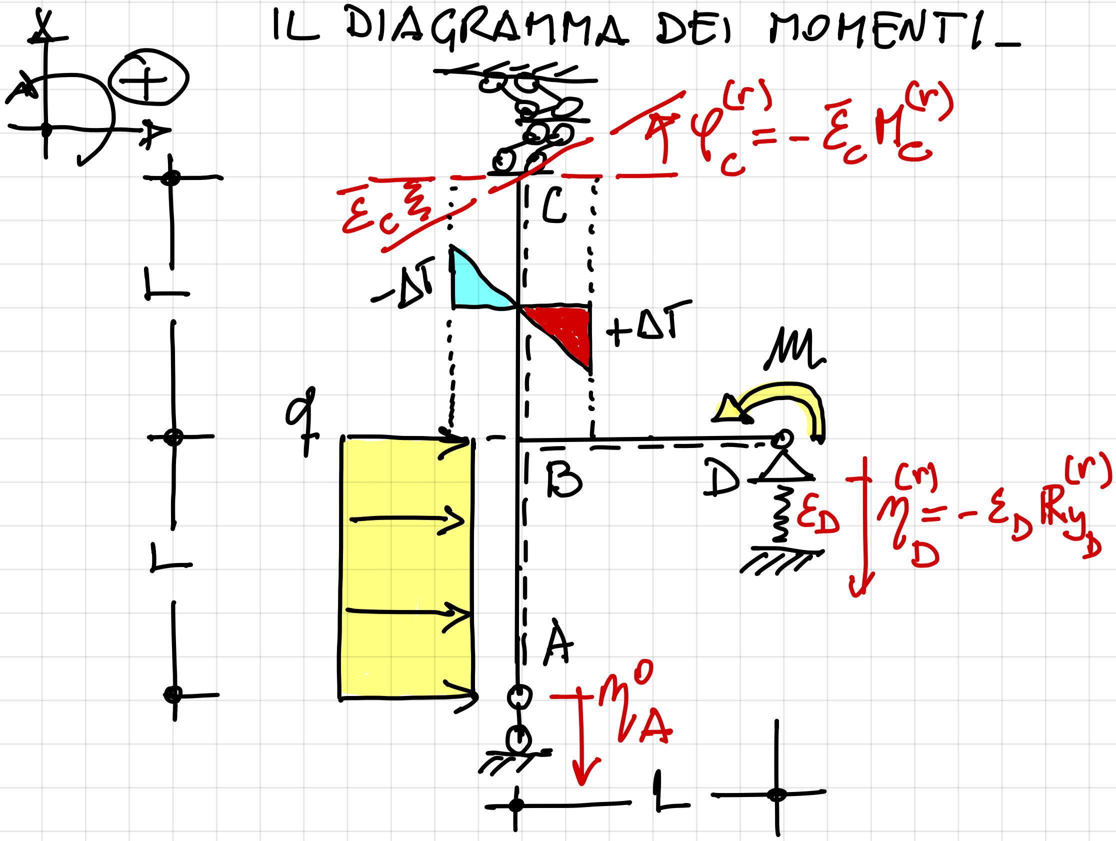
MECCANICA delle STRUTTURE – P. FUSCHI

PROVA SCRITTA del 15 GENNAIO 2025

TIPO 1

ES. #1 RISOLVERE LA STRUTTURA UNA VOLTA IPERSTATICA SEGUENTE DETERMINANDO IL DIAGRAMMA DEI MOMENTI_



Posizioni:

$$|\mathcal{M}| = qL^2$$

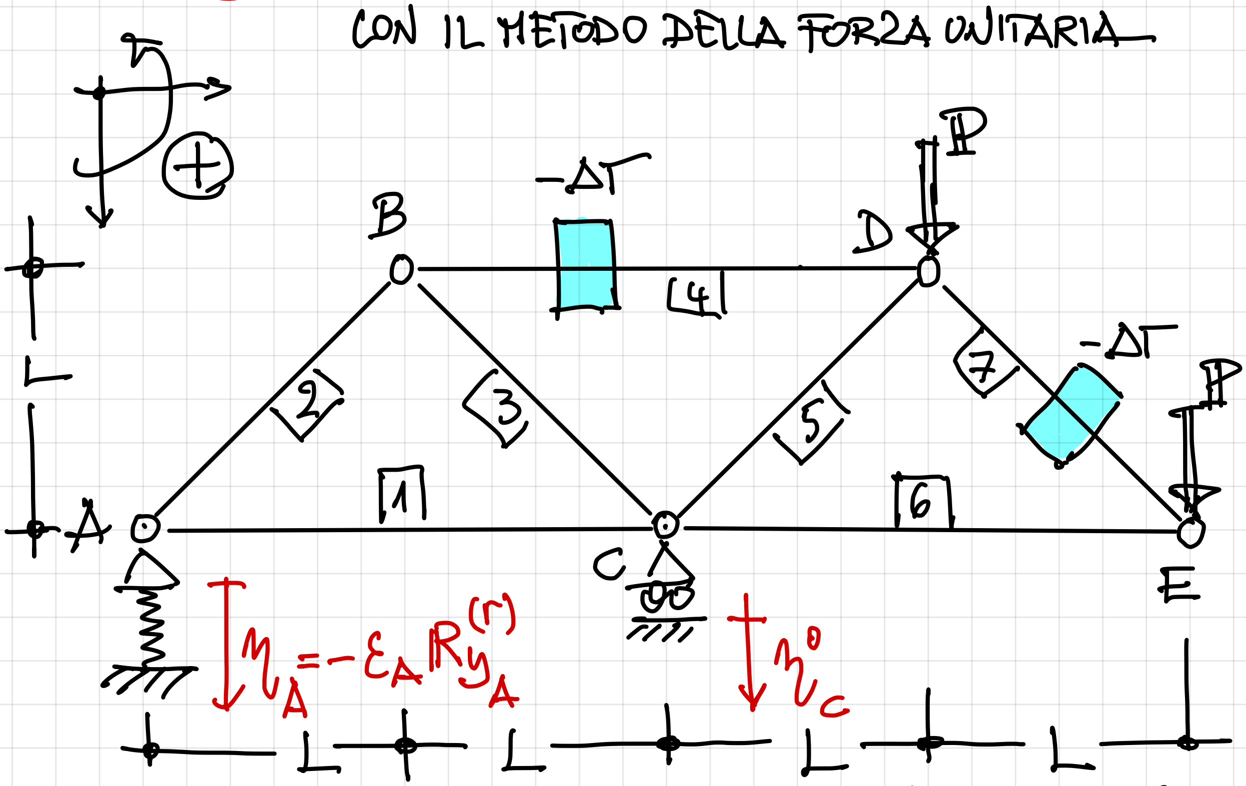
$$|\eta_A^0| = \frac{qL^4}{2EI}$$

$$|\varepsilon_C| = \frac{L}{EI}$$

$$|\varepsilon_D| = \frac{2}{3}\frac{L^3}{EI}$$

$$\left|\frac{\alpha \Delta T}{h}\right| = 2\frac{qL^2}{EI}$$

ES. #2 DETERMINARE LO SPOST. VERTICALE DELLA SEZ. E CON IL METODO DELLA FORZA UNITARIA_



Posizioni:

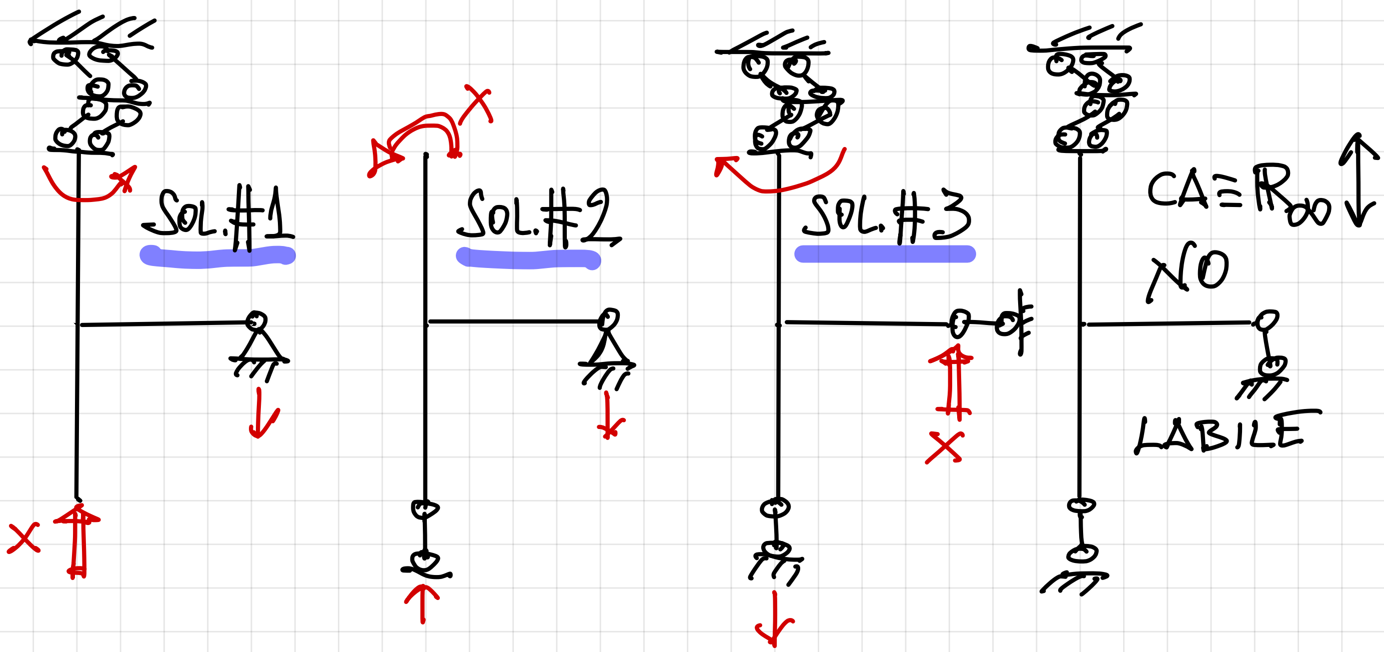
$$|\eta_C^0| = \frac{PL}{4EA}$$

$$|\varepsilon_A| = \frac{L}{3EA}$$

$$|\alpha \Delta T| = \frac{2\sqrt{2}P}{EA}$$

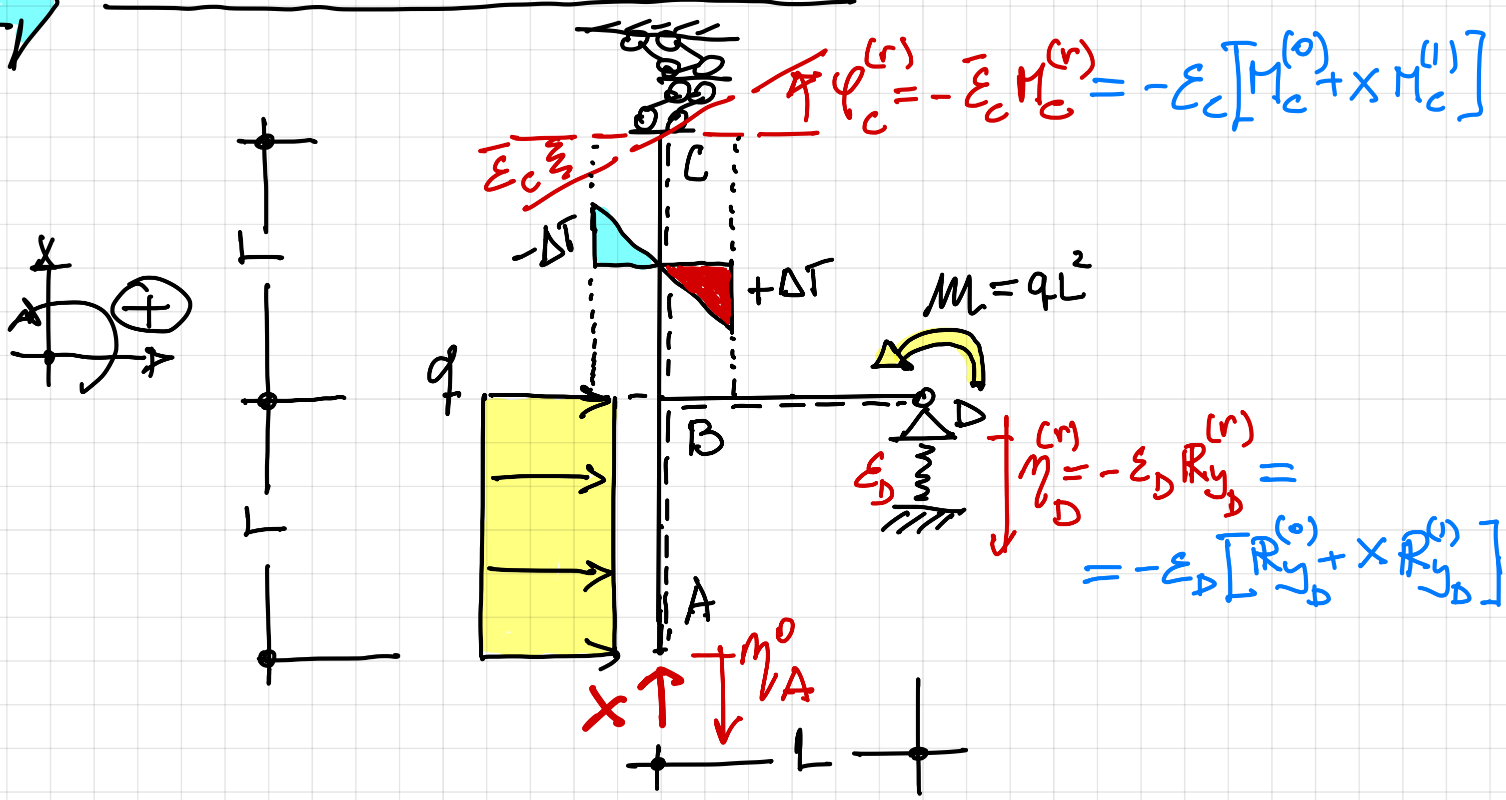
TIPO 1 – ES.#1 SOLUZIONI

SOLUZIONI POSSIBILI:



SOLUZIONE #1

SISTEMA PRINCIPALE ISOSTATICO



SCHEMA [0] SOLO CARICHI ESTERNI

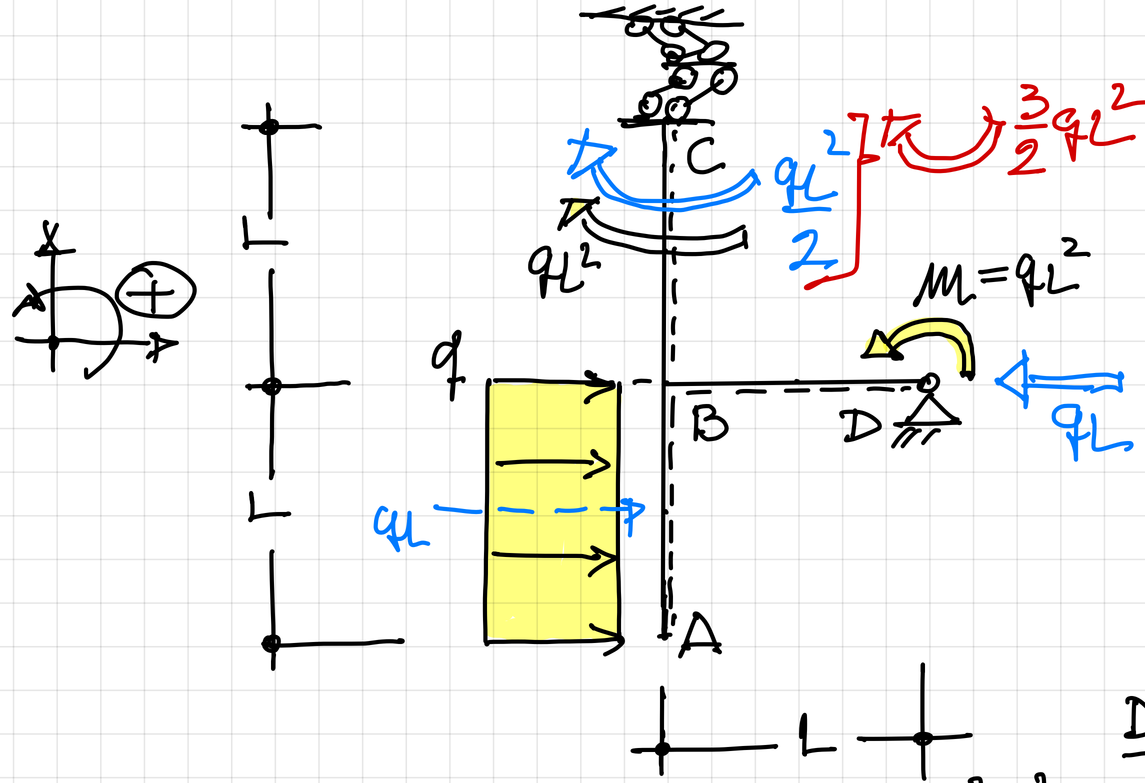
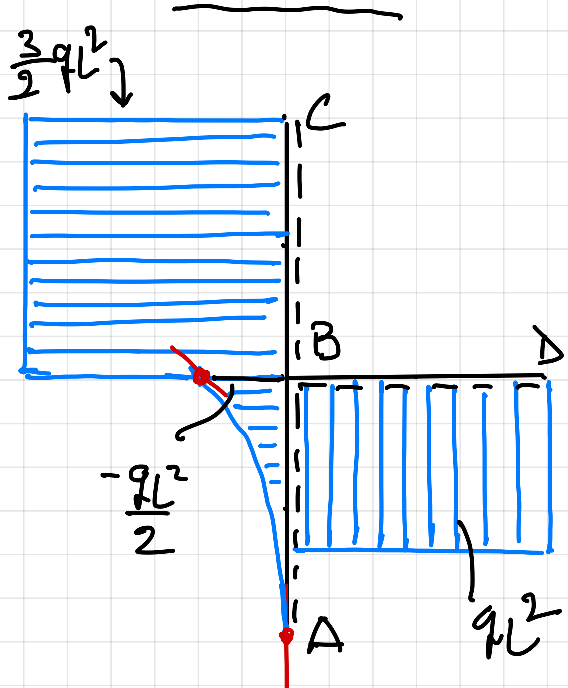


DIAGRAMMA $M^{(0)}(z)$



TRATTO AB $0 \le z \le L$

$$\boxed{M^{(0)}(z) = -q\frac{z^2}{2}}$$

$M_A = \phi$

$M_B = -\frac{qL^2}{2}$

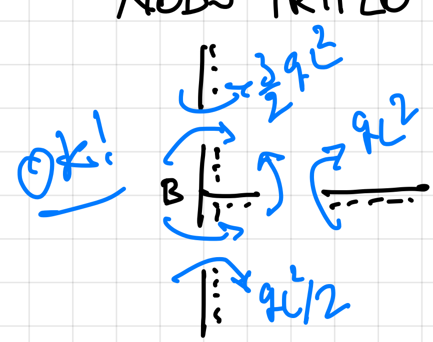
TRATTO BC $L \le z \le 2L$

$\frac{3}{2}qL^2$ $\boxed{M^{(0)}(z) = -\frac{3}{2}qL^2}$ cost

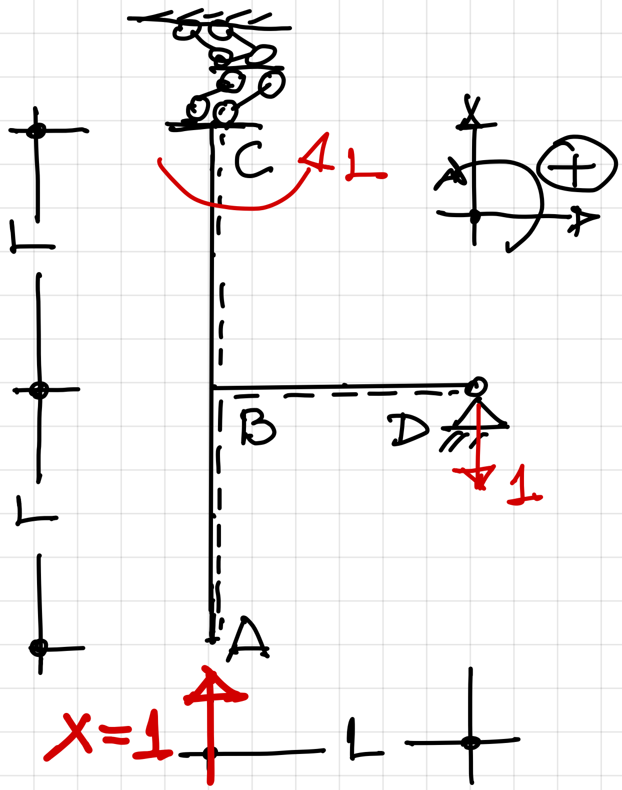
TRATTO BD $0 \le z \le L$

qL^2 $\boxed{M^{(0)}(z) = qL^2}$ cost

VERIFICA AL NODO TRIPLO B



SCHEMA [1] SOLO $X=1$



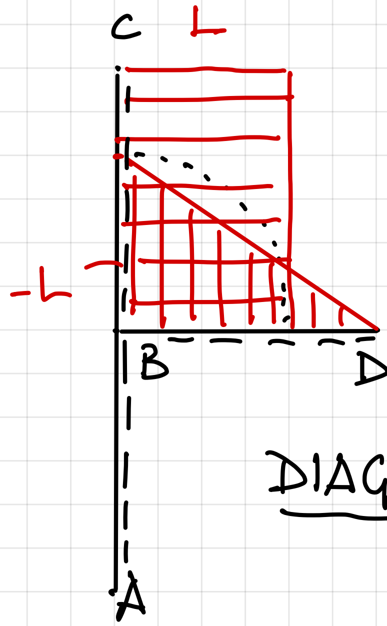
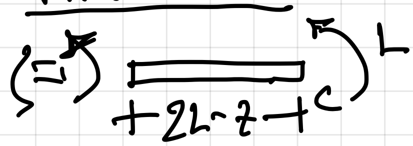


