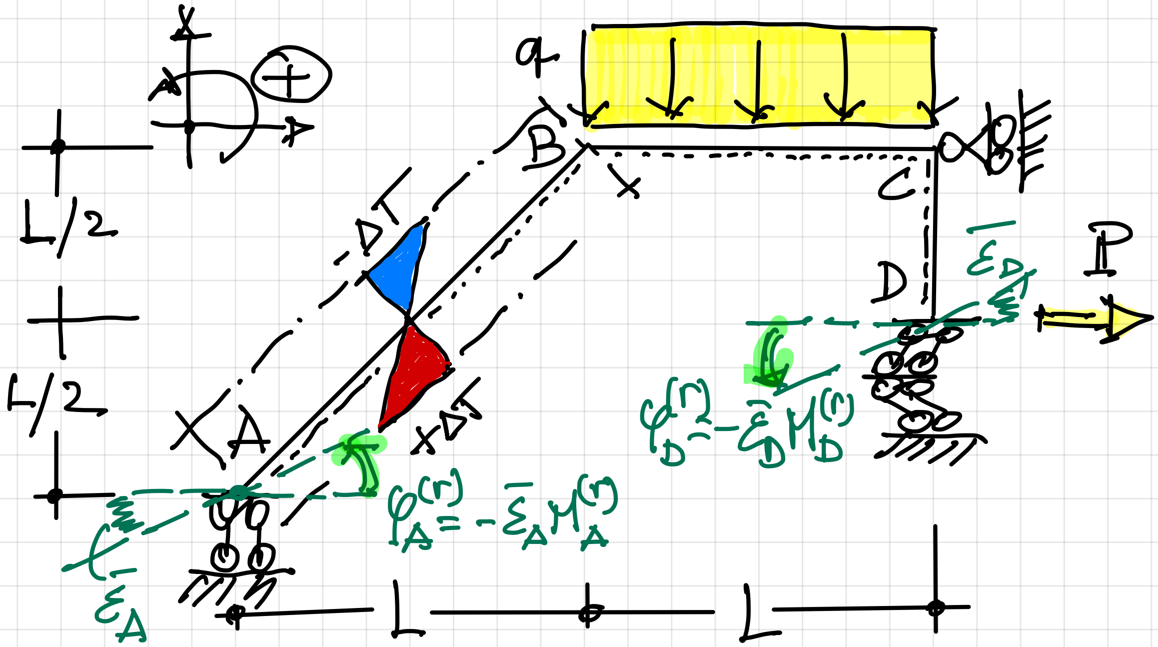
MECCANICA delle STRUTTURE – P. FUSCHI

TEST in ITINERE del 8 GENNAIO 2025

TIPO 1

ES. #1 RISOLVERE LA STRUTTURA UNA VOLTA IPERSTATICA SEGUENTE DETERMINANDO IL DIAGRAMMA DEI MOMENTI_



Posizioni:

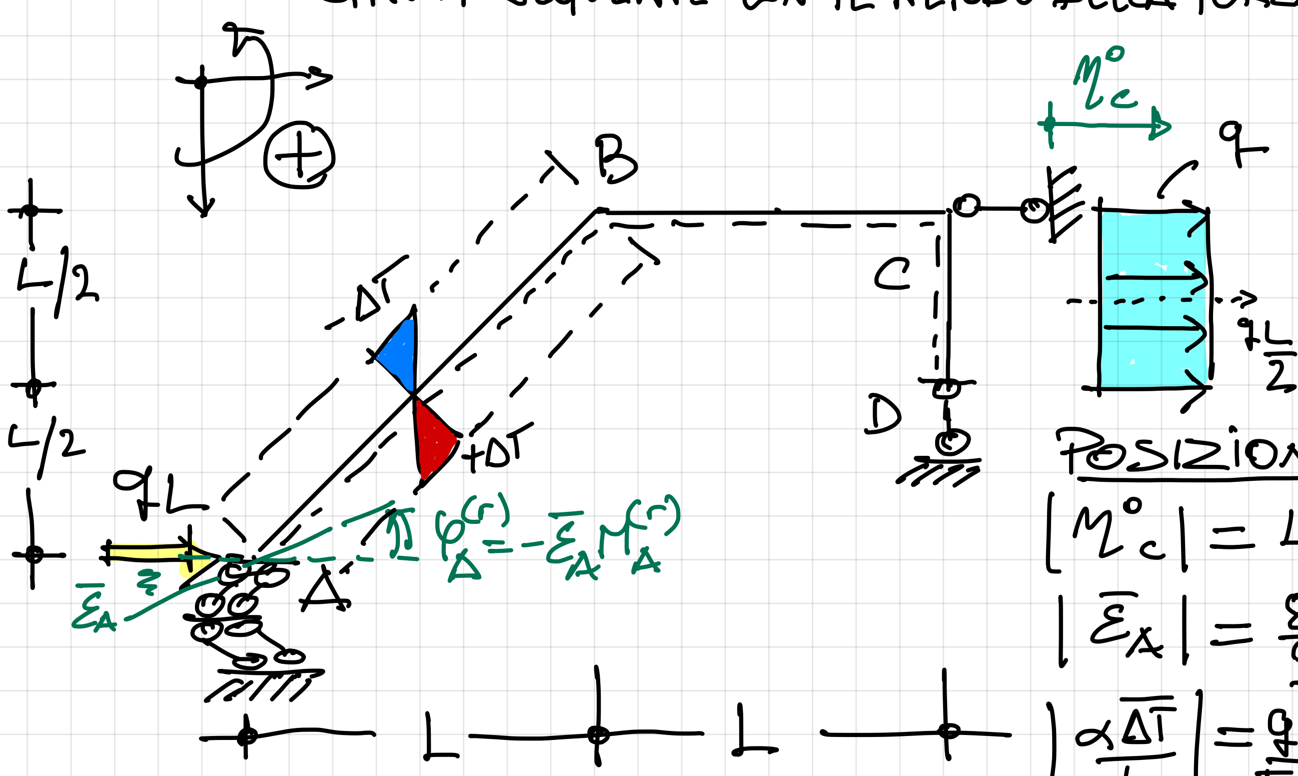
$$|P| = qL$$

$$|\bar{\varepsilon}_A| = \frac{11}{24}\frac{L}{EI}$$

$$|\bar{\varepsilon}_D| = \frac{1}{24}\frac{L}{EI}$$

$$\left|\frac{\alpha \Delta T}{h}\right| = \frac{qL^2}{2EI}$$

ES. #2 DETERMINARE LA ROTAZIONE DELLA SEZ. D DELLA STRUTT. SEGUENTE CON IL METODO DELLA FORZA UNITARIA_



Posizioni:

