \frac{\alpha \Delta T}{h}\cdot L =$$

$$= \frac{qL^3}{EI}\cdot\frac{23}{24} + \frac{\alpha \Delta T}{h}\cdot L$$

$\Rightarrow$ $L_{ve} = L_{vi}$ fornisce:

$$\varphi_D^{(r)} - \underbrace{\bar{\varepsilon}_A}_{\frac{L}{EI}} qL^2 = \frac{qL^3}{EI}\cdot\frac{23}{24} + \underbrace{\frac{\alpha \Delta T}{h}}_{\frac{qL^2}{24EI}}\cdot L$$

$$\varphi_D^{(r)} = \frac{qL^3}{EI} + \frac{23}{24}\frac{qL^3}{EI} + \frac{qL^3}{24EI} = 2\frac{qL^3}{EI}$$

POSITIVA $\Rightarrow$ ANTIORARIA