DIAGRAMMA $M^{(1)}(z)$

TRATTO AB $0 \le z \le L$

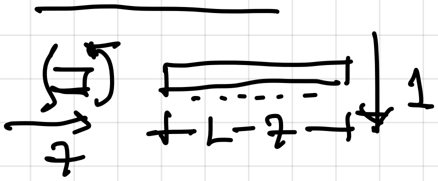
SCARICO $\boxed{M^{(1)}(z) = \emptyset}$

TRATTO BC $L \le z \le 2L$



$\boxed{M^{(1)}(z) = L}$ cost

TRATTO BD $0 \le z \le L$



$\boxed{M^{(1)}(z) = -(L-z)}$ $\begin{cases} M_B = -L \\ M_D = \emptyset \end{cases}$

$$\underline{L_{ve}} = \underset{\overset{\shortparallel}{1}}{X_i} \cdot \eta_i^{(r)} + \sum_j R_j^{(1)} \eta_j^{(r)} = 1 \cdot \left(-\eta_A^0\right) + M_C^{(1)} \varphi_C^{(r)} + R_{y_D}^{(1)} \eta_D^{(r)} =$$

$$= -\eta_A^0 + \underset{\overset{\shortparallel}{L}}{M_C^{(1)}}(-\varepsilon_C)\left[\underset{\overset{\shortparallel}{-\frac{3}{2}qL^2}}{M_C^{(0)}} + X \underset{\overset{\shortparallel}{L}}{M_C^{(1)}}\right] + \underset{\overset{\shortparallel}{-1}}{R_{y_D}^{(1)}}(-\varepsilon_D)\left[\underset{\overset{\shortparallel}{\emptyset}}{R_{y_D}^{(0)}} + X \underset{\overset{\shortparallel}{-1}}{R_{y_D}^{(1)}}\right] =$$

$$= -\eta_A^0 - \varepsilon_C L\left[-\frac{3}{2}qL^2 + XL\right] - \varepsilon_D X$$

$$L_{vi} = \int_{Str} M^{(1)} \frac{\overbrace{M^{(T)}}^{M^{0}+M^{(1)}X}}{EI}\, dStr + \int_{Str} M^{(1)} \frac{\alpha \Delta T}{h}\, dStr =$$

$$= \frac{1}{EI}\int_{Str} M^{(1)} M^{(0)}\, dStr + \frac{X}{EI}\int_{Str} \left[M^{(1)}\right]^2 dStr + \frac{\alpha \Delta T}{h}\int_{Str} M^{(1)}\, dStr =$$

$$= \frac{1}{EI}\left\{ \int_{L}^{2L} L\cdot\left(-\frac{3}{2}qL^2\right) dz + \int_{0}^{L} \overbrace{-(L-z)\, qL^2}^{-qL^3+qL^2z}\, dz \right\} +$$

$$+ \frac{X}{EI}\left\{ \int_{L}^{2L} L^2\, dz + \int_{0}^{L} \overbrace{(L-z)^2}^{L^2+z^2-2Lz}\, dz \right\} + \frac{\alpha \Delta T}{h}\int_{L}^{2L} L\, dz =$$

$$= \frac{1}{EI}\left\{ -\frac{3}{2}qL^3\cdot\left[z\right]_{L}^{2L} - qL^3\cdot L + qL^2\left[\frac{z^2}{2}\right]_{0}^{L} \right\} +$$

$$+ \frac{X}{EI}\left\{ L^2\left[z\right]_{L}^{2L} + L^2\cdot L + \left[\frac{z^3}{3}\right]_{0}^{L} - \cancel{2}L\left[\frac{z^2}{\cancel{2}}\right]_{0}^{L} \right\} + \frac{\alpha \Delta T}{h} L\cdot\left[z\right]_{L}^{2L} =$$

$$= \frac{1}{EI}\left\{ -\frac{3}{2}qL^4 - qL^4 + \frac{qL^4}{2} \right\} +$$

$$+ \frac{X}{EI}\left\{ L^3 + \cancel{L^3} + \frac{L^3}{3} - \cancel{L^3} \right\} + \frac{\alpha \Delta T}{h}\cdot L^2 =$$

$$= \frac{qL^4}{EI}\left[-\frac{3}{2} - 1 + \frac{1}{2}\right] + \frac{XL^3}{EI}\left[1 + \frac{1}{3}\right] + \alpha\frac{\Delta T}{h} L^2 =$$

$$= -2\frac{qL^4}{EI} + \frac{4}{3}\frac{XL^3}{EI} + \alpha\frac{\Delta T}{h} L^2$$

$\Rightarrow$ Lve = Lvi fornisce

$$-\eta_A^0 - \bar{\varepsilon}_c L\left[-\frac{3}{2}qL^2 + \underbrace{XL}\right] - \underbrace{\varepsilon_D X} =$$

$$= -2\frac{qL^4}{EI} + \underbrace{\frac{4}{3}\frac{XL^3}{EI}} + \alpha\frac{\Delta T}{h}L^2$$

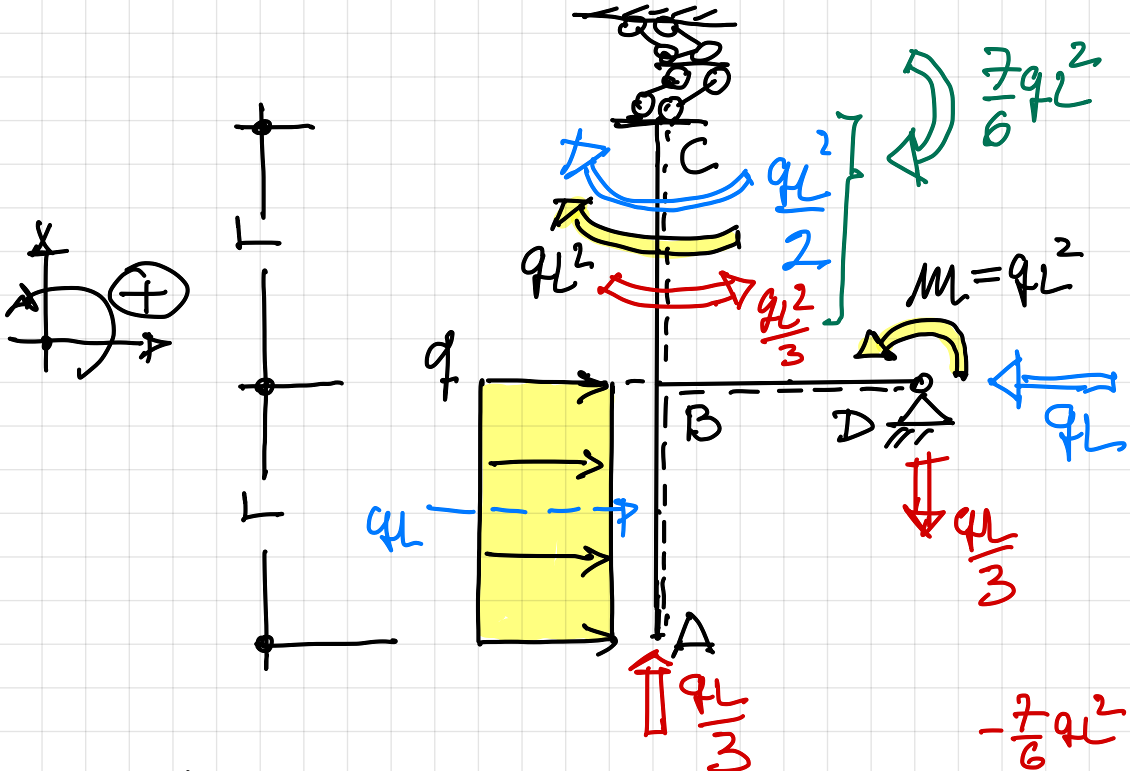
$$X\left[\frac{4}{3}\frac{L^3}{EI} + \underbrace{\varepsilon_D}_{=\frac{2L^3}{3EI}} + \underbrace{\bar{\varepsilon}_c}_{\frac{L}{EI}} L^2\right] =$$

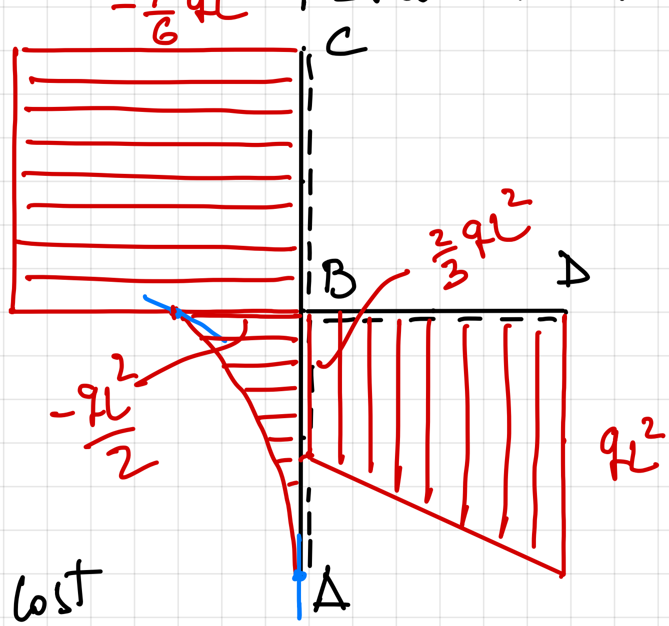
$$= -\underbrace{\eta_A^0}_{=\frac{qL^4}{2EI}} + \frac{3}{2}qL^3\underbrace{\bar{\varepsilon}_c}_{=\frac{L}{EI}} + \cancel{2\frac{qL^4}{EI}} - \cancel{\underbrace{\alpha\frac{\Delta T}{h}}_{=2\frac{qL^2}{EI}}L^2}$$

$$\cancel{\frac{L^3}{EI}}X\underbrace{\left[\frac{4}{3} + \frac{2}{3} + 1\right]}_{3} = \frac{qL^{\cancel{4}}}{\cancel{EI}}\underbrace{\left[-\frac{1}{2} + \frac{3}{2}\right]}_{1}$$

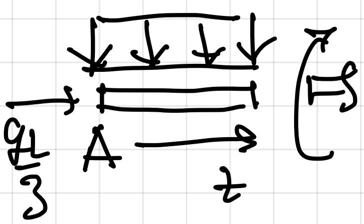
da cui $\boxed{X = \frac{qL}{3}}$ POSITIVO! VERSO IPOTIZZATO CORRETTO!

SOLUZIONE SISTEMA PRINCIPALE ISOSTATICO





TRATTO AB $0 \le z \le L$

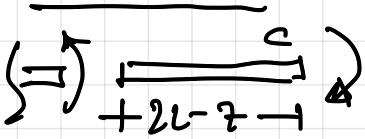


$$\boxed{M^{(r)}(z) = -\frac{qz^2}{2}}$$

$M_A = \emptyset$

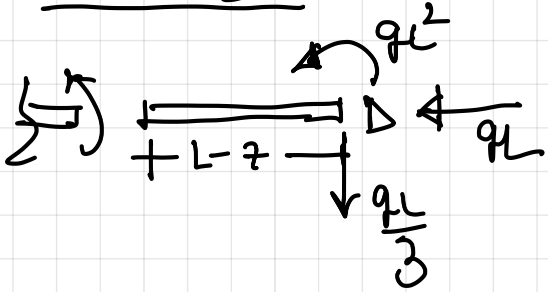
$M_B = -\frac{qL^2}{2}$

TRATTO BC $L \le z \le L$



$$\boxed{M^{(r)}(z) = -\frac{7}{6}qL^2}$$ cost

TRATTO BD $0 \le z \le L$

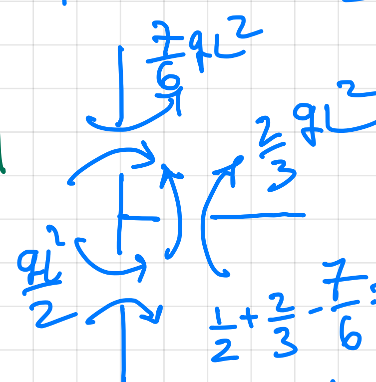


$$\boxed{M^{(r)}(z) = qL^2 - \frac{qL}{3}(L-z)}$$

$M_B = \frac{2}{3}qL^2$

$M_D = qL^2$

VERIFICA AL NODO TRIPLO B

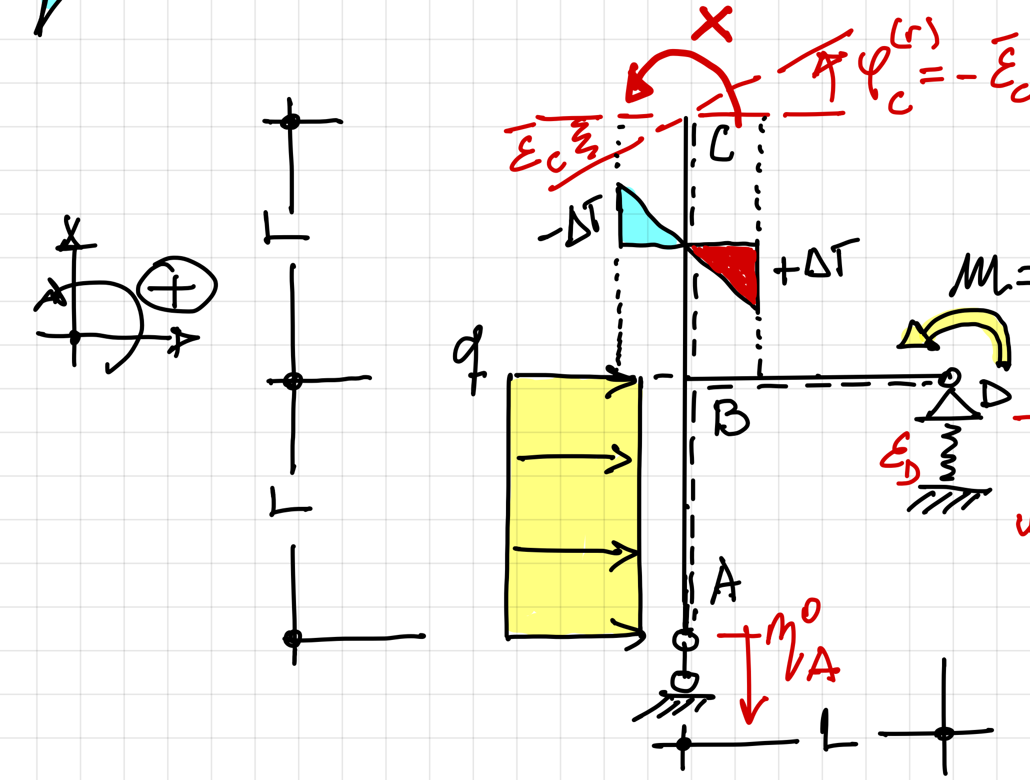


$\frac{1}{2} + \frac{2}{3} - \frac{7}{6} = \emptyset$

OK!

SOLUZIONE #2

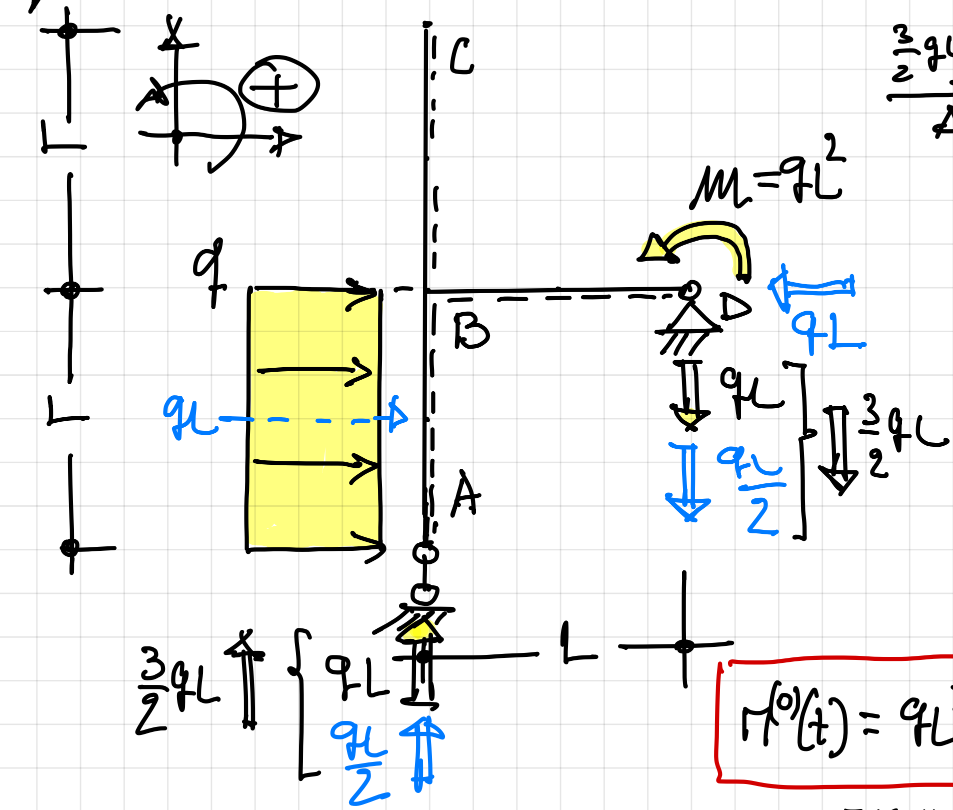
SISTEMA PRINCIPALE ISOSTATICO



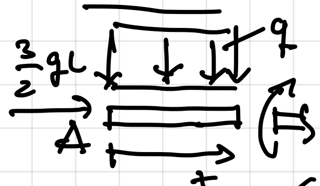
$$\varphi_C^{(r)} = -\varepsilon_C M_C^{(r)} = -\varepsilon_C X$$

$$\eta_D^{(r)} = -\varepsilon_D R_{y_D}^{(r)} = -\varepsilon_D \left[R_{y_D}^{(0)} + X R_{y_D}^{(1)} \right]$$

SCHEMA [0] SOLO CARICHI ESTERNI



TRATTO AB $0 \leq z \leq L$



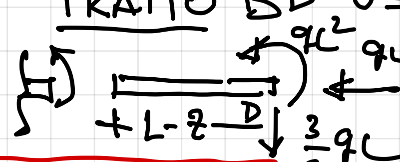
$$\boxed{M^{(0)}(z) = -q\frac{z^2}{2}}$$

$$\begin{cases} M_A = \emptyset \\ M_B = -\dfrac{qL^2}{2} \end{cases}$$

TRATTO BC
SCARICO

$$\boxed{M^{(0)}(z) = \emptyset}$$

TRATTO BD $0 \leq z \leq L$



$$\boxed{M^{(0)}(z) = qL^2 - \frac{3}{2}qL(L-z)}$$

$$\begin{cases} M_B = -\dfrac{qL^2}{2} \\ M_D = qL^2 \end{cases}$$

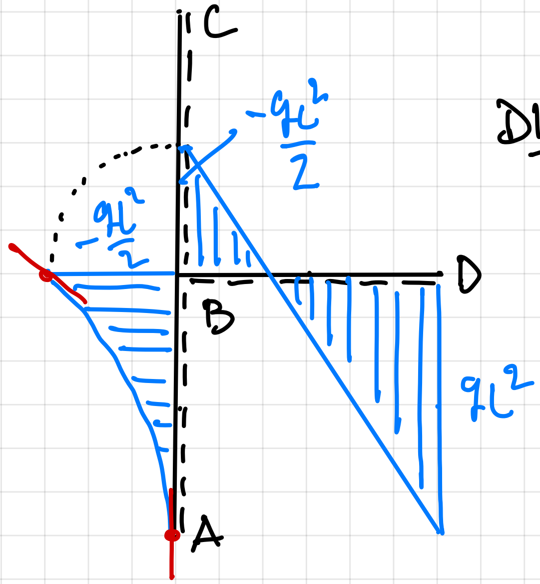
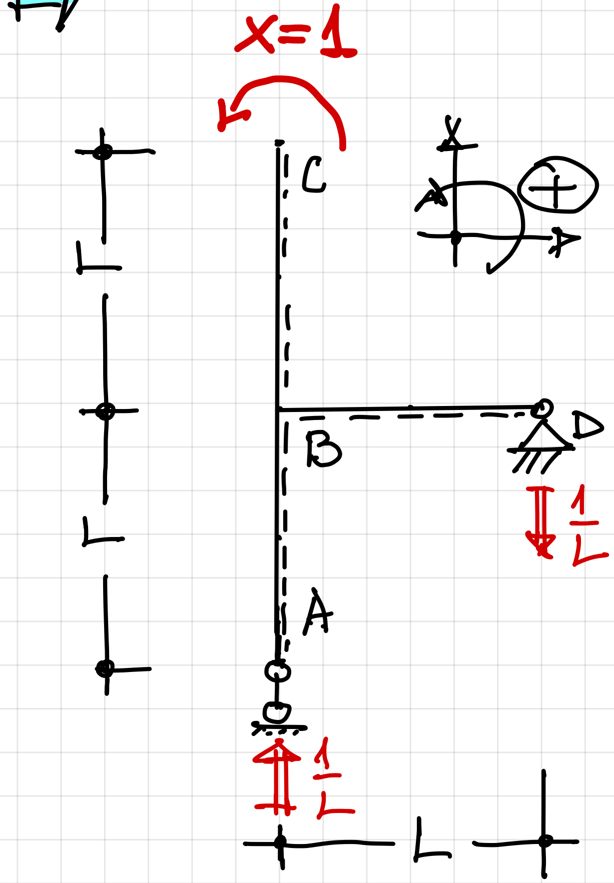


DIAGRAMMA $M^{(0)}(z)$

SCHEMA [1] solo $X=1$



TRATTO AB $0 \le z \le L$

SCARICO $\boxed{M^{(1)}(z) = \phi}$

TRATTO BC $L \le z \le 2L$

$\boxed{M^{(1)}(z) = 1}$ cost.