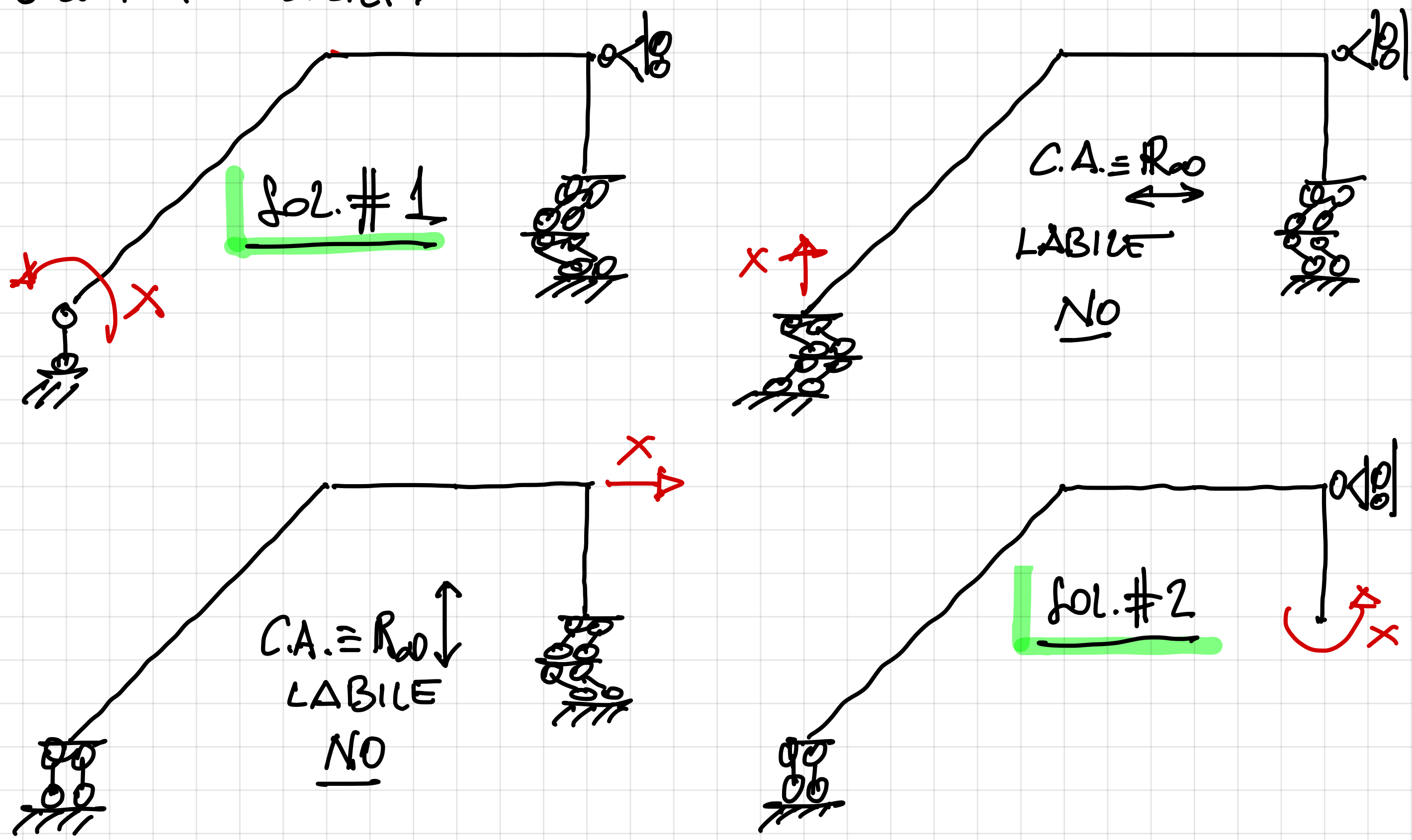
$$|M_c^0| = L^3/EI$$

$$|\bar{\varepsilon}_A| = \frac{8}{9}\frac{L}{EI}$$

$$\left|\frac{\alpha \Delta T}{h}\right| = \frac{qL^2}{EI}\left[\frac{3}{8} + \frac{41}{48\sqrt{2}}\right]$$

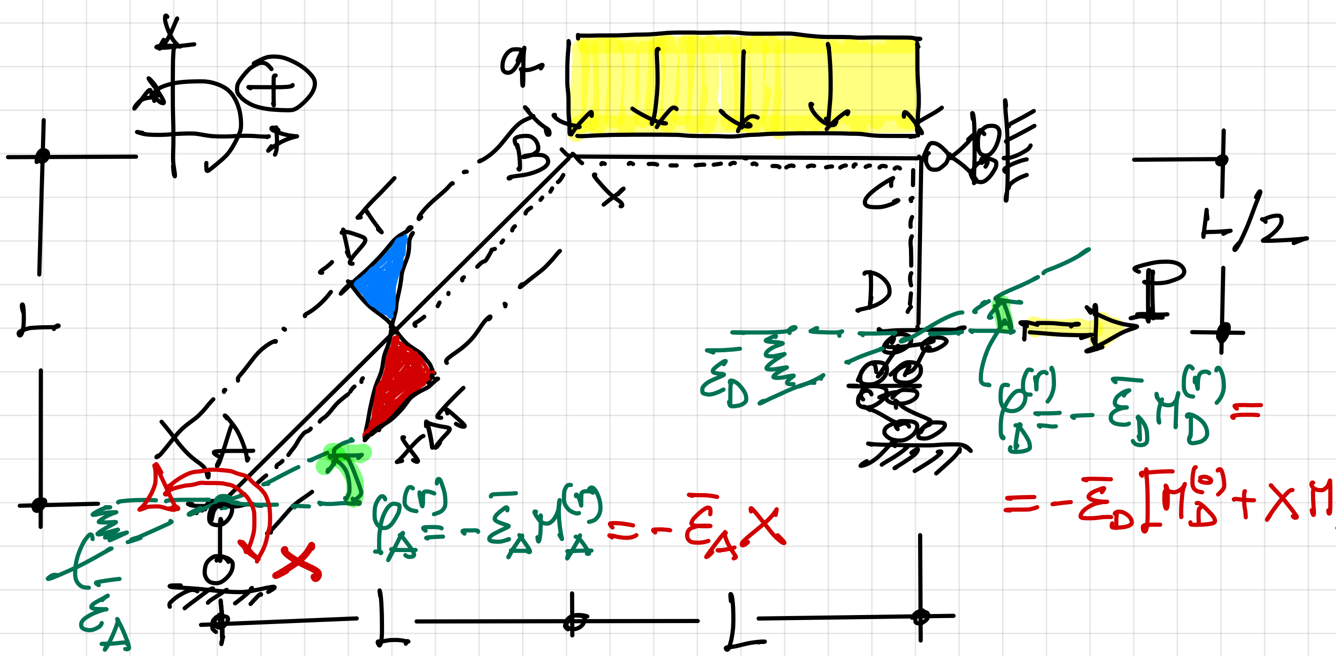
ES.#1 - tipo 1 SOLUZIONE

SOLUZIONI POSSIBILI:



SOLUZIONE #1

SISTEMA PRINCIPALE ISOSTATICO

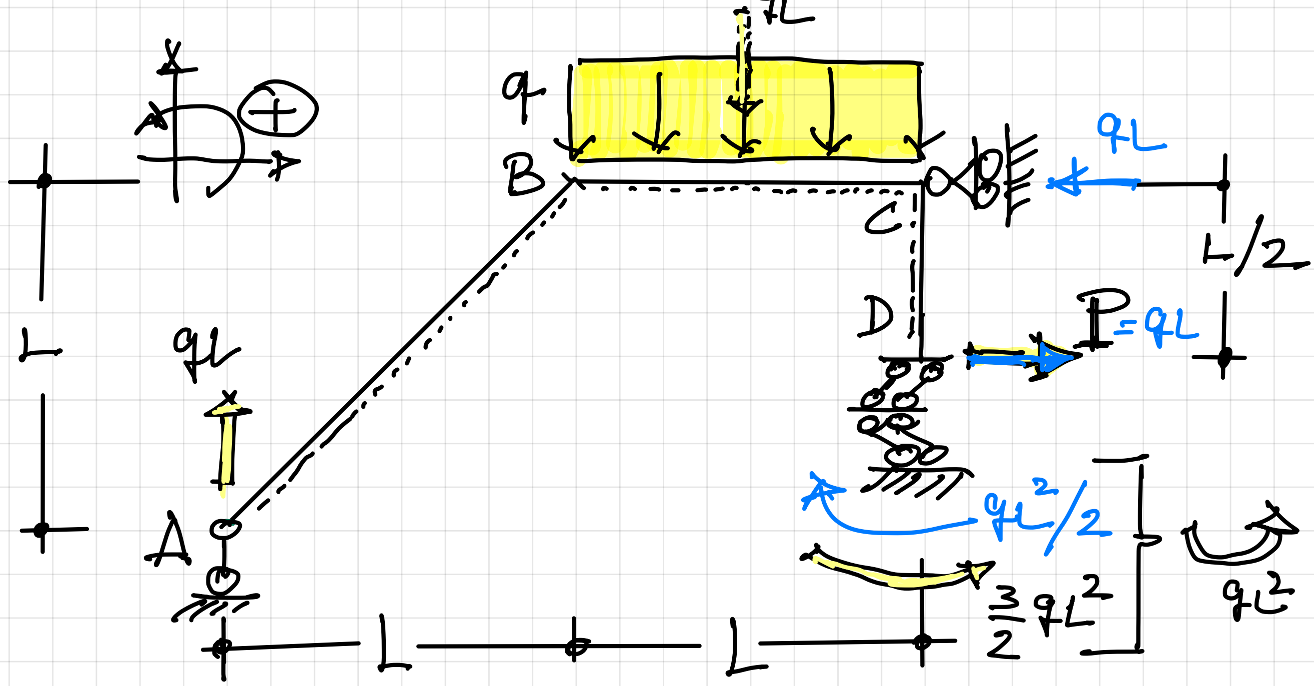


$$\varphi_A^{(r)} = -\bar{\varepsilon}_A M_A^{(r)} = -\bar{\varepsilon}_A X$$

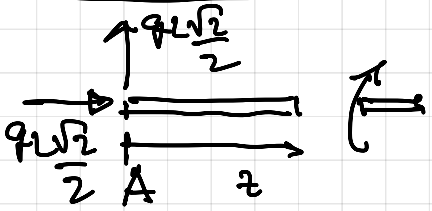
$$\varphi_D^{(r)} = -\bar{\varepsilon}_D M_D^{(r)} =$$

$$= -\bar{\varepsilon}_D \left[M_D^{(0)} + X M_D^{(1)}\right]$$

SCHEMA [0]: SOLO CARICHI ESTERNI



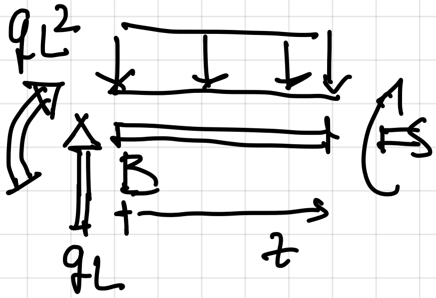
TRATTO AB $0 \le z \le L\sqrt{2}$



$$\boxed{M^{(0)}(z) = \frac{qL\sqrt{2}}{2}\cdot z}$$

$$\begin{cases} M_A = 0 \\ M_B = qL\frac{\sqrt{2}}{2}\cdot L\sqrt{2} = qL^2 \end{cases}$$

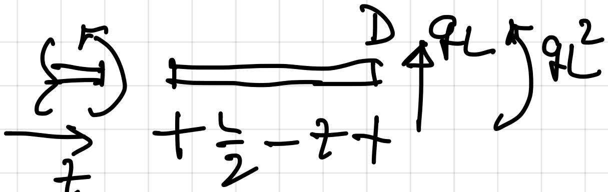
TRATTO BC $0 \le z \le L$



$$\boxed{M^{(0)}(z) = qL^2 + qL\cdot z - q\frac{z^2}{2}}$$

$$\begin{cases} M_B = qL^2 \\ M_C = 2qL^2 - \frac{qL^2}{2} = \frac{3}{2}qL^2 \end{cases}$$

TRATTO CD $0 \le z \le \frac{L}{2}$



$$\boxed{M^{(0)}(z) = qL\left(\frac{L}{2} - z\right) + qL^2}$$

$$\begin{cases} M_C = \frac{3}{2}qL^2 \\ M_D = qL^2 \end{cases}$$

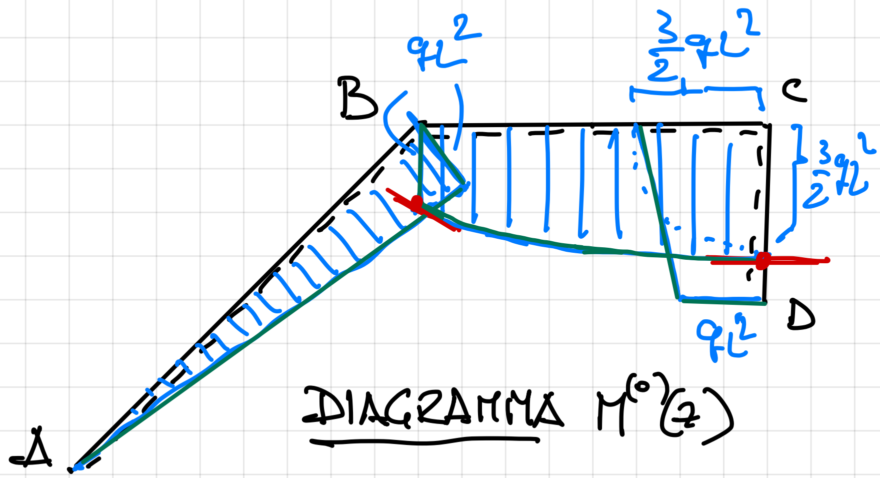
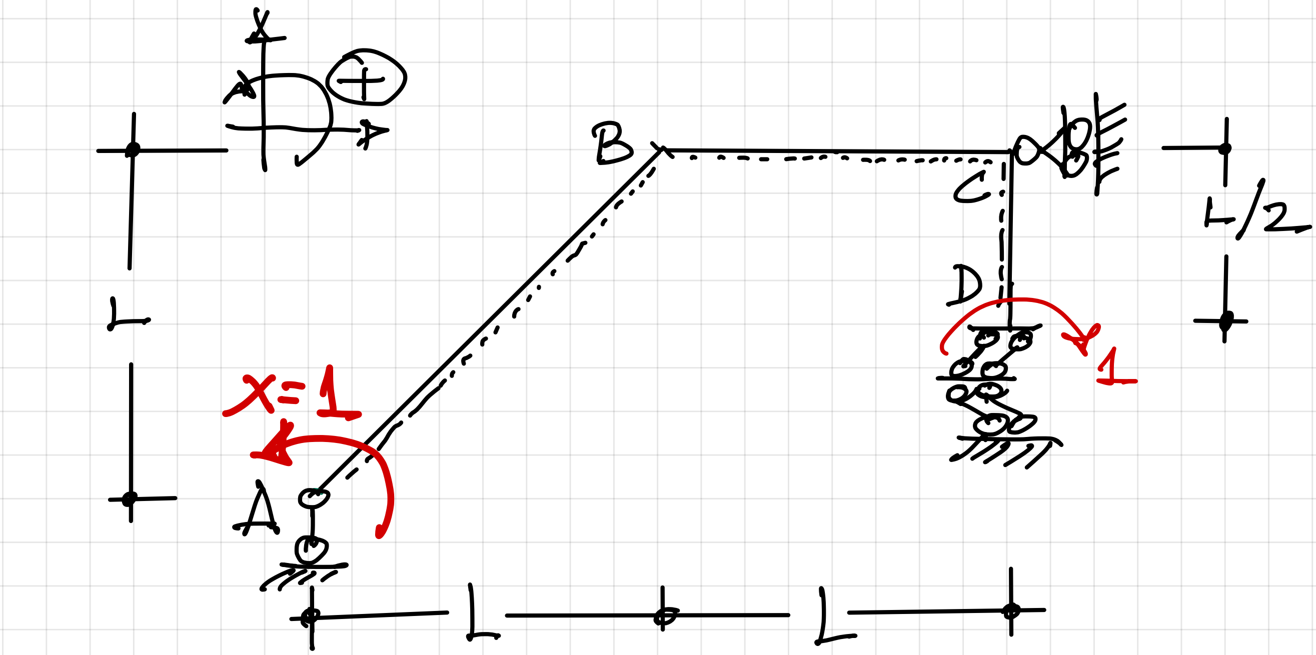
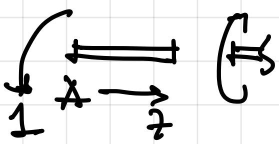