TRATTO BD $0 \le z \le L$

$\boxed{M^{(1)}(z) = -\frac{1}{L}(L-z)}$

$\begin{cases} M_B = -1 \\ M_D = \phi \end{cases}$

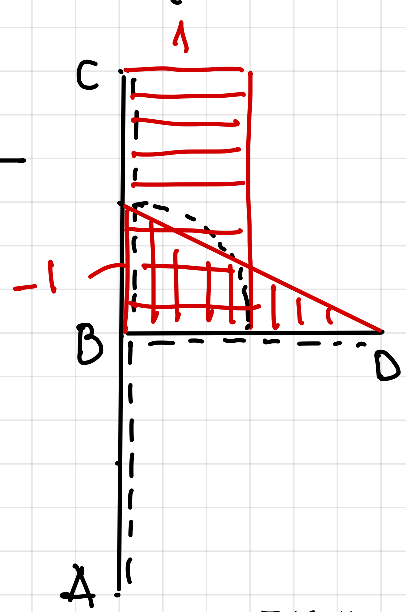


DIAGRAMMA $M^{(1)}(z)$

$$L_{ve} = \underset{=1}{X_i}\,\eta_i^{(r)} + \sum_j R_j^{(1)}\,\eta_j^{(r)} =$$

$$= 1\cdot\left(-\bar{\varepsilon}_C X\right) + \underbrace{R^{(1)}_{y_A}}_{=\frac{1}{L}}\left(-\eta^{0}_{A}\right) + \underbrace{R^{(1)}_{y_D}}_{-\frac{1}{L}}\left(-\varepsilon_D\right)\left[\underbrace{R^{(0)}_{y_D}}_{-\frac{3}{2}qL} + X\,\underbrace{R^{(1)}_{y_D}}_{-\frac{1}{L}}\right] =$$

$$= \underbrace{-\bar{\varepsilon}_C X - \frac{\eta^{0}_{A}}{L}}\;\underbrace{-\frac{\varepsilon_D}{L}\left[\frac{3}{2}qL + \frac{X}{L}\right]}$$

$$L_{vi} = \int_{str} M^{(1)}\frac{\overbrace{M^{(r)}}^{M^{(0)}+M^{(1)}X}}{EI}\,ds_{tr} + \int_{str} M^{(1)}\,\frac{\alpha\Delta T}{h}\,ds_{tr} =$$

$$= \frac{1}{EI}\int_{str} M^{(1)}M^{(0)}\,ds_{tr} + \frac{X}{EI}\int_{str}\left[M^{(1)}\right]^2 ds_{tr} + \frac{\alpha\Delta T}{h}\int_{str} M^{(1)}\,ds_{tr} =$$

$$= \frac{1}{EI}\int_0^L \overbrace{-\frac{1}{L}(L-z)\left[qL^2 - \frac{3}{2}qL(L-z)\right]}^{-qL(L-z)+\frac{3}{2}q\left(L^2+z^2-2Lz\right)}\,dz +$$

$$+ \frac{X}{EI}\left\{\int_0^L dz + \int_0^L \frac{1}{L^2}\underbrace{(L-z)^2}_{L^2+z^2-2Lz}\,dz\right\} + \frac{\alpha\Delta T}{h}\int_L^{2L} dz =$$

$$= \frac{1}{EI}\left\{-qL^2\cdot\left[z\right]_0^L + qL\left[\frac{z^2}{2}\right]_0^L + \frac{3}{2}qL^2\left[z\right]_0^L + \frac{3}{2}q\left[\frac{z^3}{3}\right]_0^L - 3qL\left[\frac{z^2}{2}\right]_0^L\right\} +$$

$$+ \frac{X}{EI}\left\{L + L + \frac{1}{L^2}\left[\frac{z^3}{3}\right]_0^L - \frac{2}{L}\left[\frac{z^2}{2}\right]_0^L\right\} + \frac{\alpha\Delta T}{h}\cdot L =$$

$$= \frac{1}{EI}\left[-\cancel{qL^3} + \cancel{\frac{qL^3}{2}} + \cancel{\frac{3}{2}qL^3} + \cancel{\frac{q}{2}L^3} - \cancel{\frac{3}{2}qL^3} \right] +$$

$$+ \frac{X}{EI}\left[2L + \frac{L}{3} - L \right] + \frac{\alpha \overline{\Delta T}}{h} \cdot L =$$

$$= \frac{XL}{EI}\left[\underbrace{2 + \frac{1}{3} - 1}_{\frac{4}{3}} \right] + \frac{\alpha \overline{\Delta T}}{h} \cdot L =$$

$\Rightarrow$ $L_{ve} = L_{vi}$ fornisce:

$$-\bar{\varepsilon}_C X - \frac{\eta_A^0}{L} - \frac{\varepsilon_D}{L}\left[\frac{3}{2}qL + \frac{X}{L} \right] = \frac{4}{3}\frac{XL}{EI} + \frac{\alpha \overline{\Delta T}}{h} \cdot L$$

$$X\left[\frac{4}{3}\frac{L}{EI} + \underbrace{\bar{\varepsilon}_C}_{\frac{L}{EI}} + \underbrace{\frac{\varepsilon_D}{L^2}}_{\frac{2}{3}\frac{L^3}{EI}} \right] = -\underbrace{\frac{\eta_A^0}{L}}_{\frac{qL^4}{2EI}} - \frac{3}{2}q \cdot \underbrace{\varepsilon_D}_{\frac{2}{3}\frac{L^3}{EI}} - \underbrace{\frac{\alpha \overline{\Delta T}}{h}}_{2\frac{qL^2}{EI}} L$$

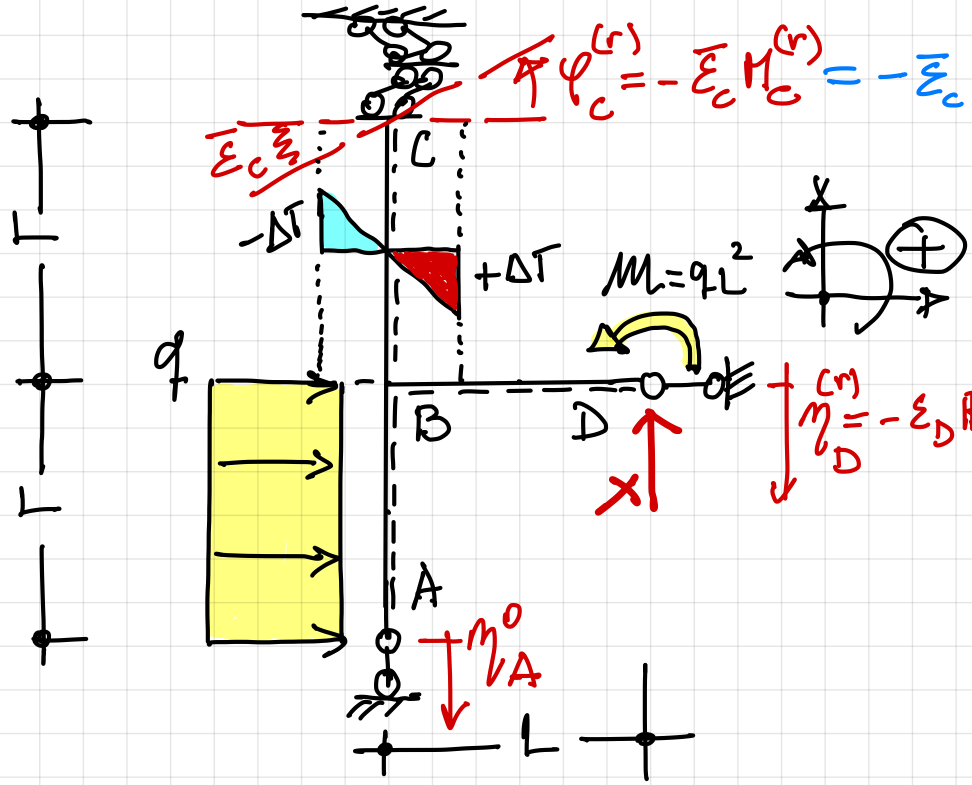
$$X \cancel{\frac{L}{EI}} \left[\underbrace{\frac{4}{3} + 1 + \frac{2}{3}}_{3} \right] = -\cancel{\frac{qL^{3\,2}}{EI}} \left[\underbrace{\frac{1}{2} + 1 + 2}_{\frac{7}{2}} \right]$$

$$X = -\frac{7}{6}qL^2$$ NEGATIVA $\Rightarrow$ VERSO OPPOSTO A QUELLO IPOTIZZATO!

Ok! cfr. RV di pag 6

SOLUZIONE #3

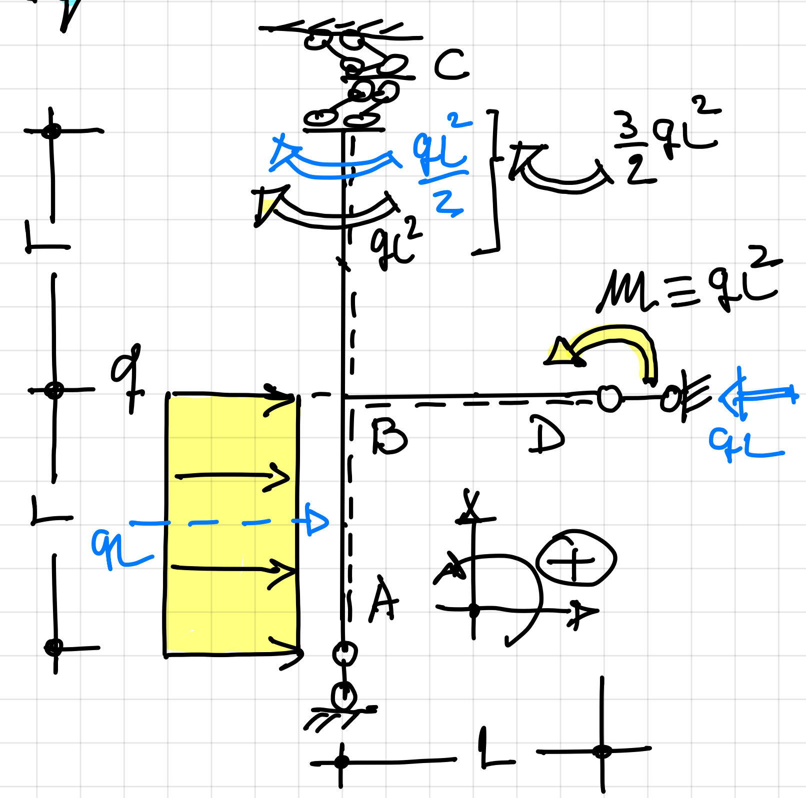
SISTEMA PRINCIPALE ISOSTATICO



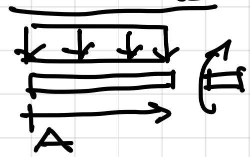
$$\varphi_C^{(r)} = -\varepsilon_C M_C^{(r)} = -\varepsilon_C \left[M_C^{(0)} + X M_C^{(1)} \right]$$

$$\eta_D^{(r)} = -\varepsilon_D R_{y_D}^{(r)} = -\varepsilon_D X$$

SCHEMA [0] SOLO CARICHI ESTERNI

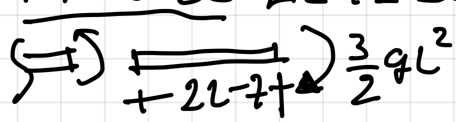


TRATTO AB $0 \le z \le L$



$$\boxed{M^{(0)}(z) = -\frac{qz^2}{2}} \qquad \begin{cases} M_A = \phi \\ M_B = -\dfrac{qL^2}{2} \end{cases}$$

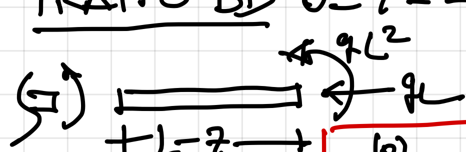
TRATTO BC $L \le z \le 2L$



$$\boxed{M^{(0)}(z) = -\frac{3}{2}qL^2}$$

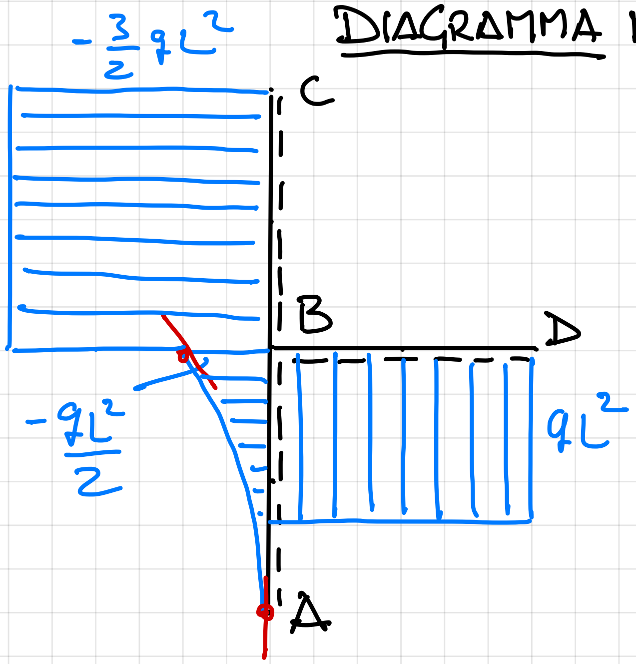
cost.

TRATTO BD $0 \le z \le L$

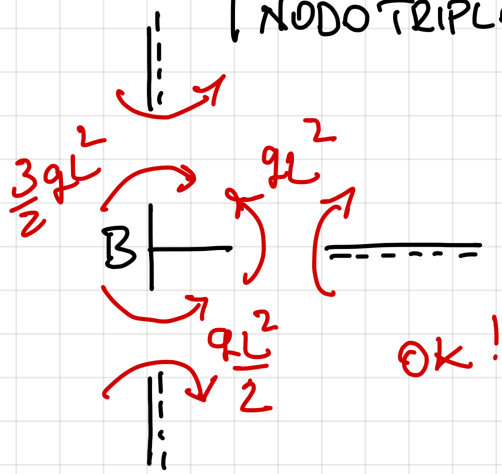


$$\boxed{M^{(0)}(z) = qL^2} \text{ cost.}$$

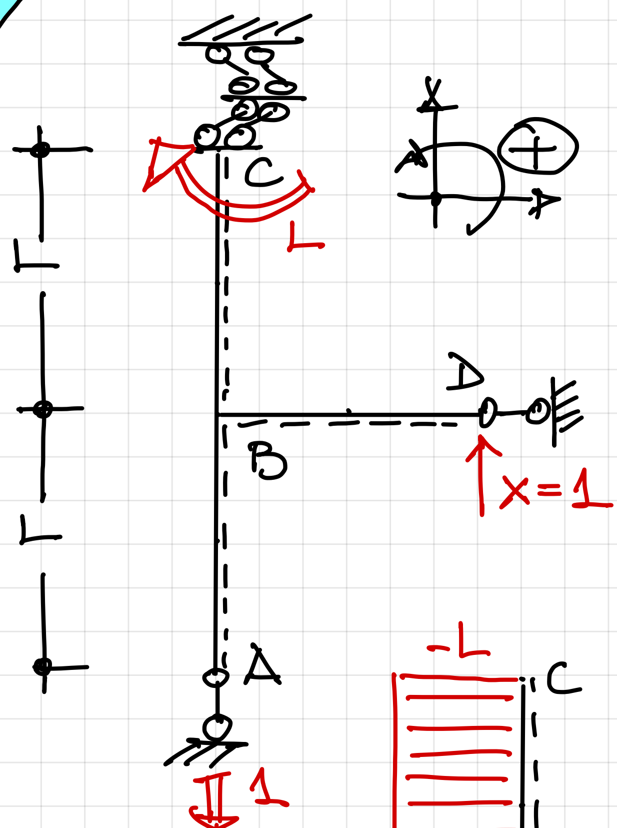
DIAGRAMMA $M^0(z)$



VERIFICA AL NODO TRIPLO B



SCHEMA [1] SOLO $X=1$



TRATTO AB $0 \le z \le L$

SCARICO $\boxed{M^{(1)}(z) = \phi}$

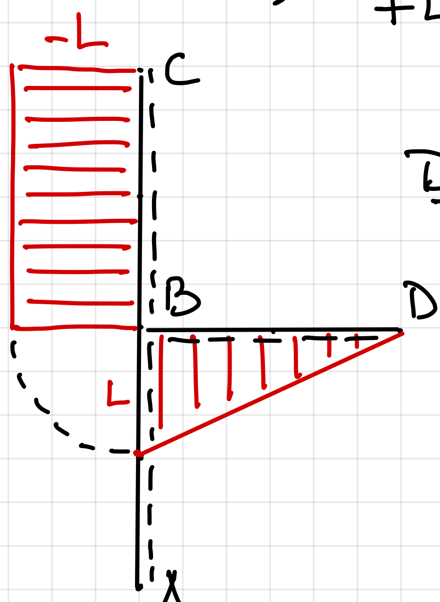
TRATTO BC $L \le z \le 2L$

$+2L-z$ C L $\boxed{M^{(1)}(z) = -L}$ cost

TRATTO BD $0 \le z \le L$

$+L-z$ 1 $\boxed{M^{(1)}(z) = L-z}$ $\begin{cases} M_B = L \\ M_D = \phi \end{cases}$

DIAGRAMMA $M^{(1)}(z)$



$$\underbrace{L_{ve}}_{} = \underset{\overset{\shortparallel}{1}}{X_i}\,\eta_i^{(r)} + \sum_j R_j^{(1)} \eta_j^{(r)} =$$

$$-\tfrac{3}{2}qL^2 \qquad -L$$

$$-\varepsilon_C \left[M_C^{(0)} + X M_C^{(1)} \right]$$

$$= 1 \cdot \underbrace{\eta_D^{(r)}}_{-\varepsilon_D X} + \underbrace{R_{y_A}^{(1)}}_{-1} \left(-\eta_A^{0} \right) + \underbrace{M_C^{(1)}}_{-L} \varphi_C^{(r)} =$$

$$= \underbrace{-\varepsilon_D X + \eta_A^{0} - \varepsilon_C L \left[\tfrac{3}{2} qL^2 + L X \right]}_{}$$

$$\underbrace{L_{vi}}_{} = \int_{str} M^{(1)} \frac{M^{(r)}}{EI}\, d str + \int_{str} M^{(1)} \frac{\alpha \Delta T}{h}\, d str =$$

$$= \frac{1}{EI} \int_{str} M^{(1)} M^{(0)}\, d str + \frac{X}{EI} \int_{str} \left[M^{(1)} \right]^2 d str + \frac{\alpha \Delta T}{h} \int_{str} M^{(1)}\, d str =$$

$$= \frac{1}{EI} \left\{ \int_L^{2L} (-L)\left(-\tfrac{3}{2} qL^2\right) dz + \int_0^L (L-z)\, qL^2\, dz \right\} +$$

$$+ \frac{X}{EI} \left\{ \int_L^{2L} L^2\, dz + \int_0^L \overbrace{(L-z)^2}^{L^2+z^2-2Lz} dz \right\} + \frac{\alpha \Delta T}{h} \int_L^{2L} (-L)\, dz =$$

$$= \frac{1}{EI} \left\{ \frac{3}{2} qL^3 \left[z \right]_L^{2L} + qL^3 \left[z \right]_0^L - qL^2 \left[\frac{z^2}{2} \right]_0^L \right\} +$$

$$+ \frac{X}{EI} \left\{ L^2 \left[z \right]_L^{2L} + L^2 \left[z \right]_0^L + \left[\frac{z^3}{3} \right]_0^L - 2L \left[\frac{z^2}{2} \right]_0^L \right\} - \frac{\alpha \Delta T}{h} L \left[z \right]_L^{2L} =$$

$$= \frac{1}{EI} \left\{ \frac{3}{2} qL^4 + qL^4 - \frac{qL^4}{2} \right\} +$$

$$+ \frac{X}{EI} \left\{ L^3 + \cancel{L^3} + \frac{L^3}{3} - \cancel{L^3} \right\} - \frac{\alpha \Delta T}{h} L^2 =$$

$$= \underbrace{\frac{2qL^4}{EI} + \frac{4}{3} L^3 \frac{X}{EI} - \frac{\alpha \Delta T}{h} L^2}_{}$$

$\Rightarrow$ $L_{ve} = L_{vi}$ fornisce

$$-\varepsilon_D X + \eta_A^0 - \bar{\varepsilon}_C L\left[\frac{3}{2}qL^2 + LX\right] = \frac{2qL^4}{EI} + \frac{4}{3}\frac{L^3 X}{EI} - \frac{\alpha\Delta T}{h}L^2$$

$$X\left[\frac{4}{3}\frac{L^3}{EI} + \underbrace{\varepsilon_D}_{\frac{2}{3}\frac{L^3}{EI}} + \underbrace{\bar{\varepsilon}_C}_{\frac{L}{EI}} L^2\right] = \underbrace{\eta_A^0}_{\frac{qL^4}{2EI}} - \underbrace{\bar{\varepsilon}_C}_{\frac{L}{EI}}\frac{3}{2}qL^3 - 2\frac{qL^4}{EI} + \underbrace{\frac{\alpha\Delta T}{h}}_{\frac{2qL^2}{EI}}L^2$$

$$X\frac{\cancel{L^3}}{\cancel{EI}}\underbrace{\left[\frac{4}{3}+\frac{2}{3}+1\right]}_{\cancel{\frac{9}{3}}\ 3} = \frac{q\cancel{L^4}}{\cancel{EI}}\underbrace{\left[\frac{1}{2}-\frac{3}{2}-\cancel{2}+\cancel{2}\right]}_{-1}$$