DIAGRAMMA $M^{(0)}(z)$

SCHEMA [1] solo $X = 1$

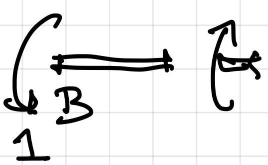


TRATTO AB $0 \le z \le L\sqrt{2}$



$\boxed{M^{(1)}(z) = -1}$ cost

TRATTO BC $0 \le z \le L$



$\boxed{M^{(1)}(z) = -1}$ cost

TRATTO CD $0 \le z \le \frac{L}{2}$

$\boxed{M^{(1)}(z) = -1}$ cost

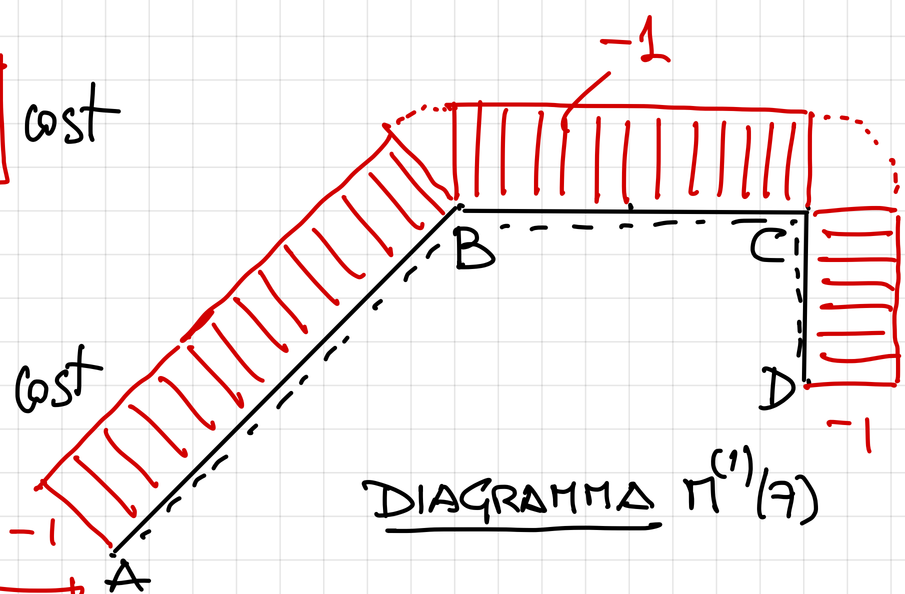


DIAGRAMMA $M^{(1)}(z)$

$$\underbrace{L_{ve}} = \underset{1}{X_i} \cdot \eta_i^{(r)} + \sum_j R_j^{(1)} \eta_j^{(r)} = 1 \cdot \underbrace{\varphi_A^{(r)}}_{-\bar{\varepsilon}_A X} + \overbrace{M_D^{(1)}}^{-1} \cdot \underbrace{\varphi_D^{(r)}}_{-\bar{\varepsilon}_D [\underbrace{M_D^{(0)}}_{qL^2} + X \underbrace{M_D^{(1)}}_{-1}]} =$$

$$= \underbrace{-\bar{\varepsilon}_A X + \bar{\varepsilon}_D [qL^2 - X]}$$

$$L_{vi} = \int_{str} M^{(1)} \frac{\overbrace{M^{(f)}}^{M^0+M^1 X}}{EI}\, ds_{tr} + \int_{str} M^{(1)} \frac{\alpha \overline{\Delta T}}{h}\, ds_{tr} =$$

$$= \frac{1}{EI}\int_{str} M^{(1)} M^{(0)}\, ds_{tr} + \frac{X}{EI}\int_{str} \left[M^{(1)}\right]^2 ds_{tr} + \frac{\alpha \overline{\Delta T}}{h}\int_{str} M^{(1)}\, ds_{tr} =$$

$$= \frac{1}{EI}\left\{\int_0^{L\sqrt{2}} -\frac{qL\sqrt{2}}{2} z\, dz + \int_0^{L}\left[-qL^2 - qLz + \frac{qz^2}{2}\right] dz + \right.$$

$$\left. + \int_0^{\frac{L}{2}}\left[-qL\left(\frac{L}{2}-z\right) - qL^2\right] dz\right\} +$$

$$+ \frac{X}{EI}\left\{\int_0^{L\sqrt{2}} dz + \int_0^{L} dz + \int_0^{\frac{L}{2}} dz\right\} + \frac{\alpha \overline{\Delta T}}{h}\int_0^{L\sqrt{2}} -1\cdot dz =$$

$$= \frac{1}{EI}\left\{-\frac{qL\sqrt{2}}{2}\left[\frac{z^2}{2}\right]_0^{L\sqrt{2}} - qL^2\left[z\right]_0^{L} - qL\left[\frac{z^2}{2}\right]_0^{L} + \frac{q}{2}\left[\frac{z^3}{3}\right]_0^{L} + \right.$$

$$\left. - \frac{qL^2}{2}\left[z\right]_0^{\frac{L}{2}} + qL\left[\frac{z^2}{2}\right]_0^{\frac{L}{2}} - qL^2\left[z\right]_0^{\frac{L}{2}}\right\} +$$

$$+ \frac{X}{EI}\left\{L\sqrt{2} + L + \frac{L}{2}\right\} - \frac{\alpha \overline{\Delta T}}{h}\cdot L\sqrt{2} =$$

$$= \frac{1}{EI}\left\{-\frac{qL\sqrt{2}}{2}\,\frac{L^2\cancel{2}}{\cancel{2}} - qL^3 - \frac{qL^3}{2} + \frac{qL^3}{6} - \frac{qL^3}{4} + \frac{qL^3}{8} - \frac{qL^3}{2}\right\} +$$

$$+ \frac{XL}{EI}\left[\sqrt{2} + \frac{3}{2}\right] - \frac{\alpha \overline{\Delta T}}{h}\cdot L\sqrt{2} =$$

$$= -\frac{qL^3}{EI}\left[\frac{\sqrt{2}}{2} + \underbrace{1 + \frac{1}{2} - \frac{1}{6} + \frac{1}{4} - \frac{1}{8} + \frac{1}{2}}_{\frac{24+12-4+6-3+12}{24} = 47/24}\right] + \frac{XL}{EI}\left[\sqrt{2} + \frac{3}{2}\right] - \frac{\alpha \overline{\Delta T}}{h} L\sqrt{2}$$

⇒ $L_{ve} = L_{vi}$ fornisce

$$-\bar{\varepsilon}_A X + \bar{\varepsilon}_D\left[qL^2 - X\right] = -\frac{qL^3}{EI}\left[\frac{\sqrt{2}}{2}+\frac{47}{24}\right] + \frac{XL}{EI}\left[\sqrt{2}+\frac{3}{2}\right] - \frac{\alpha\Delta T}{h}L\sqrt{2}$$

$$X\left[-\overbrace{\bar{\varepsilon}_A}^{\frac{11}{24}\frac{L}{EI}} - \underbrace{\bar{\varepsilon}_D}_{\frac{1}{24}\frac{L}{EI}} - \frac{L}{EI}\left(\sqrt{2}+\frac{3}{2}\right)\right] = -\frac{qL^3}{EI}\left[\frac{\sqrt{2}}{2}+\frac{47}{24}\right] - \overbrace{\frac{\alpha\Delta T}{h}L\sqrt{2}}^{\frac{qL^2}{2EI}} - \overbrace{\bar{\varepsilon}_D}^{=\frac{1}{24}\frac{L}{EI}} qL^2$$

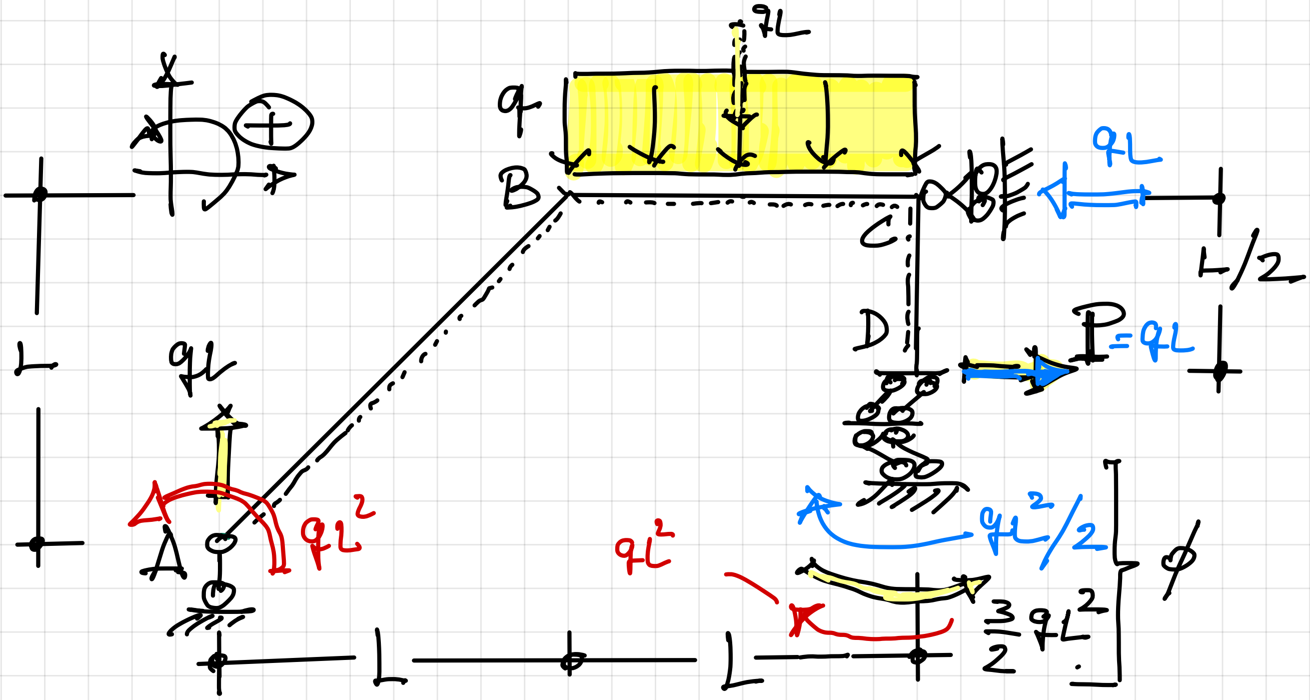
$$-X\cancel{\frac{L}{EI}}\left[\underbrace{\frac{11}{24}+\frac{1}{24}+\sqrt{2}+\frac{3}{2}}_{\sqrt{2}+2}\right] = -\frac{qL^{\cancel{3}\,2}}{\cancel{EI}}\left[\underbrace{\frac{\sqrt{2}}{2}+\frac{47}{24}+\frac{\sqrt{2}}{2}+\frac{1}{24}}_{\sqrt{2}+2}\right]$$

da cui

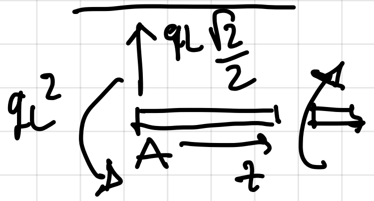
$$\boxed{X = qL^2}$$

⇒ POSITIVO! verso ipotizzato corretto

SOLUZIONE SISTEMA PRINCIPALE ISOSTATICO



TRATTO AB $0 \le z \le L\sqrt{2}$

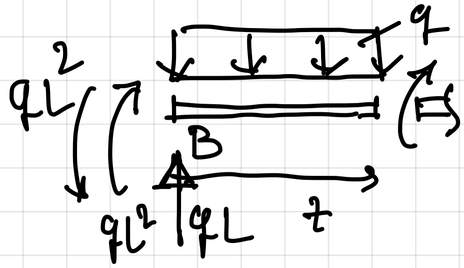


$$M^{(r)}(z) = -qL^2 + \frac{qL\sqrt{2}}{2}\cdot z$$

$$M_A = -qL^2$$

$$M_B = -qL^2 + \frac{qL\cancel{\sqrt{2}}}{\cancel{2}}\, L\cancel{\sqrt{2}} = \phi$$

TRATTO BC $0 \le z \le L$

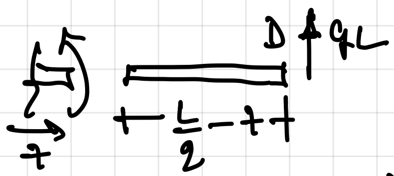


$$M^{(r)}(z) = qL\cdot z - \frac{qz^2}{2}$$

$$M_B = \phi$$

$$M_C = \frac{qL^2}{2}$$

TRATTO CD $0 \le z \le \frac{L}{2}$



$$M^{(r)}(z) = qL\left(\frac{L}{2} - z\right)$$

$$M_C = qL^2/2$$

$$M_D = \phi$$

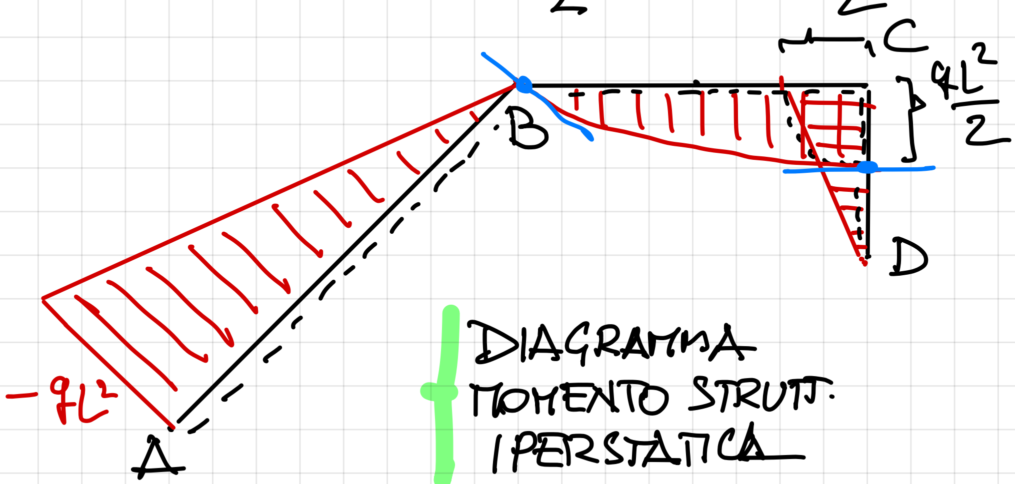
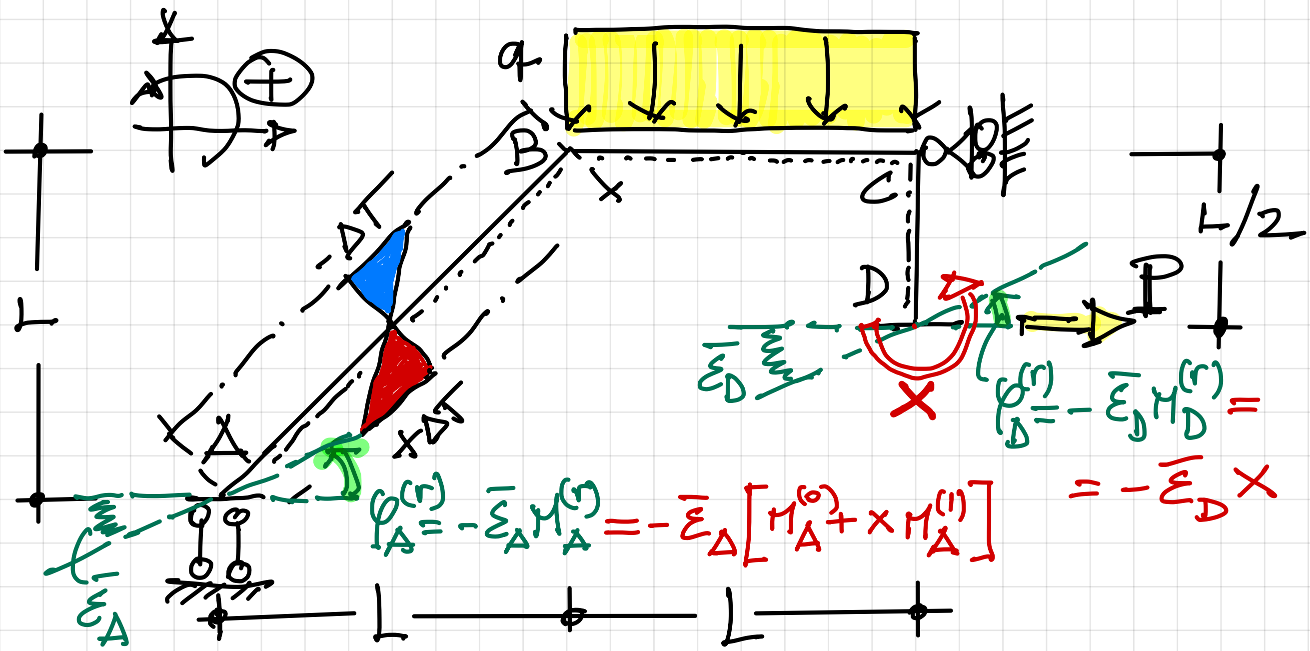