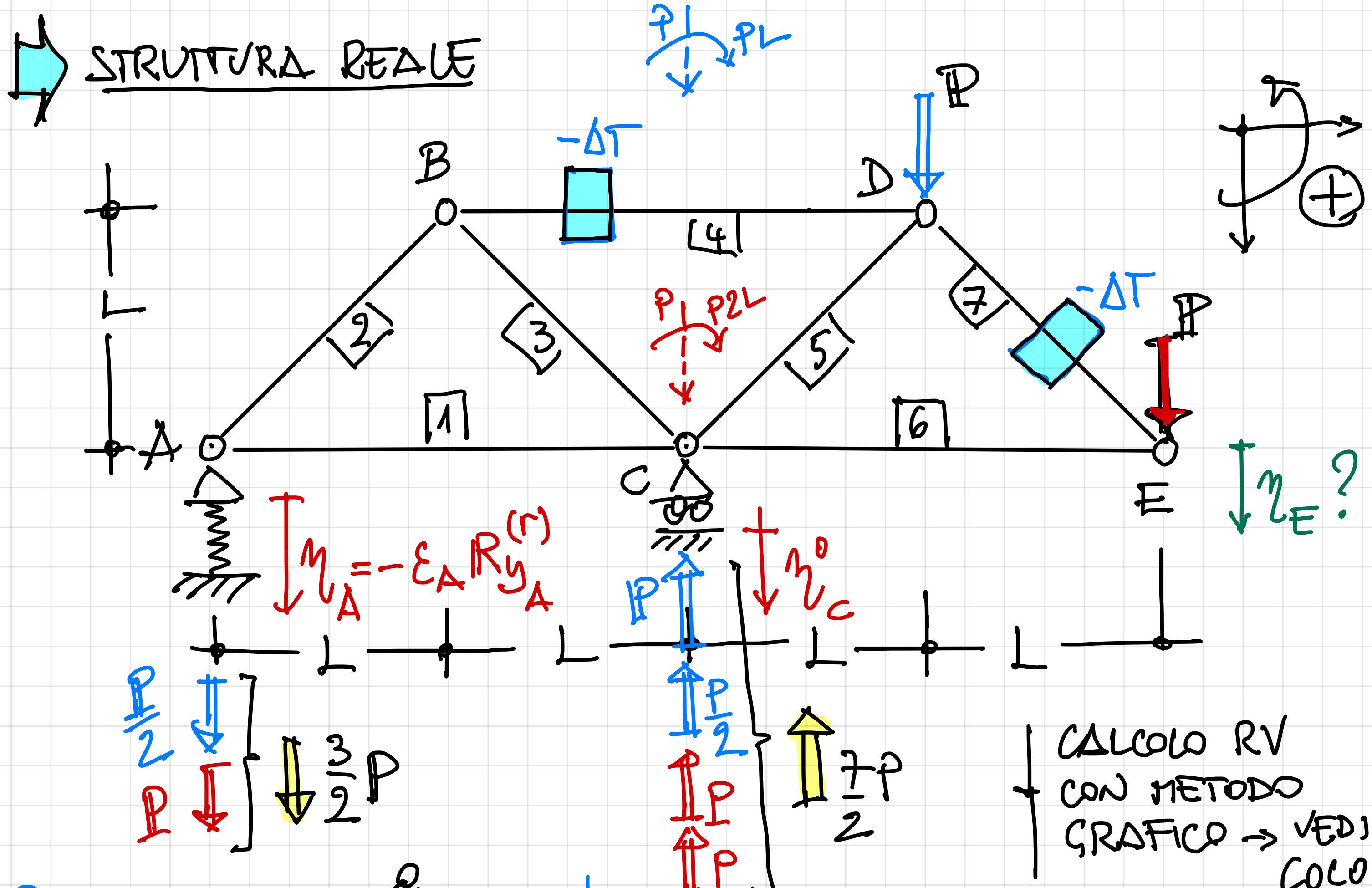
da cui $\boxed{X = -\frac{qL}{3}}$

NEGATIVA!
VERSO OPPOSTO
A QUELLO IPOTIZZATO!

OK! cfr. RV di p. 6

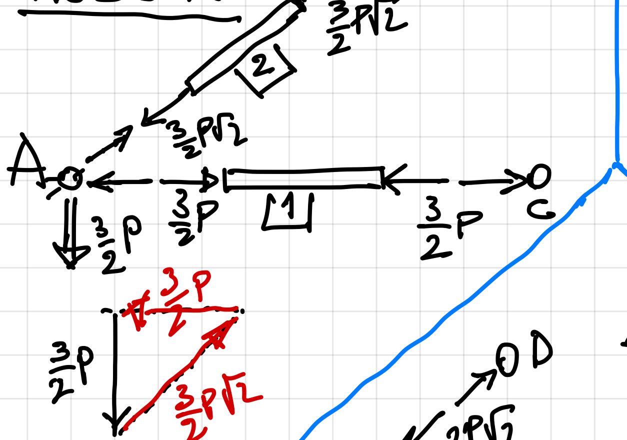
TIPO 1 – ES. #2 SOLUZIONE

STRUTTURA REALE

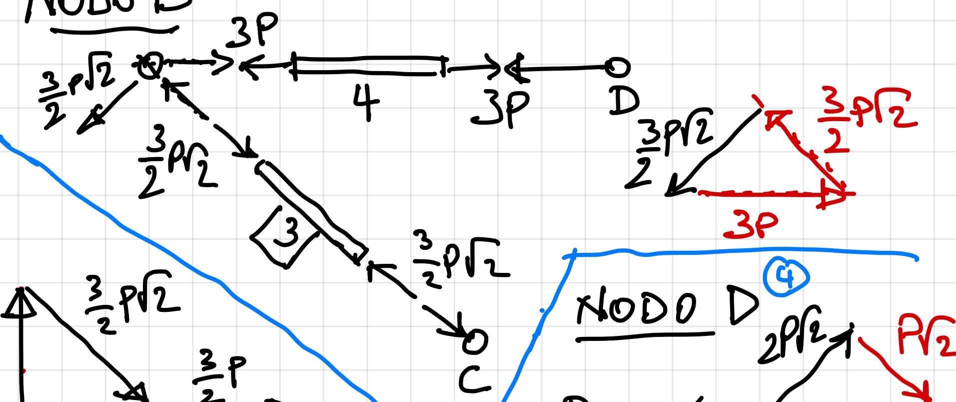


CALCOLO RV CON METODO GRAFICO → VEDI COLORI

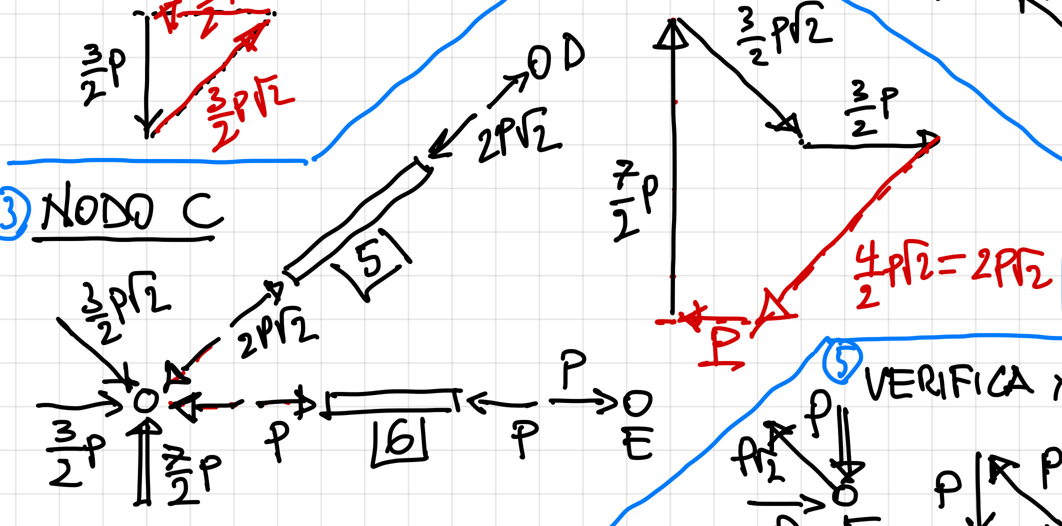
① NODO A



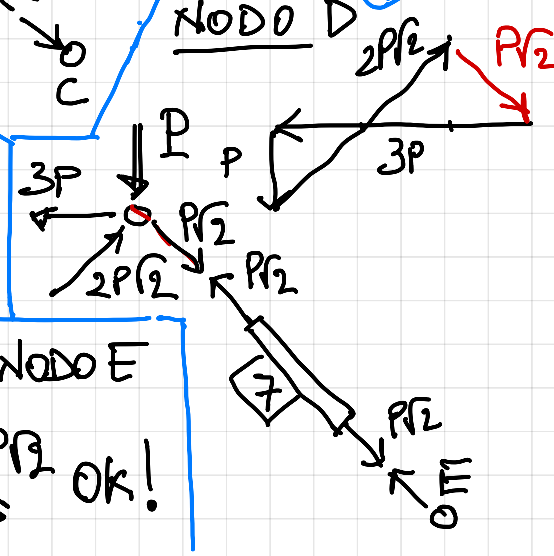
② NODO B



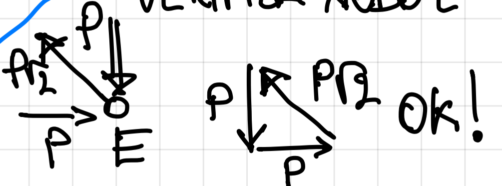
③ NODO C



④ NODO D



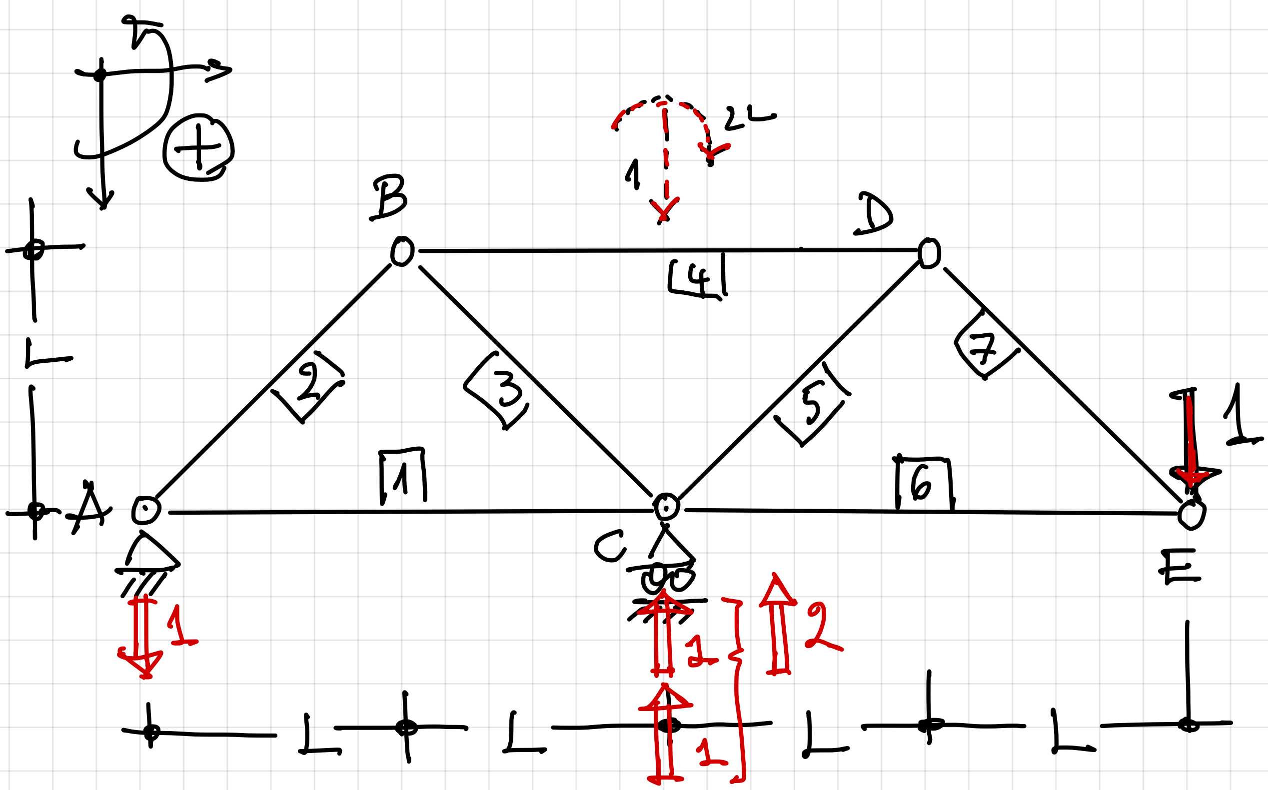
⑤ VERIFICA NODO E

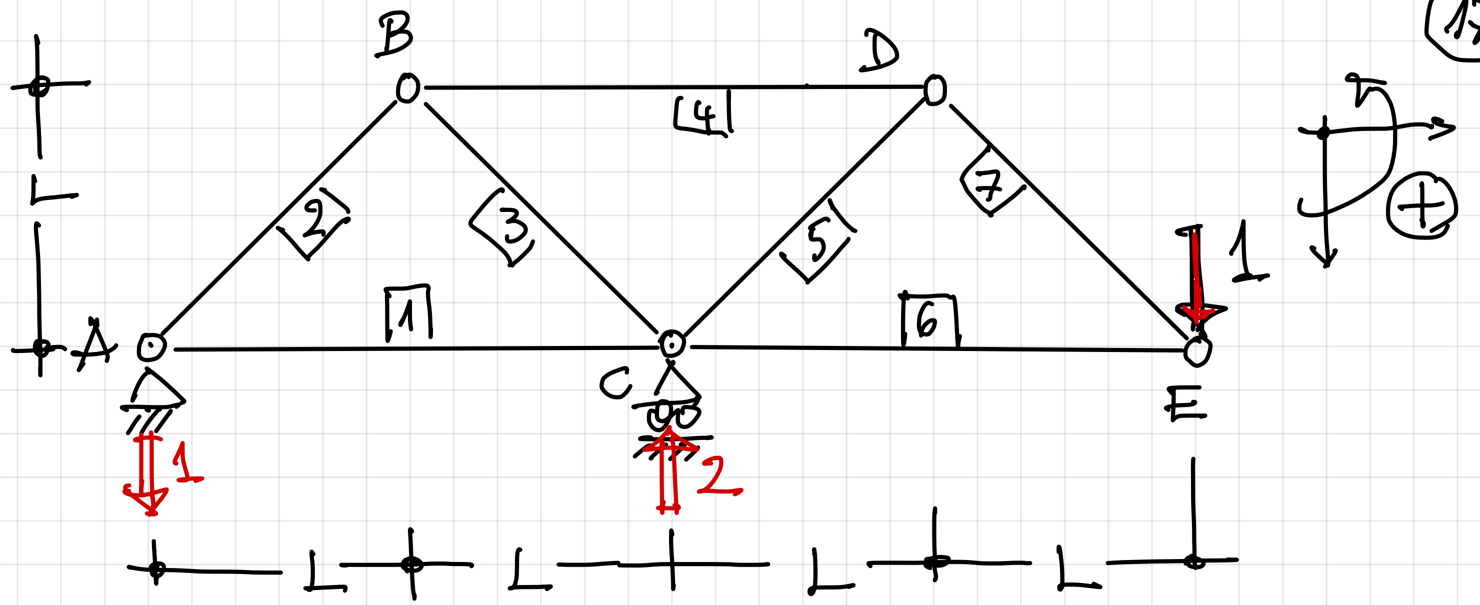


SFORZI NORMALI NELLA STRUTTURA REALE

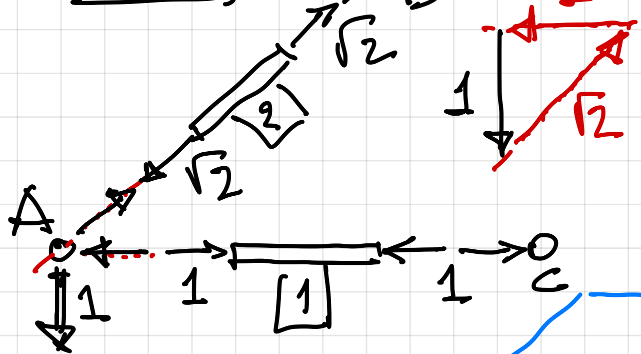
ASTA	$N^{(r)}$	COMPORT. MECCANICO
1	$-\frac{3}{2}P$	puntone
2	$\frac{3}{2}P\sqrt{2}$	tirante
3	$-\frac{3}{2}P\sqrt{2}$	puntone
4	$3P$	tirante
5	$-2P\sqrt{2}$	puntone
6	$-P$	" "
7	$P\sqrt{2}$	tirante

STRUTTURA FITTIZIA PER IL CALCOLO DELLO SPOSTAMENTO VERTICALE DELLA SEZ. E

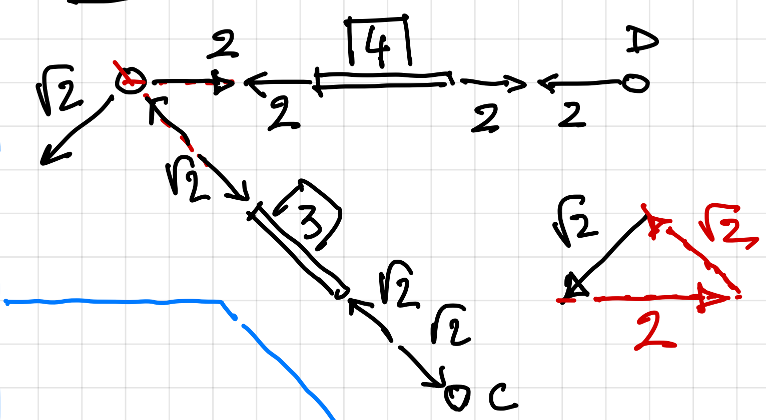




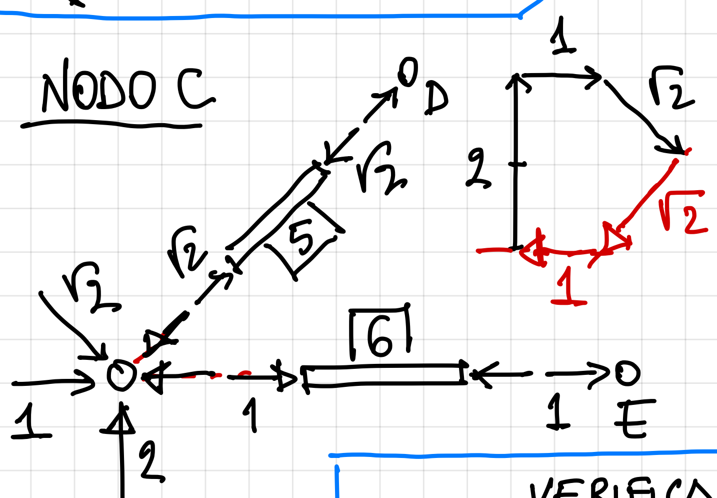
NODO A



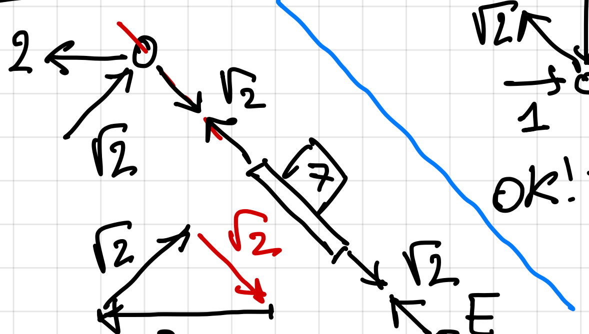
NODO B



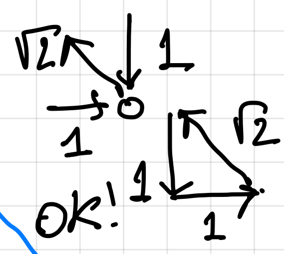
NODO C



NODO D



VERIFICA AL NODO E



SFORZI NORMALI NELLA STR. FITT.