DIAGRAMMA MOMENTO STRUTT. IPERSTATICA

SOLUZIONE #2

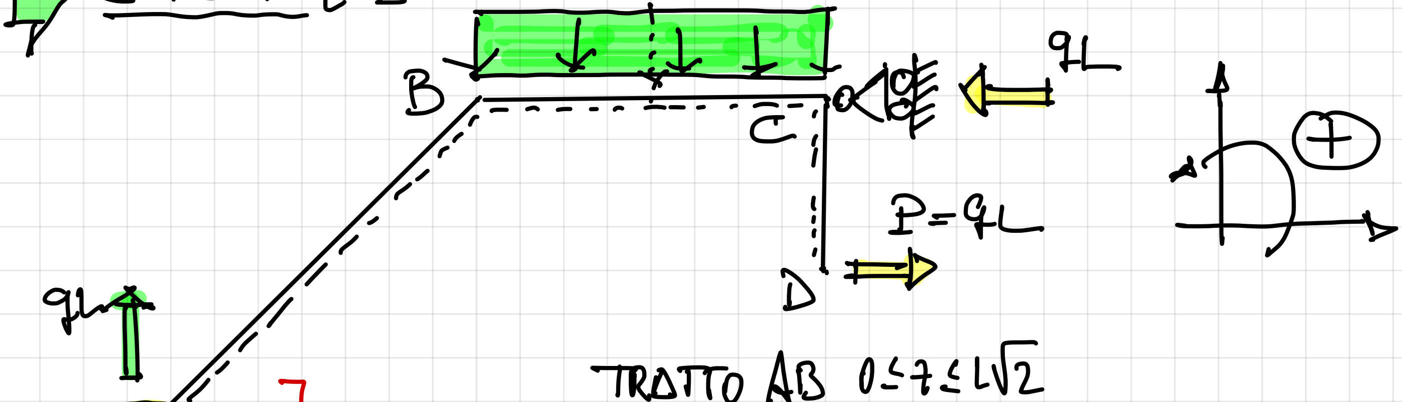
SISTEMA PRINCIPALE ISOSTATICO

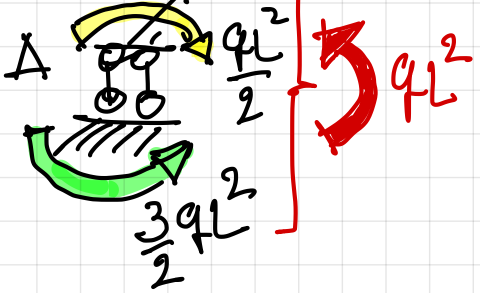


$$\varphi_A^{(r)} = -\bar{\varepsilon}_A M_A^{(r)} = -\bar{\varepsilon}_A \left[M_A^{(0)} + X\, M_A^{(1)} \right]$$

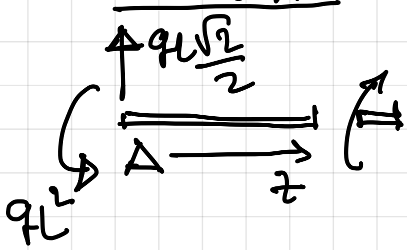
$$\varphi_D^{(r)} = -\bar{\varepsilon}_D M_D^{(r)} = -\bar{\varepsilon}_D X$$

SCHEMA [0] SOLO CARICHI ESTERNI





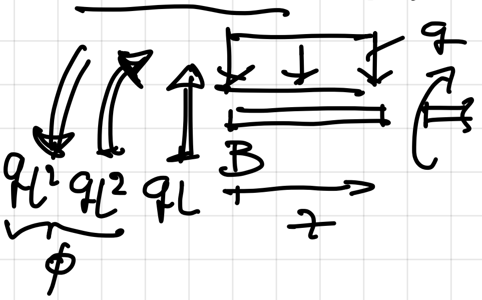
TRATTO AB $0 \le z \le L\sqrt{2}$



$$\boxed{M^{(0)}(z) = -qL^2 + qL\frac{\sqrt{2}}{2} \cdot z}$$

$$\begin{cases} M_A = -qL^2 \\ M_B = \emptyset \end{cases}$$

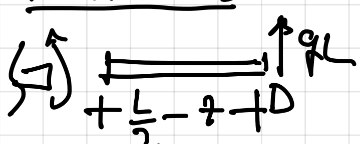
TRATTO BC $0 \le z \le L$



$$\boxed{M^{(0)}(z) = qL \cdot z - \frac{qz^2}{2}}$$

$$\begin{cases} M_B = \emptyset \\ M_C = \frac{qL^2}{2} \end{cases}$$

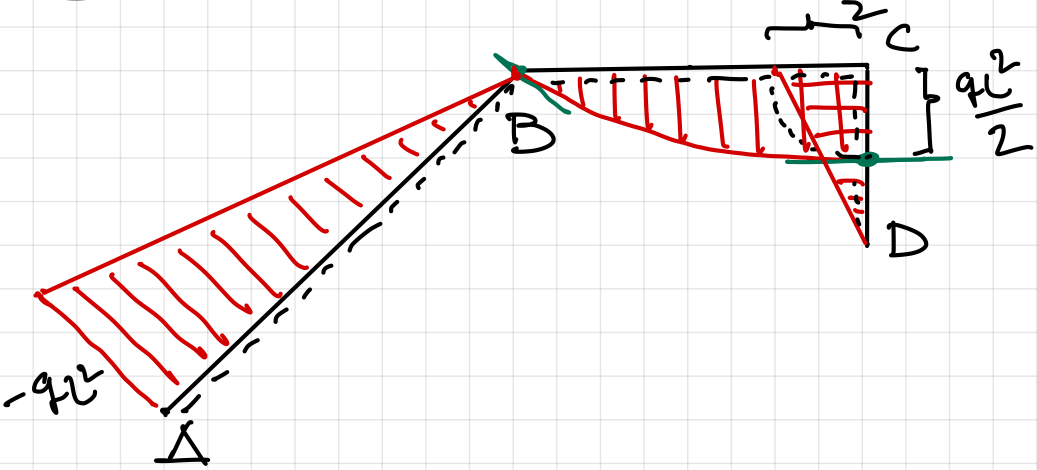
TRATTO CD $0 \le z \le \frac{L}{2}$



$$\boxed{M^{(0)}(z) = qL\left(\frac{L}{2} - z\right)}$$

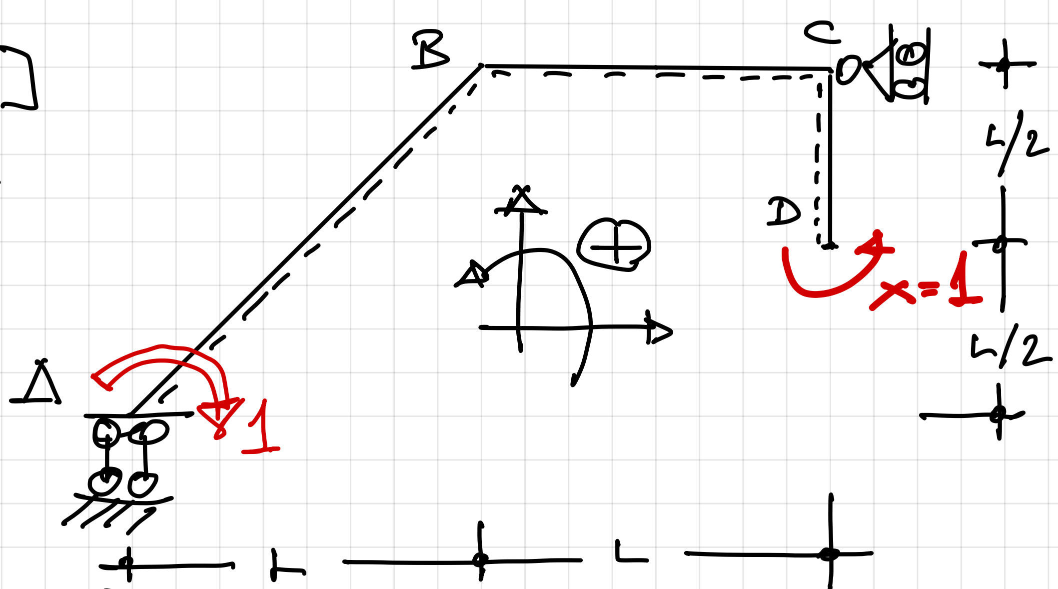
$$\begin{cases} M_C = \frac{qL^2}{2} \\ M_D = \emptyset \end{cases}$$

DIAGRAMMA $M^0(z)$



SCHEMA [1]

solo $X = 1$



TRATTO AB $0 \leq z \leq L\sqrt{2}$

$\boxed{M^{(1)}(z) = 1}$ cost

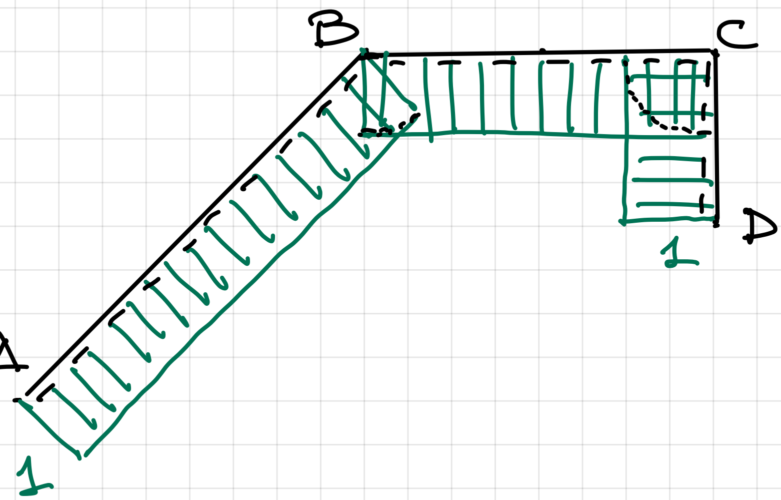
TRATTO BC $0 \leq z \leq L$

$\boxed{M^{(1)}(z) = 1}$ cost

TRATTO CD $0 \leq z \leq \frac{L}{2}$

$\boxed{M^{(1)}(z) = 1}$ cost

DIAGRAMMA $M^{(1)}(z)$



$$L_{ve} = \underset{\overset{\shortparallel}{1}}{X_i}\,\eta_i^{(r)} + \sum_j R_j^{(1)} \eta_j^{(r)} = 1 \cdot \underbrace{\varphi_D^{(r)}}_{\overset{\shortparallel}{-\bar{\varepsilon}_D X}} + \underbrace{M_A^{(1)}}_{\overset{\shortparallel}{-1}} \underbrace{\varphi_A^{(r)}}_{-\bar{\varepsilon}_A\left[\underbrace{M_A^{(0)}}_{qL^2} + X \underbrace{M_A^{(1)}}_{-1}\right]} =$$

$$= -\bar{\varepsilon}_D X + \bar{\varepsilon}_A\left[qL^2 - X\right]$$

$$L_{vi} = \int_{str} M^{(1)} \frac{\overbrace{M^{(r)}}^{M^0 + M' X}}{EI}\, ds_{tr} + \int_{str} M^{(1)} \frac{\alpha \Delta T}{h}\, ds_{tr} =$$

$$= \frac{1}{EI}\int_{str} M^{(1)} M^{(0)}\, ds_{tr} + \frac{X}{EI}\int_{str}\left[M^{(1)}\right]^2 ds_{tr} + \frac{\alpha \Delta T}{h}\int_{str} M^{(1)}\, ds_{tr} =$$

$$= \frac{1}{EI}\left[\int_0^{L\sqrt{2}}\left[-qL^2 + qL\frac{\sqrt{2}}{2}\cdot z\right] dz + \int_0^{L}\left[qL\cdot z - \frac{qz^2}{2}\right] dz + \int_0^{L/2} qL\left(\frac{L}{2} - z\right) dz\right] +$$

$$+ \frac{X}{EI}\left[\int_0^{L\sqrt{2}} dz + \int_0^{L} dz + \int_0^{L/2} dz\right] + \frac{\alpha \Delta T}{h}\int_0^{L\sqrt{2}} dz =$$

$$= \frac{1}{EI}\left[-qL^2\cdot L\sqrt{2} + qL\frac{\sqrt{2}}{2}\left[\frac{z^2}{2}\right]_0^{L\sqrt{2}} + qL\cdot\frac{L^2}{2} - \frac{q}{2}\frac{L^3}{3} + \frac{qL^2}{2}\cdot\frac{L}{2} - qL\left[\frac{z^2}{2}\right]_0^{\frac{L}{2}}\right] +$$

$$+ \frac{X}{EI}\left[L\sqrt{2} + L + \frac{L}{2}\right] + \frac{\alpha \Delta T}{h}\cdot L\sqrt{2} =$$

$$= \frac{1}{EI}\left\{ -qL^3\sqrt{2} + \frac{qL^3\sqrt{2}}{2} + \frac{qL^3}{2} - \frac{qL^3}{6} + \frac{qL^3}{4} - \frac{qL^3}{8} \right\} +$$

$$+ \frac{XL}{EI}\left[\sqrt{2} + \frac{3}{2}\right] + \frac{\alpha \Delta T}{h} L\sqrt{2}$$

$$= \frac{qL^3}{EI}\left\{ -\sqrt{2} + \frac{\sqrt{2}}{2} + \underbrace{\frac{1}{2} - \frac{1}{6} + \frac{1}{4} - \frac{1}{8}}_{\frac{12-4+6-3}{24} = \frac{11}{24}} \right\} + \frac{XL}{EI}\left[\sqrt{2} + \frac{3}{2}\right] + \frac{\alpha \Delta T}{h} \cdot L\sqrt{2}$$

$L_{ve} = L_{vi}$ fornisce

$$-\bar{\mathcal{E}}_D X + \bar{\mathcal{E}}_A\left[qL^2 - X\right] = \frac{qL^3}{EI}\left[-\frac{\sqrt{2}}{2} + \frac{11}{24}\right] + \frac{XL}{EI}\left[\sqrt{2} + \frac{3}{2}\right] + \frac{\alpha \Delta T}{h} L\sqrt{2}$$

$$X\Big[\underbrace{-\bar{\mathcal{E}}_D}_{\frac{1}{24}\frac{L}{EI}} \underbrace{-\bar{\mathcal{E}}_A}_{\frac{11}{24}\frac{L}{EI}} - \frac{L}{EI}\left(\sqrt{2} + \frac{3}{2}\right)\Big] = \frac{qL^3}{EI}\left[-\frac{\sqrt{2}}{2} + \frac{11}{24}\right] - \underbrace{\bar{\mathcal{E}}_A}_{\frac{11}{24}\frac{L}{EI}} qL^2 + \underbrace{\frac{\alpha \Delta T}{h}}_{\frac{qL^2}{2EI}} L\sqrt{2}$$

$$\frac{L}{EI} \cdot X\left[-\frac{1}{24} - \frac{11}{24} - \sqrt{2} - \frac{3}{2}\right] =$$