ASTA	$N^{(f)}$	COMP. MECCANICO
1	-1	puntone
2	$\sqrt{2}$	tirante
3	$-\sqrt{2}$	puntone
4	2	tirante
5	$-\sqrt{2}$	puntone
6	-1	" "
7	$\sqrt{2}$	tirante

$$L_{ve} = 1\cdot\eta_E + \sum_j R_j^{(1)}\eta_j^{(r)} = \eta_E + \underbrace{R_{y_A}^{(1)}}_{1}\underbrace{\eta_A^{(r)}}_{-\varepsilon_A \underbrace{R_{y_A}^{(r)}}_{\frac{3}{2}P}} + \underbrace{R_{y_C}^{(1)}}_{-2}\underbrace{\eta_C^0}_{+\eta_C^0} =$$

$$= \eta_E - \frac{3}{2}P\varepsilon_A - 2\eta_C^0$$

$$L_{vi} = \sum_i N_i^{(f)}\frac{N_i^{(r)}L_i}{EA} + \underbrace{\sum_j N_j^{(f)}\alpha\Delta T_j L_j}_{\text{ASTE \#4 e \#7}} =$$

$$= (-1)\left(-\frac{3}{2}P\right)\frac{2L}{EA} + \sqrt{2}\left(\frac{3}{2}P\sqrt{2}\right)\frac{L\sqrt{2}}{EA} + \left(-\sqrt{2}\right)\left(-\frac{3}{2}P\sqrt{2}\right)\frac{L\sqrt{2}}{EA} +$$

$$+ 2(3P)\frac{2L}{EA} + \left(-\sqrt{2}\right)\left(-2P\sqrt{2}\right)\frac{L\sqrt{2}}{EA} + (-1)(-P)\frac{2L}{EA} +$$

$$+ \sqrt{2}\left(P\sqrt{2}\right)\frac{L\sqrt{2}}{EA} + 2(-\alpha\Delta T)2L + \sqrt{2}(-\alpha\Delta T)L\sqrt{2} =$$

$$= \frac{PL}{EA}\left[\underline{3} + 3\sqrt{2} + 3\sqrt{2} + \underline{12} + 4\sqrt{2} + \underline{2} + 2\sqrt{2}\right] +$$

$$- \alpha\Delta T\left[4L + 2L\right] =$$

$$= \frac{PL}{EA}\left[17 + 12\sqrt{2}\right] - 6L\underbrace{\alpha\Delta T}_{=\frac{2\sqrt{2}P}{EA}}$$

$L_{ve} = L_{vi}$ fornisce

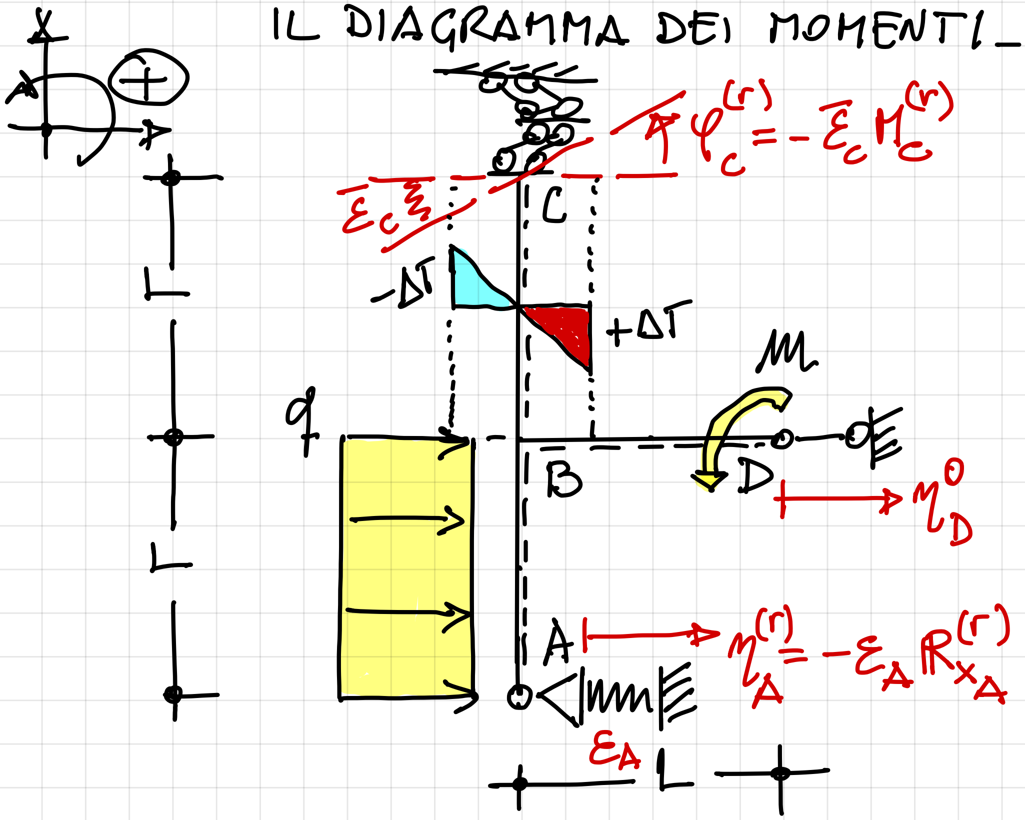
$$\eta_E - \frac{3}{2}P\underbrace{\varepsilon_A}_{\frac{L}{3EA}} - 2\underbrace{\eta_C^0}_{\frac{PL}{4EA}} = \frac{PL}{EA}\left[17 + \cancel{12\sqrt{2}}\right] - \cancel{6L\underbrace{\alpha\Delta T}_{=\frac{2\sqrt{2}P}{EA}}}$$

da cui $\boxed{\eta_E = \frac{PL}{EA}\left[\frac{1}{2} + \frac{1}{2} + 17\right] = 18\frac{PL}{EA}}$ POSITIVO! VERSO IL BASSO!

MECCANICA delle STRUTTURE – P. FUSCHI
PROVA SCRITTA del 15 GENNAIO 2025

TIPO 2

ES. #1 RISOLVERE LA STRUTTURA UNA VOLTA IPERSTATICA SEGUENTE DETERMINANDO IL DIAGRAMMA DEI MOMENTI.



Posizioni:

$$|\mathcal{M}| = qL^2$$

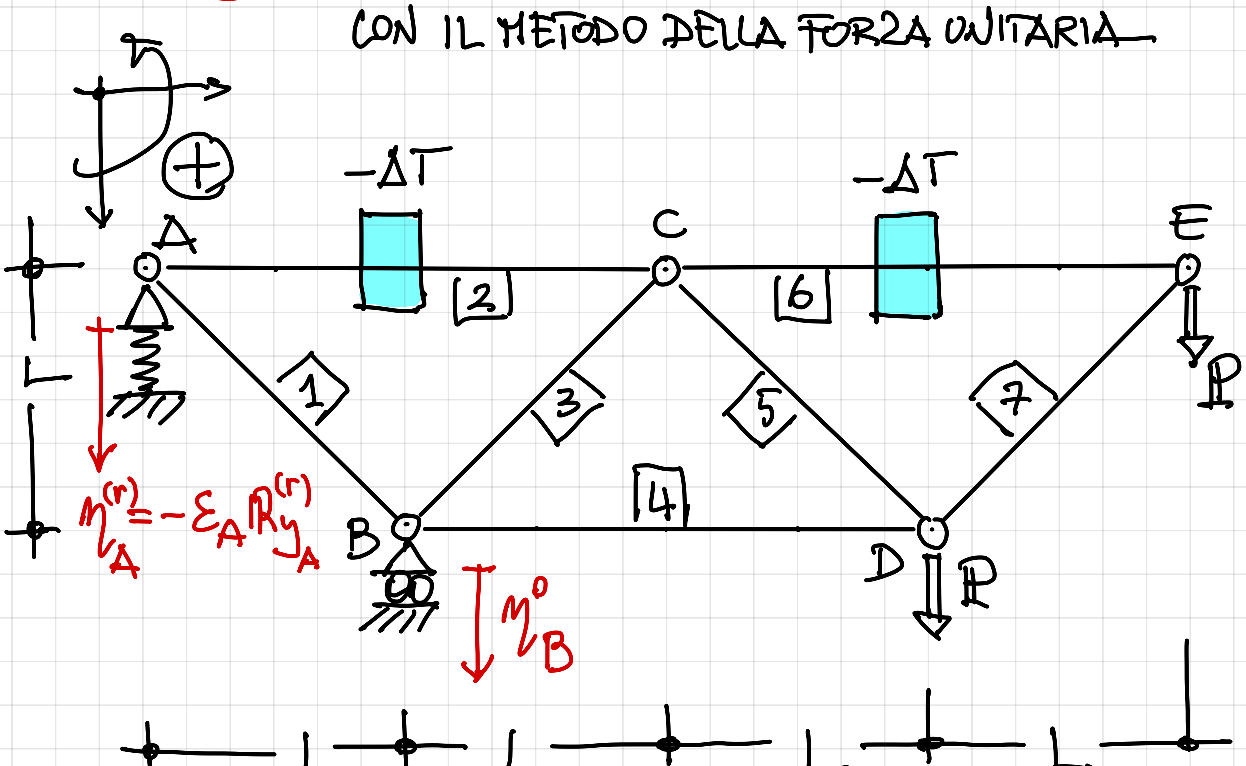
$$|\eta_D^0| = \frac{qL^4}{2EI}$$

$$|\mathcal{E}_c| = \frac{L}{EI}$$

$$|\mathcal{E}_A| = \frac{2}{3}\frac{L^3}{EI}$$

$$\left|\frac{\alpha \Delta T}{h}\right| = \frac{13}{8}\frac{qL^2}{EI}$$

ES. #2 DETERMINARE LO SPOST. VERTICALE DELLA SEZ. E CON IL METODO DELLA FORZA UNITARIA.



Posizioni:

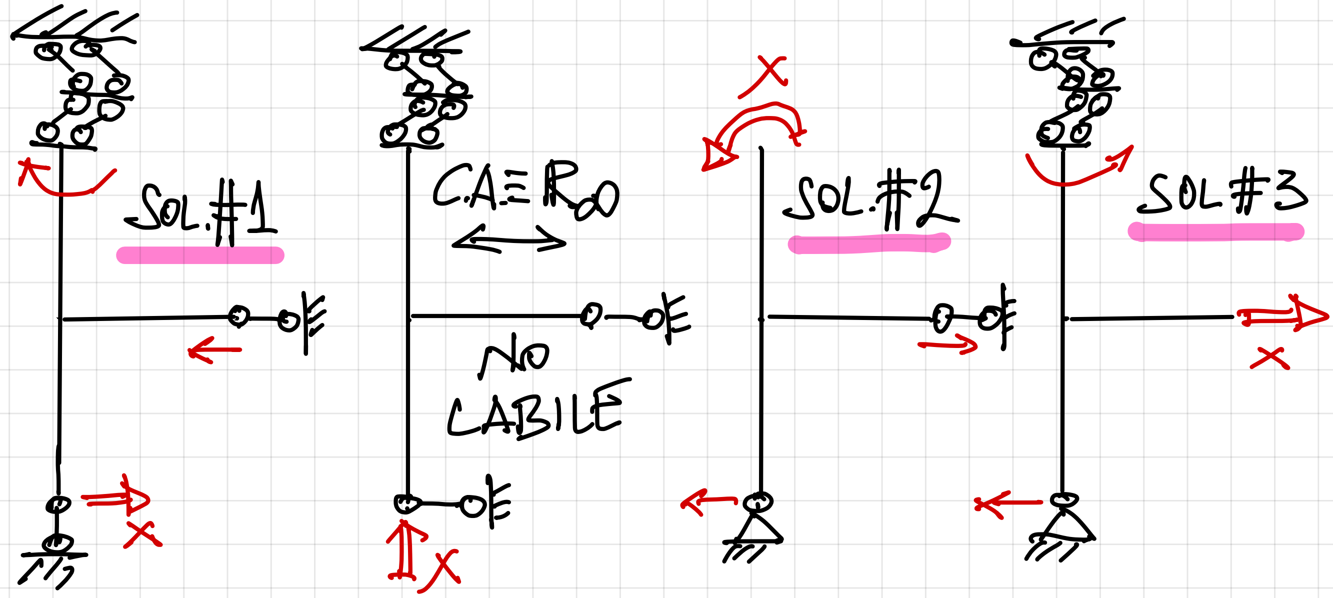
$$|\eta_B^0| = \frac{PL}{4EA}$$

$$|\mathcal{E}_A| = \frac{L}{15EA}$$

$$|\alpha \Delta T| = 5\sqrt{2}\frac{P}{EA}$$

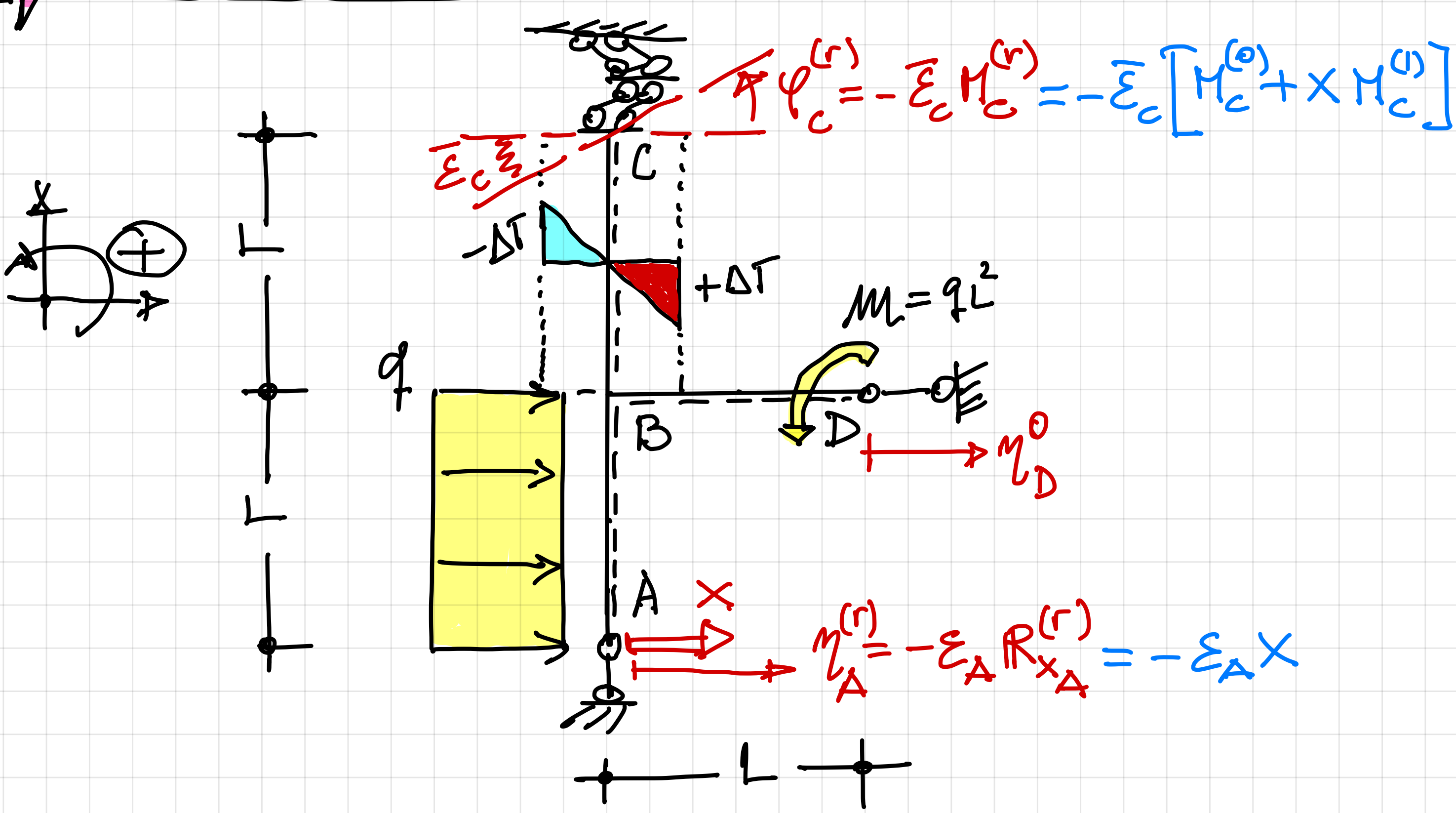
TIPO 2 - ES.#1 SOLUZIONI

SOLUZIONI POSSIBILI:



SOLUZIONE #1

SISTEMA PRINCIPALE ISOSTATICO



SCHEMA [0] SOLO CARICHI ESTERNI

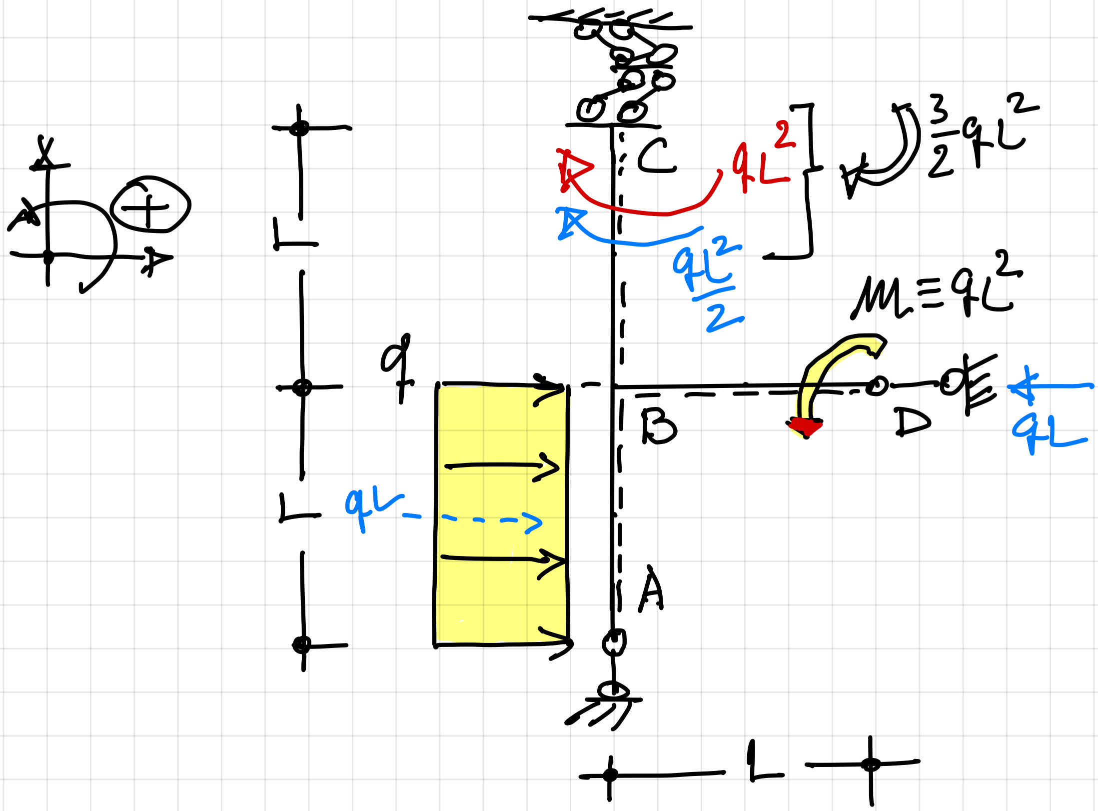
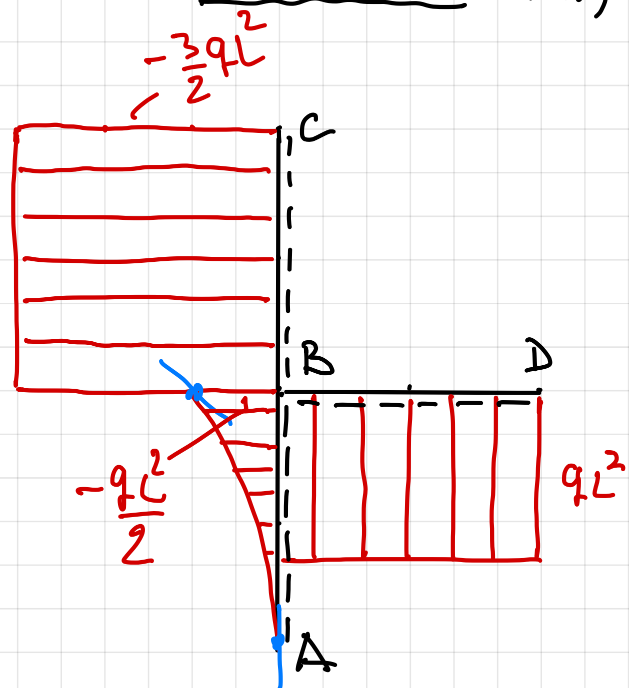
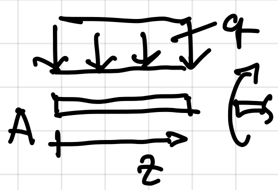


DIAGRAMMA $M^{(0)}(z)$



TRATTO AB $0 \leq z \leq L$

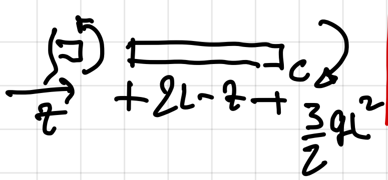


$$\boxed{M^{(0)}(z) = -\frac{q z^2}{2}}$$

$M_A = \phi$

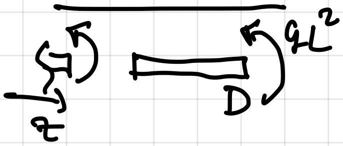
$M_B = -\frac{qL^2}{2}$

TRATTO BC $L \leq z \leq 2L$

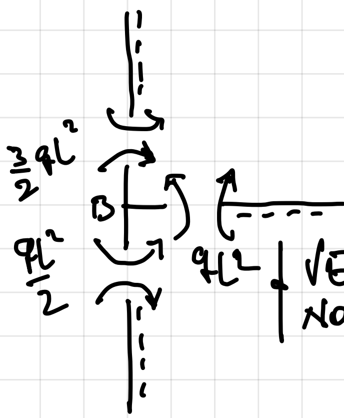


$$\boxed{M^{(0)}(z) = -\frac{3}{2}qL^2}$$ cost.

TRATTO BD $0 \leq z \leq L$

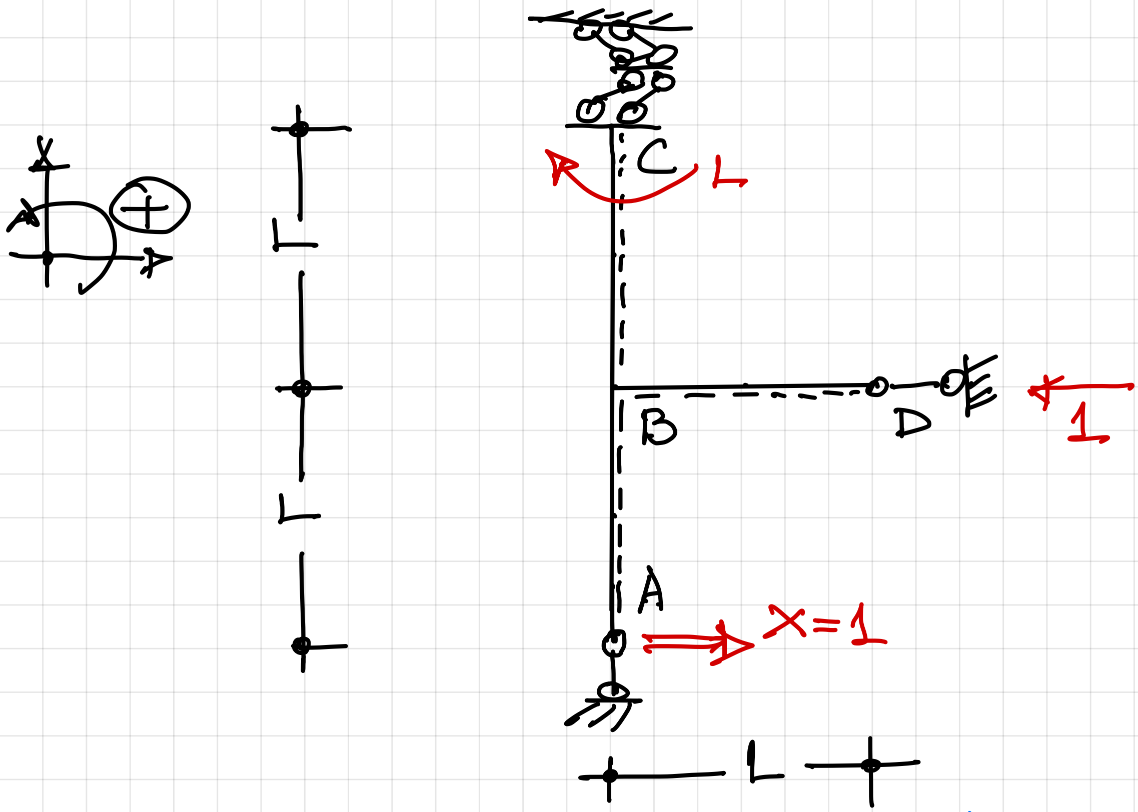


$$\boxed{M^{(0)}(z) = qL^2}$$ cost.



VERIFICA AL NODO TRIPLO B

SCHEMA [1] SOLO X = 1



TRATTO AB $0 \le z \le L$

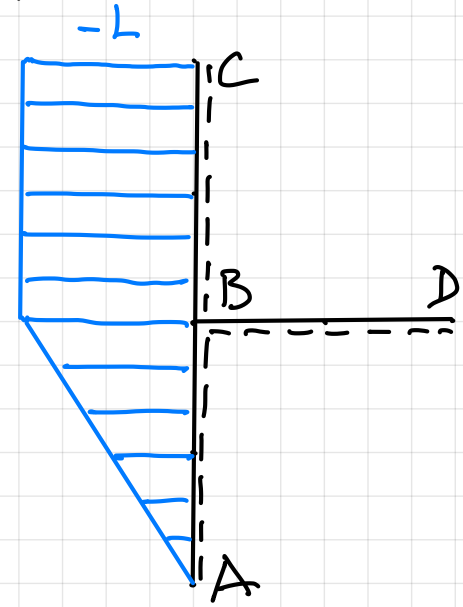
$$\boxed{M^{(1)}(z) = -z} \qquad M_A = \phi, \quad M_B = -L$$

TRATTO BC $L \le z \le 2L$

$$\boxed{M^{(1)}(z) = -L} \quad \text{cost.}$$

TRATTO BD $0 \le z \le L$

SCARICO $\boxed{M^{(1)}(z) = \phi}$



$$L_{ve} = X_i\,\eta_i^{(r)} + \sum_j R_j^{(1)}\,\eta_j^{(r)} = 1 \cdot \underbrace{\eta_A^{(r)}}_{-\varepsilon_A X} + \underbrace{R_{X_D}^{(1)}}_{-1}\,\underbrace{\eta_D}_{+\eta_D^0} + \overbrace{M_c^{(1)}}^{-L}\,\underbrace{\varphi_c^{(r)}}_{-\bar{\varepsilon}_c\left[M_c^{(0)} + X M_c^{(1)}\right]} =$$

$$M_c^{(0)} = -\frac{3}{2}qL^2 \qquad M_c^{(1)} = -L$$

$$= -\varepsilon_A X - \eta_D^0 - L\,\bar{\varepsilon}_c\left[\frac{3}{2}qL^2 + XL\right]$$

$$L_{vi} = \int_{str} M^{(1)} \frac{\overset{M^{0}+M^{(1)}X}{M^{(T)}}}{EI} \, d\,str + \int_{str} M^{(1)} \frac{\alpha \Delta T}{h} \, d\,str =$$

$$= \frac{1}{EI} \int_{str} M^{(1)} M^{(0)} \, d\,str + \int_{str} \left[M^{(1)}\right]^2 d\,str + \frac{\alpha \Delta T}{h} \int_{str} M^{(1)} \, d\,str =$$

$$= \frac{1}{EI} \left[\int_0^L (-z)\left(-\frac{qz^2}{2}\right) dz + \int_L^{2L} (-L)\left(-\frac{3}{2} qL^2\right) dz \right] +$$

$$+ \frac{X}{EI} \left[\int_0^L z^2 \, dz + \int_L^{2L} L^2 \, dz \right] + \frac{\alpha \Delta T}{h} \int_L^{2L} -L \, dz =$$

$$= \frac{1}{EI} \left[\frac{q}{2} \left[\frac{z^4}{4}\right]_0^L + \frac{3}{2} qL^3 \left[z\right]_L^{2L} \right] +$$

$$+ \frac{X}{EI} \left[\left[\frac{z^3}{3}\right]_0^L + L^2 \left[z\right]_L^{2L} \right] - \frac{\alpha \Delta T}{h} L \left[z\right]_L^{2L} =$$

$$= \frac{1}{EI} \left[\frac{qL^4}{8} + \frac{3}{2} qL^4 \right] + \frac{X}{EI} \left[\frac{L^3}{3} + L^3 \right] - \frac{\alpha \Delta T}{h} L^2 =$$

$$= \underbrace{\frac{qL^4}{EI} \frac{13}{8} + \frac{X L^3}{EI} \frac{4}{3} - \frac{\alpha \Delta T}{h} L^2}$$

$L_{ve} = L_{vi}$ fornisce

$$-\varepsilon_A X - \eta_D^0 - L \varepsilon_C \left[\frac{3}{2} qL^2 + XL \right] =$$

$$= \frac{qL^4}{EI} \frac{13}{8} + \frac{X L^3}{EI} \frac{4}{3} - \frac{\alpha \Delta T}{h} L^2$$

$$X\left[\frac{4}{3}\frac{L^3}{EI}+\underbrace{\varepsilon_A}_{\frac{2}{3}\frac{L^3}{EI}}+\underbrace{\bar{\varepsilon}_c}_{\frac{L}{EI}}L^2\right]=-\underbrace{\eta_D^0}_{\frac{qL^4}{2EI}}-\frac{3}{2}qL^3\underbrace{\bar{\varepsilon}_c}_{\frac{L}{EI}}-\cancel{\frac{qL^4}{EI}\frac{13}{8}}+\cancel{\underbrace{\alpha\frac{\Delta T}{h}}_{\frac{13}{8}\frac{qL^2}{EI}}L^2}$$

$$\cancel{\frac{L^3}{EI}}X\underbrace{\left[\frac{4}{3}+\frac{2}{3}+1\right]}_{3}=-\cancel{\frac{qL^4}{EI}}\underbrace{\left[\frac{1}{2}+\frac{3}{2}\right]}_{2}$$

$$\boxed{X=-\frac{2}{3}qL}\Rightarrow$$ NEGATIVA!

VERSO OPPOSTO A QUELLO IPOTIZZATO

SOLUZIONE SISTEMA PRINCIPALE ISOSTATICO

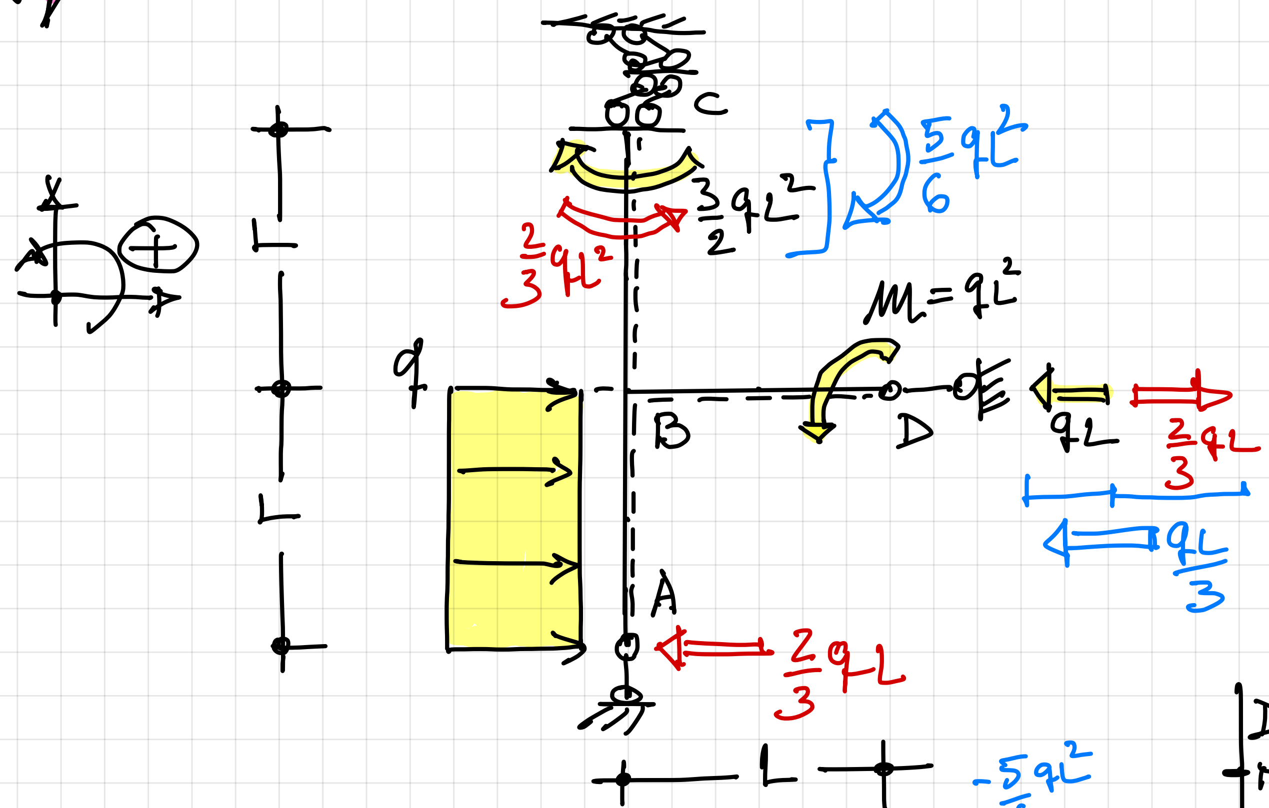
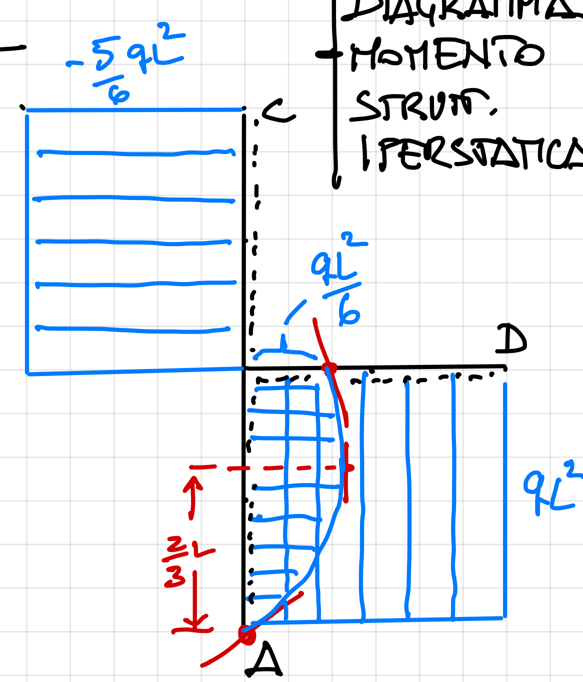
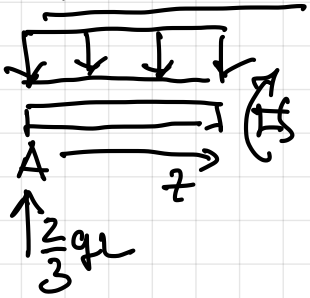


DIAGRAMMA MOMENTO STRUTT. IPERSTATICA



TRATTO AB $0 \le z \le L$

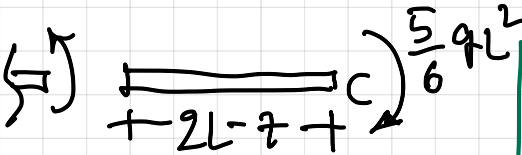


$$\boxed{M^{(r)}(z) = \frac{2}{3}qL\cdot z - \frac{qz^2}{2}}$$

$$M_A = \phi$$

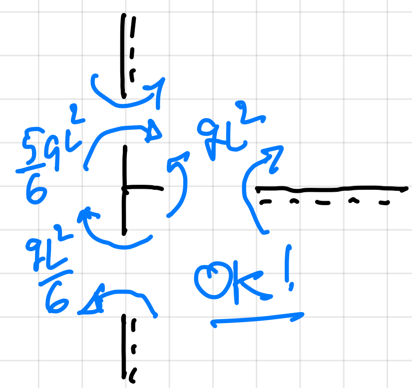
$$M_B = \frac{qL^2}{6}$$

TRATTO BC $L \le z \le 2L$

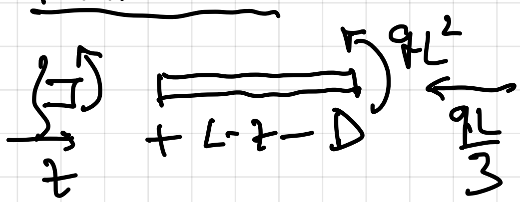


$$\boxed{M^{(r)}(z) = -\frac{5}{6}qL^2} \quad \text{cost.}$$

VERIFICA AL NODO TRIPLO B



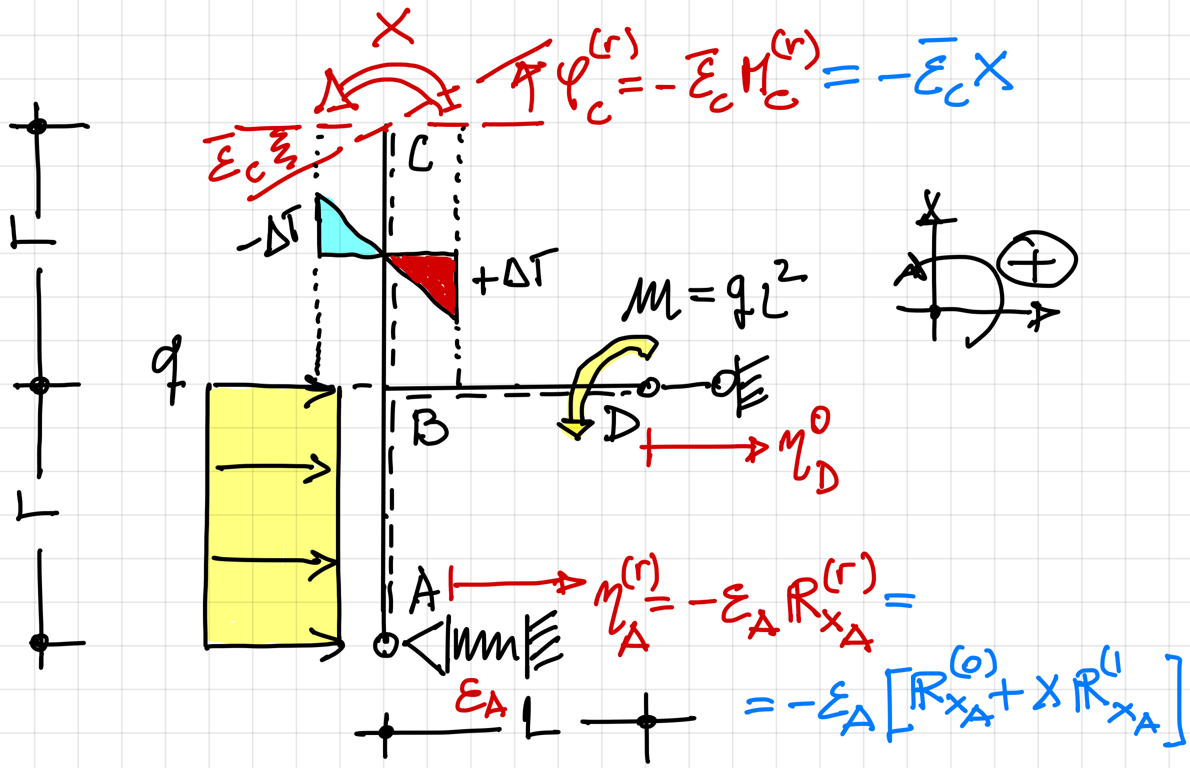
TRATTO BD $0 \le z \le L$



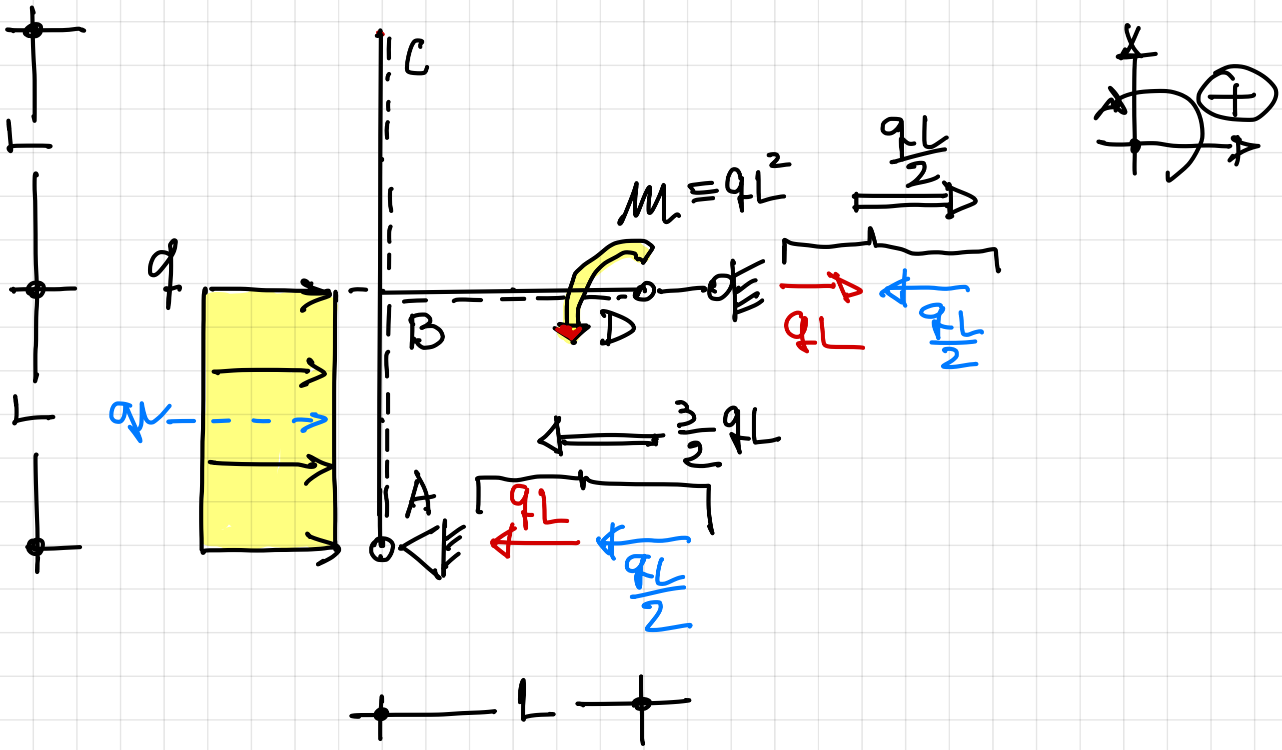
$$\boxed{M^{(r)}(z) = qL^2} \quad \text{cost.}$$

SOLUZIONE #2

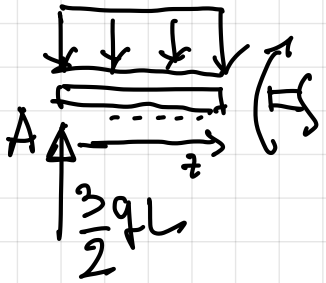
SISTEMA PRINCIPALE ISOSTATICO



SCHEMA [0] SOLO CARICHI ESTERNI



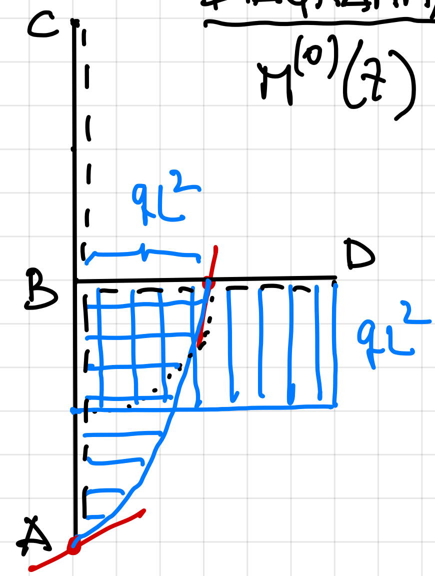
TRATTO AB $0 \le z \le L$



$$\boxed{M^{(0)}(z) = \frac{3}{2} qL \cdot z - q\frac{z^2}{2}}$$

$$\begin{cases} M_A = \emptyset \\ M_B = qL^2 \end{cases}$$

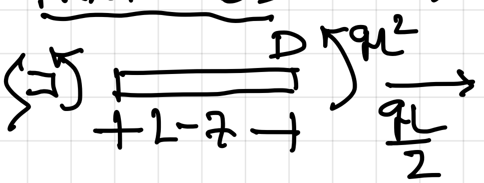
DIAGRAMMA $M^{(0)}(z)$



TRATTO BC $L \le z \le 2L$

SCARICO $\boxed{M^{(0)}(z) = \emptyset}$

TRATTO BD $0 \le z \le L$



$\boxed{M^{(0)}(z) = qL^2}$ cost.

SCHEMA [1] SOLO X = 1

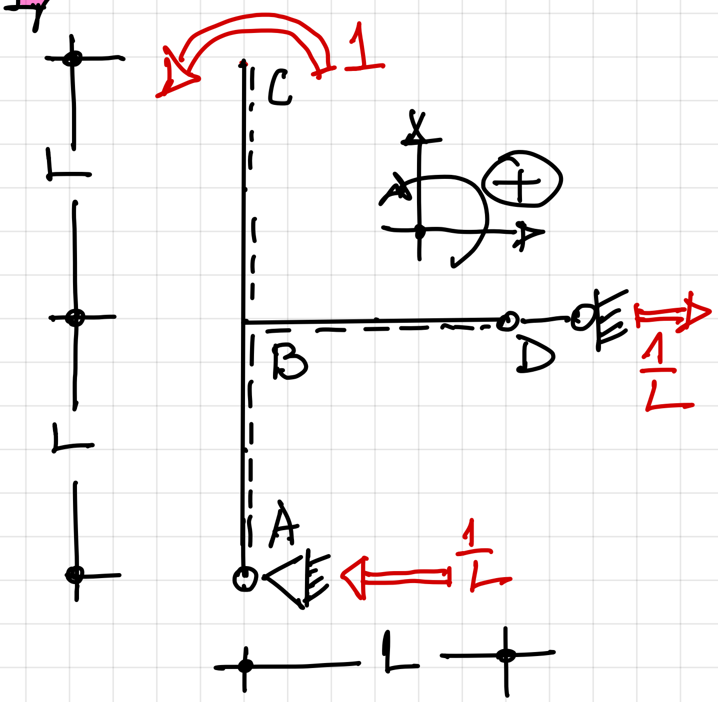
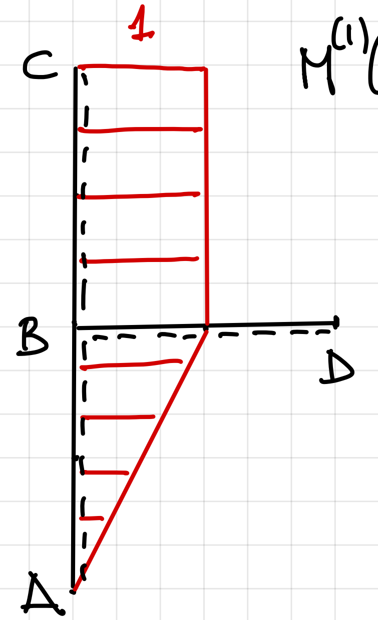
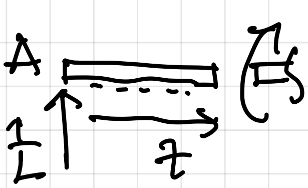


DIAGRAMMA $M^{(1)}(z)$

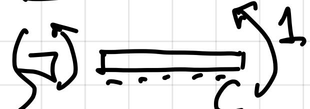


TRATTO AB $0 \le z \le L$



$\boxed{M^{(1)}(z) = \frac{z}{L}}$ $\begin{cases} M_A = \emptyset \\ M_B = 1 \end{cases}$

TRATTO BC $L \le z \le 2L$



$\boxed{M^{(1)}(z) = 1}$ cost.

TRATTO BD

SCARICO!

$\boxed{M^{(1)}(z) = \emptyset}$

$$L_{ve} = \underset{\overset{\parallel}{1}}{X_i}\,\eta_i^{(r)} + \sum_j R_j^{(1)}\,\eta_j^{(r)} =$$

$$= -\bar{\varepsilon}_c X + \underbrace{R_{x_A}^{(1)}}_{-\frac{1}{L}}\,\underbrace{\eta_A^{(r)}}_{\hookrightarrow\, -\varepsilon_A\left[\underbrace{R_{x_A}^{(0)}}_{-\frac{3}{2}qL} + X\,\underbrace{R_{x_A}^{(1)}}_{-\frac{1}{L}}\right]} + \overbrace{R_{x_D}^{(1)}}^{\frac{1}{L}}\,\overbrace{\eta_D^{0}}^{+\eta_D^0} =$$

$$= -\bar{\varepsilon}_c X - \frac{\varepsilon_A}{L}\left[\frac{3}{2}qL + \frac{X}{L}\right] + \frac{\eta_D^0}{L}$$

$$L_{vi} = \int_{Str} M^{(1)}\frac{M^{(r)}}{EI}\,ds + \int_{Str} M^{(1)}\,\frac{\alpha\,\Delta T}{h}\,ds =$$

$$= \frac{1}{EI}\int_{Str} M^{(1)}M^{(0)}\,ds + \frac{X}{EI}\int_{Str}\left[M^{(1)}\right]^2 ds + \frac{\alpha\,\Delta T}{h}\int_{Str} M^{(1)}\,ds =$$

$$= \frac{1}{EI}\int_0^L \frac{z}{L}\left[\frac{3}{2}qL\cdot z - \frac{qz^2}{2}\right]dz + \qquad \frac{3}{2}qz^2 - \frac{qz^3}{2L}$$

$$+ \frac{X}{EI}\left[\int_0^L\left(\frac{z}{L}\right)^2 dz + \int_L^{2L} 1\cdot dz\right] + \frac{\alpha\,\Delta T}{h}\int_L^{2L} 1\,dz =$$

$$= \frac{1}{EI}\left\{\frac{3}{2}q\left[\frac{z^3}{3}\right]_0^L - \frac{q}{2L}\left[\frac{z^4}{4}\right]_0^L\right\} +$$

$$+ \frac{X}{EI}\left\{\frac{1}{L^2}\cdot\left[\frac{z^3}{3}\right]_0^L + \left[z\right]_L^{2L}\right\} + \frac{\alpha\,\Delta T}{h}\left[z\right]_L^{2L} =$$

$$= \frac{1}{EI}\left[\frac{q}{2}L^3 - \frac{q}{8}L^3\right] + \frac{X}{EI}\left[\frac{L}{3} + L\right] + \alpha\,\frac{\Delta T}{h}\cdot L =$$

$$= \frac{qL^3}{EI}\,\frac{3}{8} + \frac{XL}{EI}\,\frac{4}{3} + \frac{\alpha\,\Delta T}{h}\,L$$

$\Rightarrow$ $L_{ve} = L_{vi}$ fornisce

$$-\bar{\varepsilon}_c X - \frac{\varepsilon_A}{L}\left[\frac{3}{2}qL + \frac{X}{L}\right] + \frac{\eta_D^0}{L} =$$

$$= \frac{qL^3}{EI}\frac{3}{8} + \frac{XL}{EI}\frac{4}{3} + \frac{\alpha\Delta T}{h}L$$

$$X\left[\frac{4}{3}\frac{L}{EI} + \underbrace{\bar{\varepsilon}_c}_{\frac{L}{EI}} + \frac{\overbrace{\varepsilon_A}^{\frac{2}{3}\frac{L^3}{EI}}}{L^2}\right] = -\overbrace{\varepsilon_A}^{\frac{2}{3}\frac{L^3}{EI}}\frac{3}{2}q - \frac{qL^3}{EI}\frac{3}{8} - \underbrace{\frac{\alpha\Delta T}{h}}_{\frac{13}{8}\frac{qL^2}{EI}}L + \frac{\overbrace{\eta_D^0}^{\frac{qL^4}{2EI}}}{L}$$

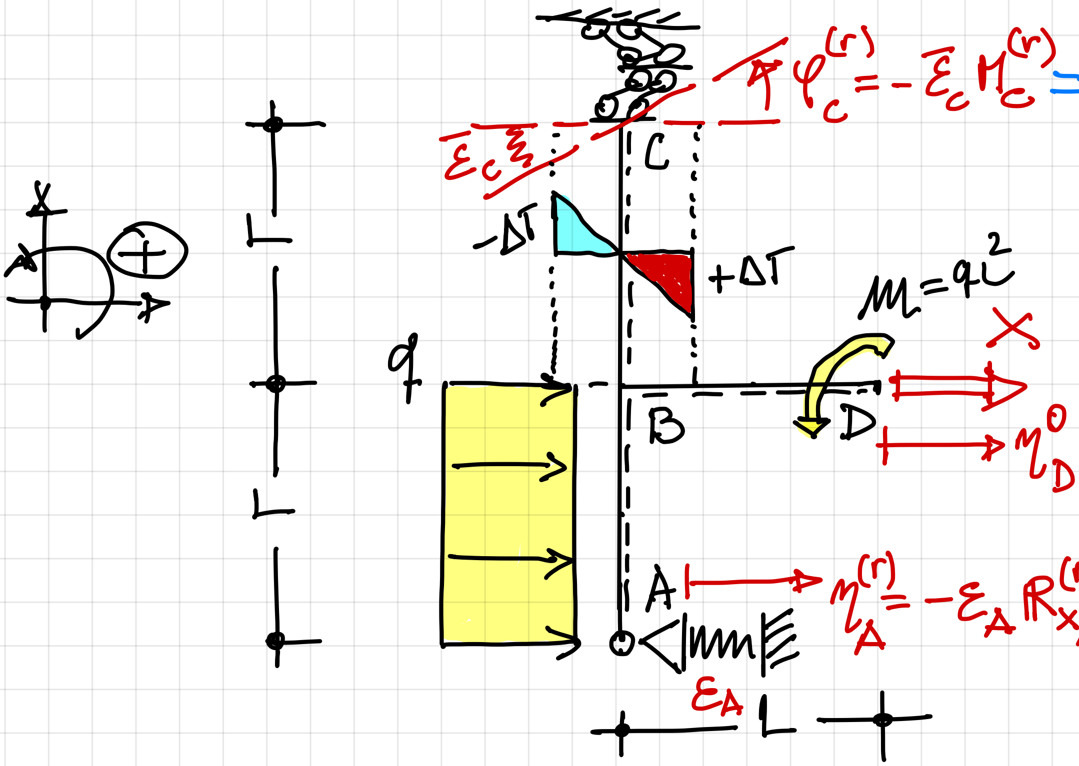
$$X\frac{\cancel{L}}{\cancel{EI}}\underbrace{\left[\frac{4}{3} + 1 + \frac{2}{3}\right]}_{=3} = -\frac{qL^{\cancel{3}\,2}}{\cancel{EI}}\underbrace{\left\{1 + \frac{3}{8} + \frac{13}{8} - \frac{1}{2}\right\}}_{\frac{5}{2}}$$

$$\boxed{X = -\frac{5}{6}qL^2}$$

$\Rightarrow$ NEGATIVA ORARIA! OK cfr. RV di p. VI

SOLUZIONE #3

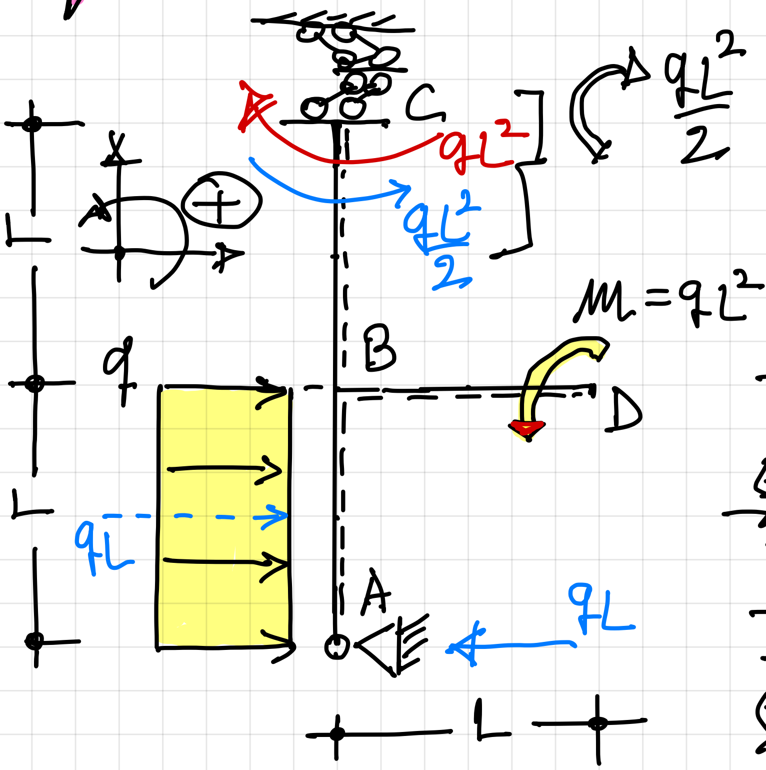
SISTEMA PRINCIPALE ISOSTATICO



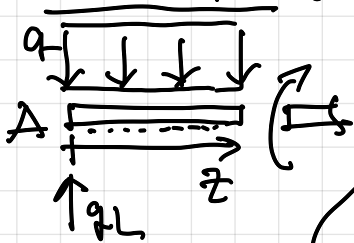
$$\varphi_C^{(r)} = -\varepsilon_C M_C^{(r)} = -\varepsilon_C\left[M_C^{(0)} + X M_C^{(1)}\right]$$

$$\eta_A^{(r)} = -\varepsilon_A R_{x_A}^{(r)} = -\varepsilon_A\left[R_{x_A}^{(0)} + X R_{x_A}^{(1)}\right]$$

SCHEMA [0] SOLO CARICHI ESTERNI



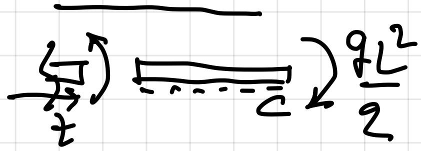
TRATTO AB $0 \le z \le L$



$$\boxed{M^{(0)}(z) = qL\cdot z - \frac{qz^2}{2}}$$

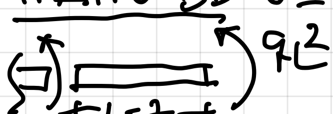
$$\begin{cases} M_A = \phi \\ M_B = \dfrac{qL^2}{2} \end{cases}$$

TRATTO BC $L \le z \le 2L$



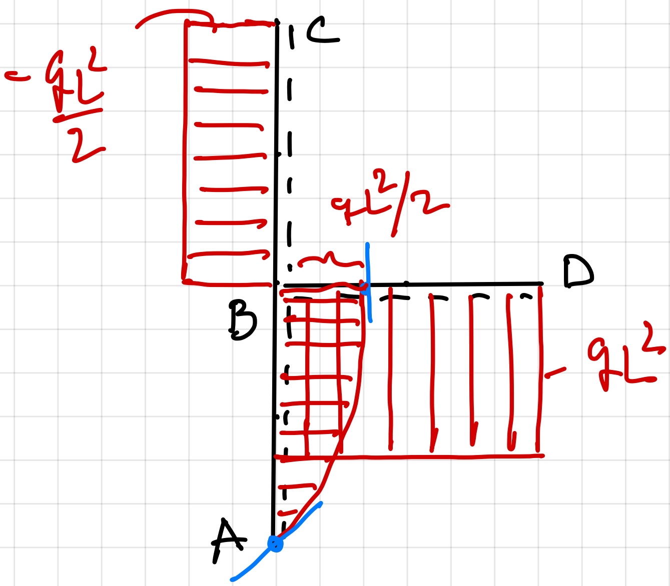
$$\boxed{M^{(0)}(z) = -\frac{qL^2}{2}}$$ cost.

TRATTO BD $0 \le z \le L$

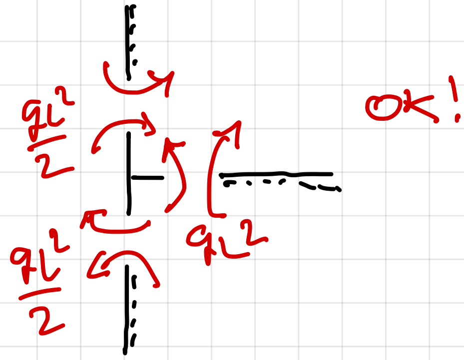


$$\boxed{M^{(0)}(z) = qL^2}$$ cost

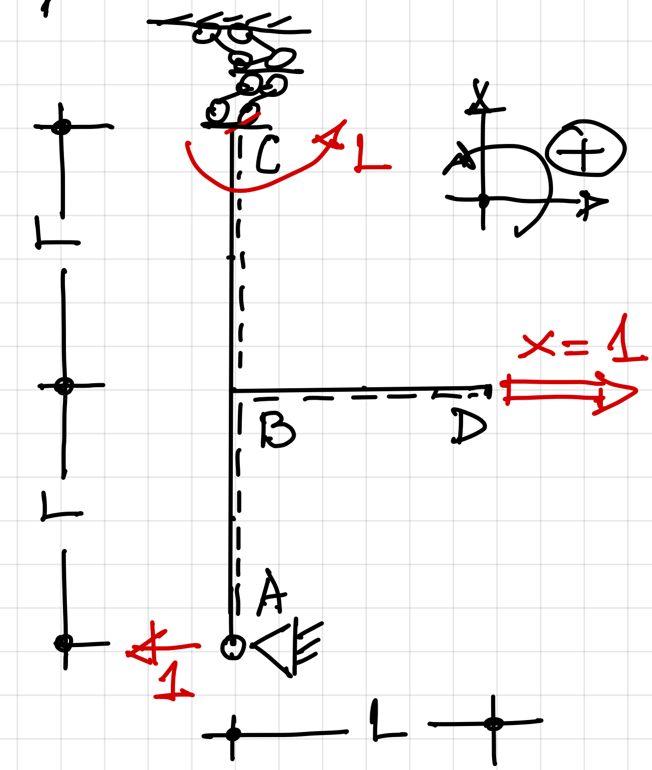
DIAGRAMMA $M^{(0)}(z)$



VERIFICA AL NODO TRIPLO B



SCHEMA [1] SOLO $X=1$



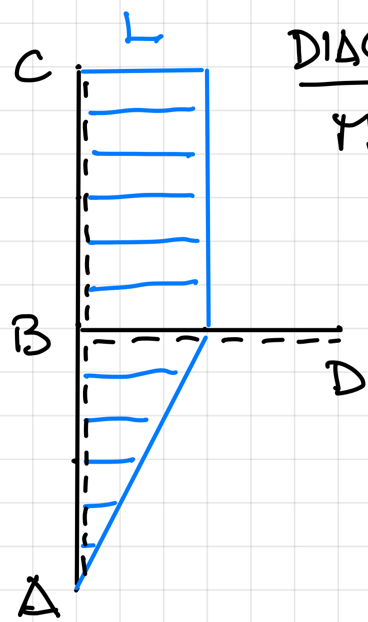
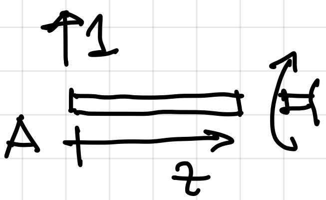


DIAGRAMMA $M^{(1)}(z)$

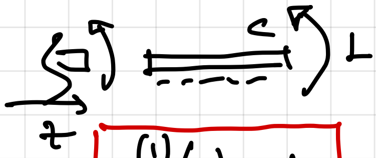
TRATTO AB $0 \le z \le L$



$\boxed{M^{(1)}(z) = z}$

$M_A = \phi$
$M_B = L$

TRATTO BC $L \le z \le 2L$



$\boxed{M^{(1)}(z) = L}$ cost.

TRATTO BD SCARICO

$\boxed{M^{(1)}(z) = \phi}$

$$\underbrace{L_{ve}} = \underset{\overset{\parallel}{1}}{X_i}\,\eta_i^{(r)} + \sum_j R_j^{(1)}\eta_j^{(r)} =$$

$$= 1\cdot\eta_D^{0} + \underbrace{R_{X_A}^{(1)}}_{-1}\,\underbrace{\eta_A^{(r)}}_{-\varepsilon_A\left[\underbrace{R_{X_A}^{(0)}}_{-qL} + X\underbrace{R_{X_A}^{(1)}}_{-1}\right]} + \overbrace{M_c^{(1)}}^{L}\,\overbrace{\varphi_c^{(r)}}^{-\varepsilon_c\left[\underbrace{M_c^{(0)}}_{-\frac{qL^2}{2}} + X\underbrace{M_c^{(1)}}_{L}\right]} =$$

$$= \underbrace{\eta_D^{0} - \varepsilon_A\left[qL + X\right] - L\varepsilon_c\left[-\frac{qL^2}{2} + XL\right]}$$

$$\underbrace{L_{vi}} = \int_{str} M^{(1)}\frac{M^{(r)}}{EI}\,ds + \int_{str} M^{(1)}\frac{\alpha\Delta T}{h}\,ds =$$

$$= \frac{1}{EI}\int_{str} M^{(1)}M^{(0)}\,ds + \frac{X}{EI}\int_{str}\left[M^{(1)}\right]^2 ds + \frac{\alpha\Delta T}{h}\int_{str} M^{(1)}\,ds =$$

$$= \frac{1}{EI}\left\{\int_0^L z\left[qL\cdot z - \frac{qz^2}{2}\right]dz + \int_L^{2L} L\left[-\frac{qL^2}{2}\right]dz\right\} +$$

$$+ \frac{X}{EI}\left\{\int_0^L z^2\,dz + \int_L^{2L} L^2\,dz\right\} + \frac{\alpha\Delta T}{h}\int_L^{2L} L\,dz =$$

$$= \frac{1}{EI}\left\{qL\left[\frac{z^3}{3}\right]_0^L - \frac{q}{2}\left[\frac{z^4}{4}\right]_0^L - \frac{qL^3}{2}\left[z\right]_L^{2L}\right\} +$$

$$+ \frac{X}{EI}\left\{\left[\frac{z^3}{3}\right]_0^L + L^2\left[z\right]_L^{2L}\right\} + \frac{\alpha\Delta T}{h}L\left[z\right]_L^{2L} =$$

$$= \frac{1}{EI}\left\{\frac{qL^4}{3} - \frac{qL^4}{8} - \frac{qL^4}{2}\right\} + \frac{X}{EI}\left\{\frac{L^3}{3} + L^3\right\} + \frac{\alpha\Delta T}{h}L^2 =$$

$$= \underbrace{-\frac{qL^4}{EI}\frac{7}{24} + \frac{XL^3}{EI}\cdot\frac{4}{3} + \frac{\alpha\Delta T}{h}\cdot L^2}$$

$\Rightarrow$ $L_{ve} = L_{vi}$ fornisce

$$\eta_D^0 - \varepsilon_A\left[qL + X\right] - L\bar{\varepsilon}_c\left[-\frac{qL^2}{2} + XL\right] =$$

$$= -\frac{qL^4}{EI}\,\frac{7}{24} + \frac{XL^3}{EI}\cdot\frac{4}{3} + \frac{\alpha\Delta T}{h}\cdot L^2$$

$$X\left[\frac{4}{3}\frac{L^3}{EI} + \underbrace{\varepsilon_A}_{\frac{2}{3}\frac{L^3}{EI}} + \underbrace{\bar{\varepsilon}_c}_{\frac{L}{EI}}L^2\right] =$$

$$= \underbrace{\eta_D^0}_{\frac{qL^4}{2EI}} - \underbrace{\varepsilon_A}_{\frac{2}{3}\frac{L^3}{EI}}qL + \frac{qL^3}{2}\underbrace{\bar{\varepsilon}_c}_{\frac{L}{EI}} + \frac{qL^4}{EI}\cdot\frac{7}{24} - \underbrace{\frac{\alpha\Delta T}{h}}_{\frac{13}{8}\frac{qL^2}{EI}}L^2$$

$$X\,\cancel{\frac{L^3}{EI}}\,\underbrace{\left[\frac{4}{3} + \frac{2}{3} + 1\right]}_{3} = \frac{qL^{\cancel{4}}}{\cancel{EI}}\,\underbrace{\left[\frac{1}{2} - \frac{2}{3} + \frac{1}{2} + \frac{7}{24} - \frac{13}{8}\right]}_{-1}$$

da cui:

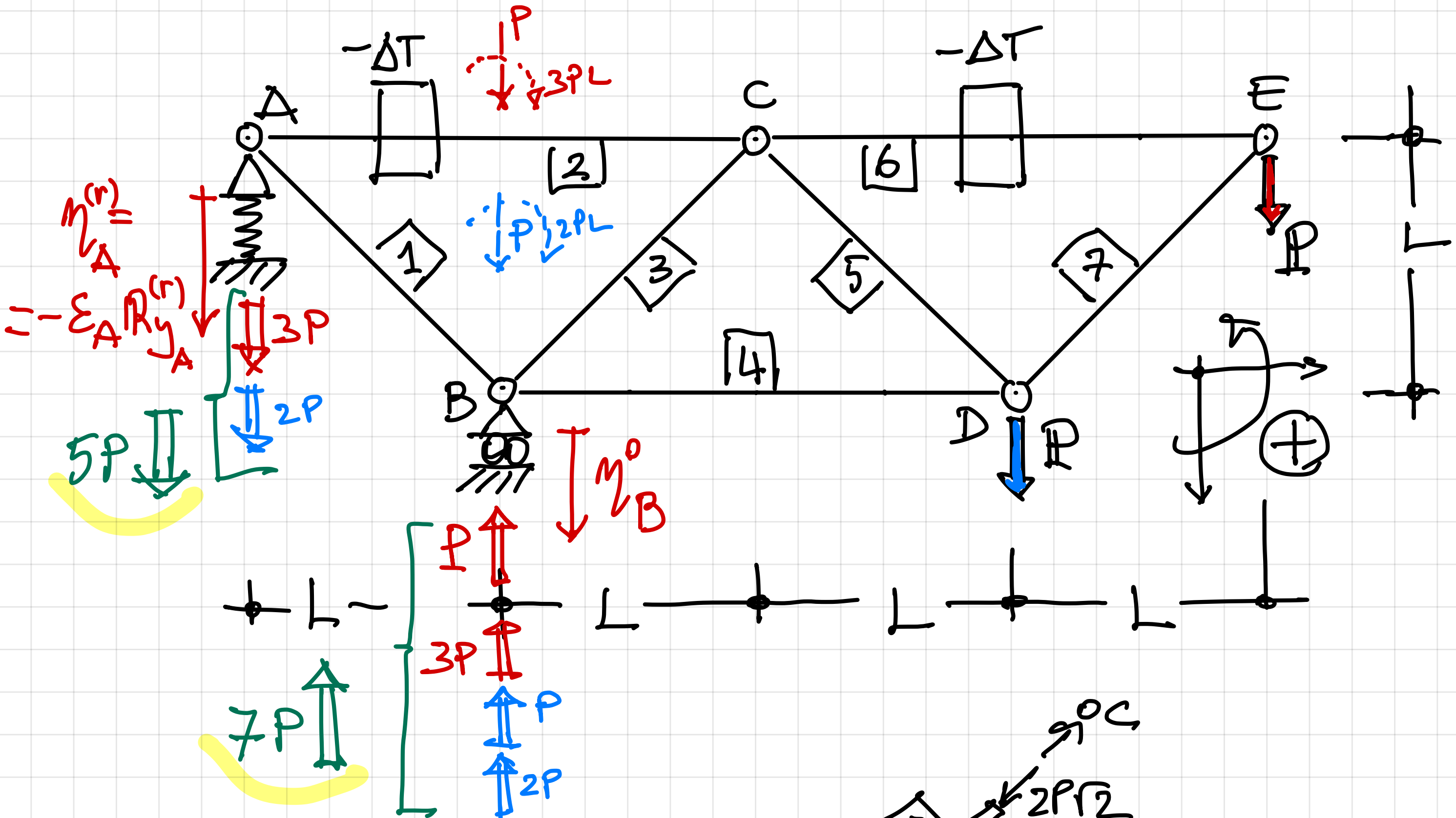
$$\boxed{X = -\frac{qL}{3}}$$

$\Rightarrow$ NEGATIVA! VERSO OPPOSTO A QUELLO IPOTIZZATO! OK! cfr. RV di pag. VI

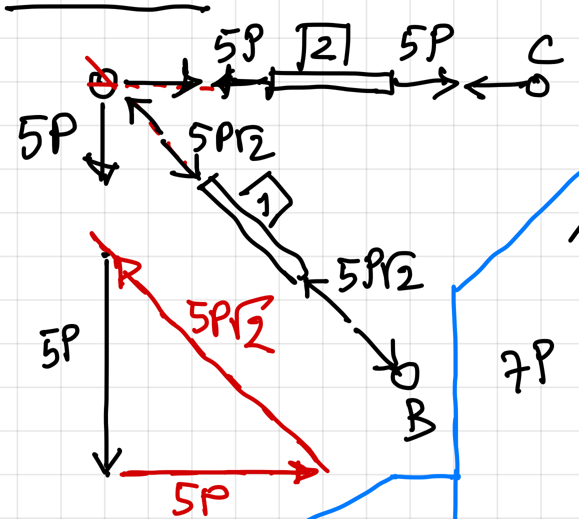
TIPO 2 - ES.#2 SOLUZIONE

XV

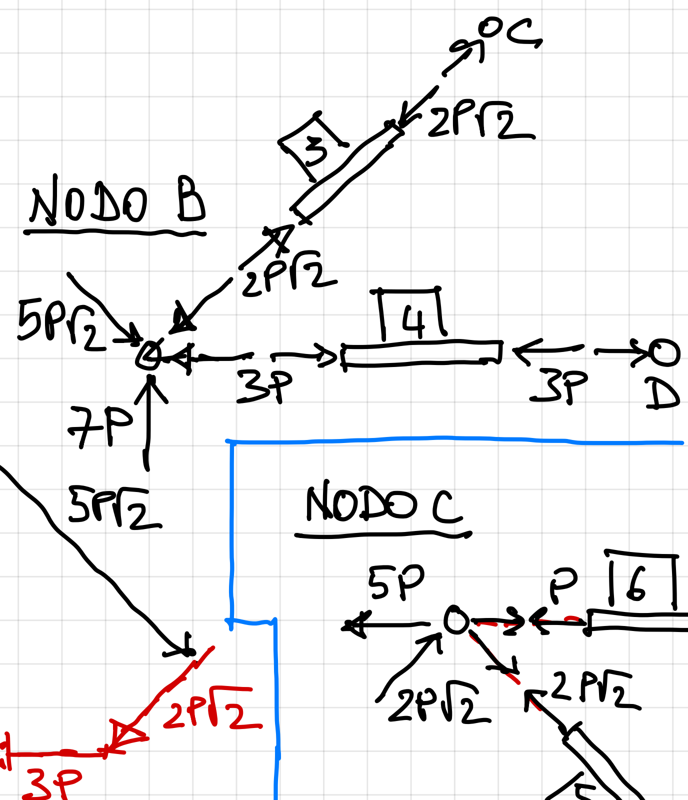
STRUTTURA REALE



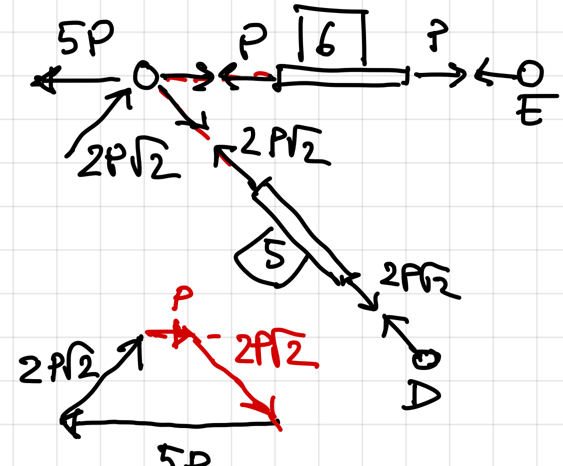
NODO A



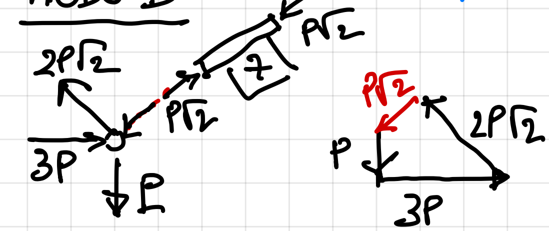
NODO B



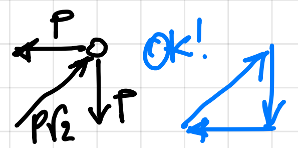
NODO C



NODO D



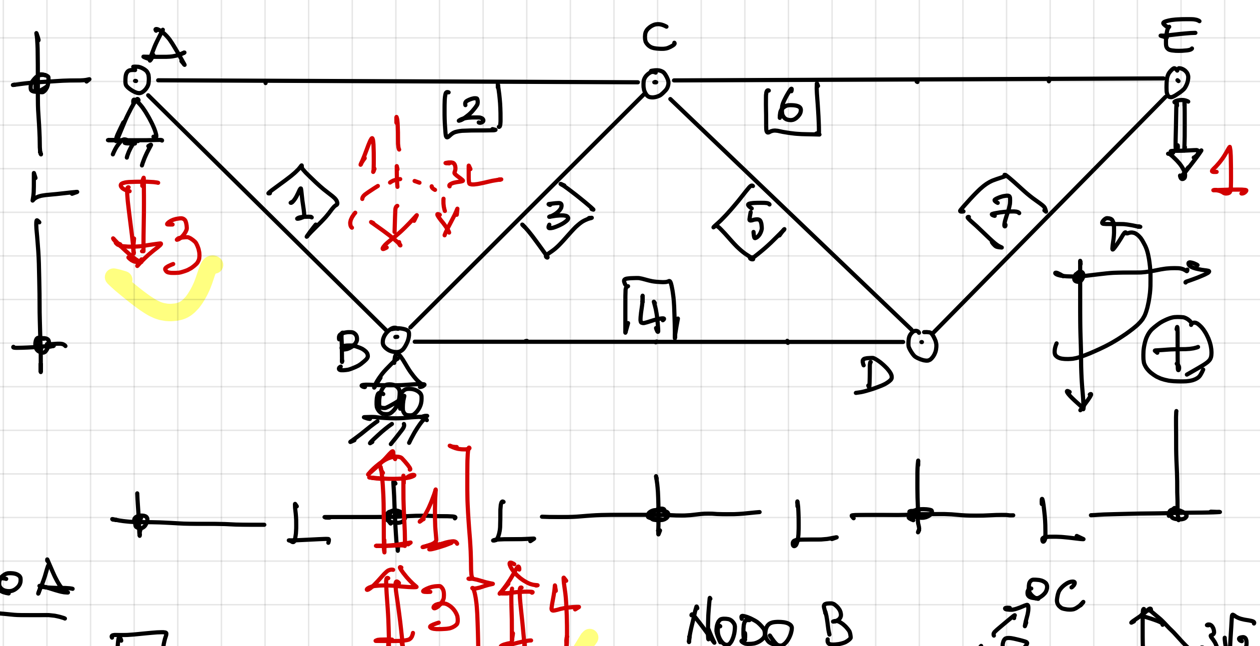
VERIFICA NODO E

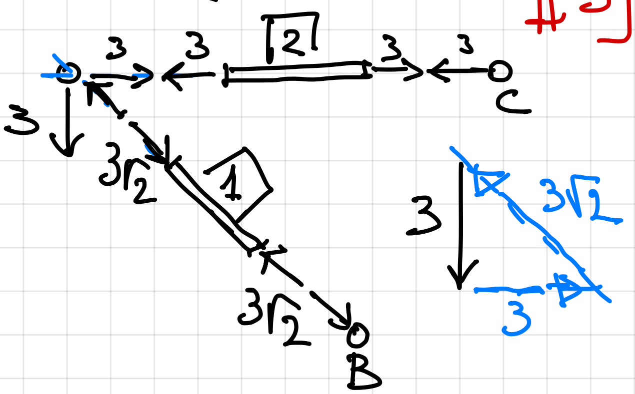


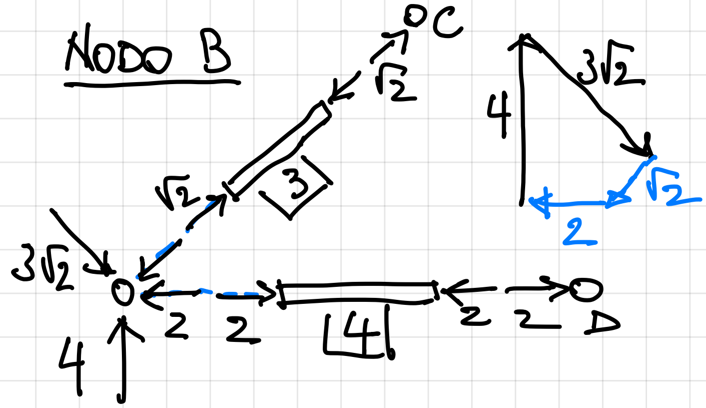
SFORZI NORMALI NELLA STRUTTURA REALE

ASTA	$N^{(r)}$	COMPORT. MECCANICO
1	$-5P\sqrt{2}$	puntone
2	$5P$	tirante
3	$-2P\sqrt{2}$	puntone
4	$-3P$	" "
5	$2P\sqrt{2}$	tirante
6	P	" "
7	$-P\sqrt{2}$	puntone

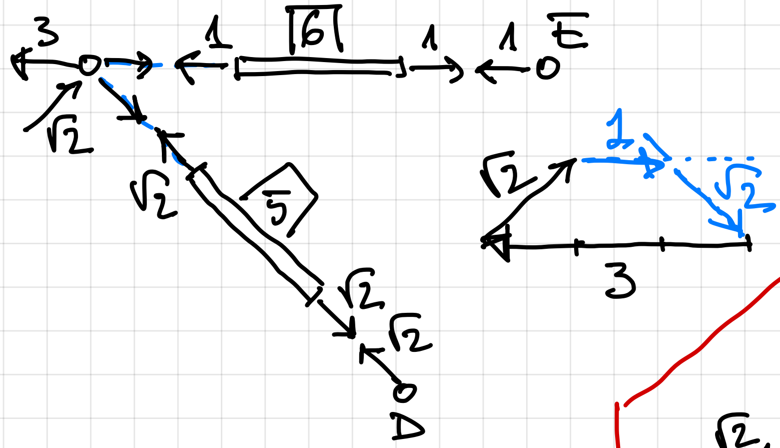
STRUTTURA FITTIZIA PER IL CALCOLO DELLO SPOSTAMENTO VERTICALE DEL NODO E



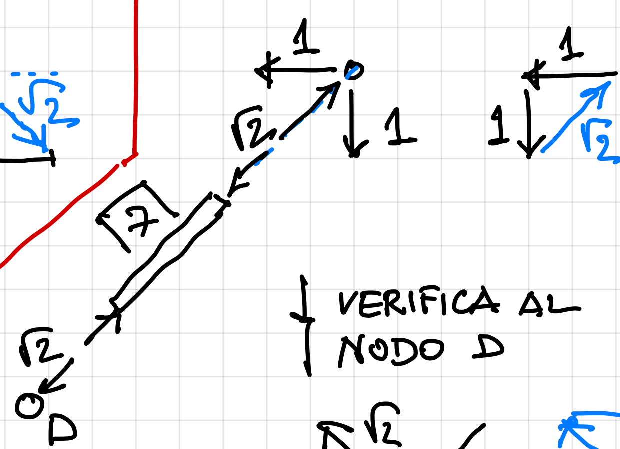




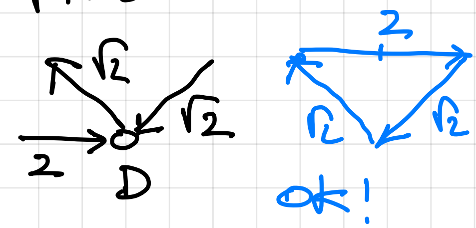
NODO C



NODO E



VERIFICA AL NODO D



SFORZI NORMALI NELLA STRUTTURA FITTIZIA

ASTA	$N^{(f)}$	COMPORT. MECC.
1	$-3\sqrt{2}$	puntone
2	3	tirante
3	$-\sqrt{2}$	puntone
4	-2	" "
5	$\sqrt{2}$	tirante
6	1	" "
7	$-\sqrt{2}$	puntone

$$L_{ve} = 1\cdot\eta_E + \sum_j R_j^{(1)} \eta_j^{(r)} = 1\cdot\eta_E + \underbrace{R_{y_A}^{(1)}}_{3} \overbrace{\eta_A^{(r)}}^{-\varepsilon_A \overbrace{R_{y_A}^{(r)}}^{5P}} + \underbrace{R_{y_B}^{(1)}}_{-4} \eta_B^0 =$$

$$= \eta_E - \varepsilon_A 15P - 4\eta_B^0$$

$$\Rightarrow \underbrace{L_{vi}} = \sum_i N_i^{(t)} \frac{N_i^{(r)} L_i}{EA} + \underbrace{\sum_j N_j^{(t)} \alpha \Delta T L_j}_{\text{ASTE \#2 e \#6}} =$$

$$= \frac{1}{EA} \Big\{ -3\sqrt{2}\left(-5P\sqrt{2}\right) L\sqrt{2} + 3\left(5P\right) 2L + \left(-\sqrt{2}\right)\left(-2P\sqrt{2}\right) L\sqrt{2} +$$

$$+ \left(-2\right)\left(-3P\right) 2L + \sqrt{2}\left(2P\sqrt{2}\right) L\sqrt{2} + 1\left(P\right) 2L +$$

$$+ \left(-\sqrt{2}\right)\left(-P\sqrt{2}\right) L\sqrt{2} \Big\} - \alpha\Delta T \cdot \left[3(2L) + 1(2L)\right] =$$

$$= \frac{PL}{EA}\left\{+30\sqrt{2} + 30 + 4\sqrt{2} + 12 + 4\sqrt{2} + 2 + 2\sqrt{2}\right\} +$$

$$- \alpha\Delta T\left[6L + 2L\right] =$$

$$= \underbrace{\frac{PL}{EA}\left[40\sqrt{2} + 44\right] - \alpha\Delta T \cdot 8L}$$

$\Rightarrow$ $L_{ve} = L_{vi}$ fornisce:

$$\eta_E - \underbrace{\varepsilon_A}_{\frac{L}{15EA}} 15P - 4 \underbrace{\eta_B^0}_{\frac{PL}{4EA}} = \frac{PL}{EA}\left[\cancel{40\sqrt{2}} + 44\right] \cancel{- \underbrace{\alpha\Delta T}_{5\sqrt{2}\frac{P}{EA}} 8L}$$

$$\boxed{\eta_E = \frac{PL}{EA}\left[44 + 1 + 1\right] = \underline{46\,\frac{PL}{EA}}} \Rightarrow \text{POSITIVO! VERSO IL BASSO}$$