$$= \frac{qL^3}{EI}\Big[\underbrace{-\frac{\sqrt{2}}{2} + \frac{11}{24} - \frac{11}{24} + \frac{\sqrt{2}}{2}}_{\emptyset}\Big]$$

da cui

$$\boxed{X = \emptyset}$$

OK! CORRETTO!
cfr. RV di pag. 7

ES. #2 - tipo 1 SOLUZIONE

STRUTTURA REALE

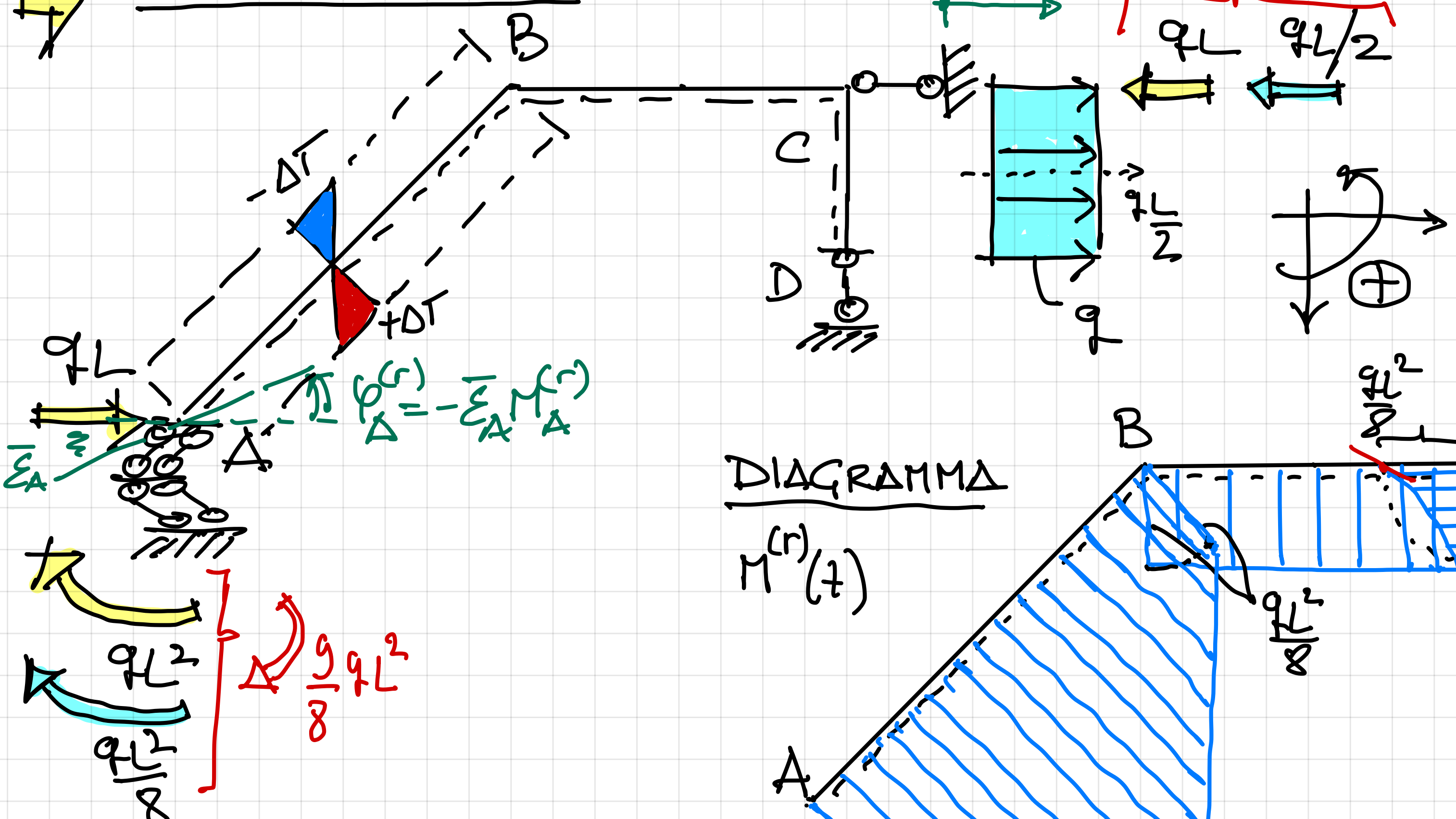
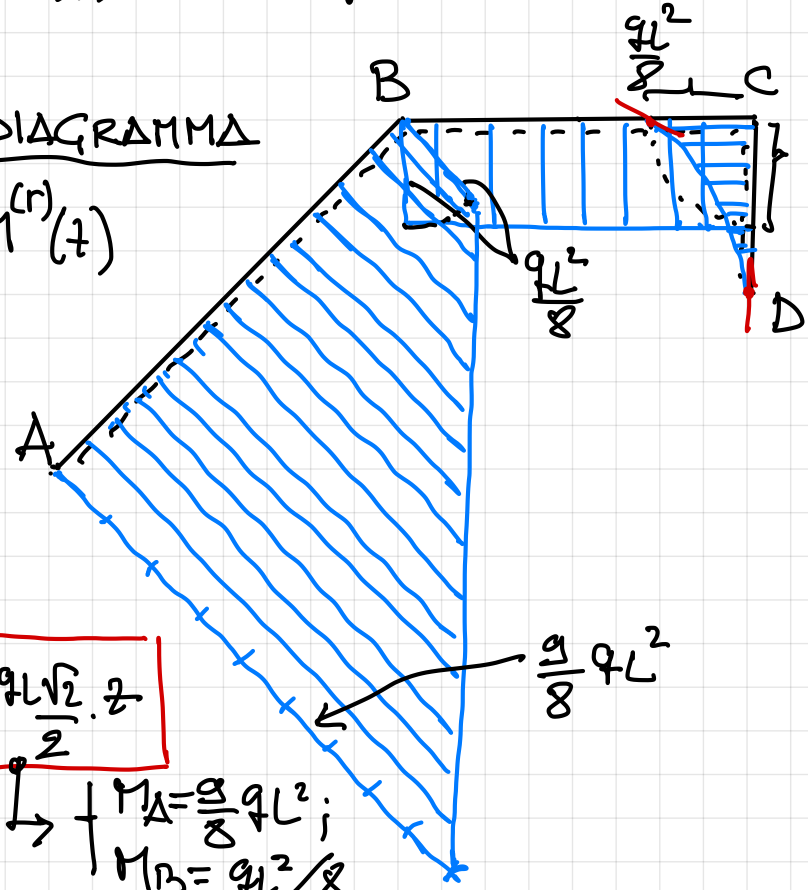
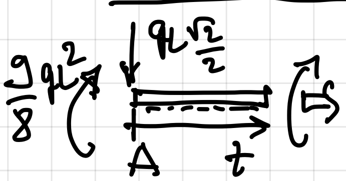


DIAGRAMMA $M^{(r)}(z)$



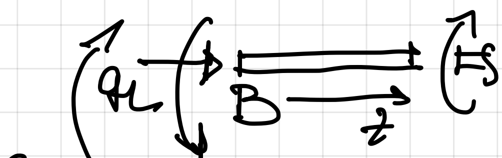
TRATTO AB $0 \le z \le L\sqrt{2}$



$$\boxed{M^{(r)}(z) = \frac{9}{8}qL^2 - \frac{qL\sqrt{2}}{2} \cdot z}$$

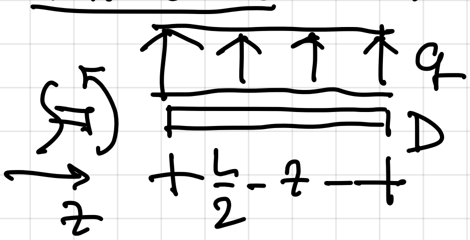
$$\begin{cases} M_A = \frac{9}{8}qL^2; \\ M_B = qL^2/8 \end{cases}$$

TRATTO BC $0 \le z \le L$



$$\boxed{M^{(r)}(z) = \frac{qL^2}{8}} \text{ cost}$$

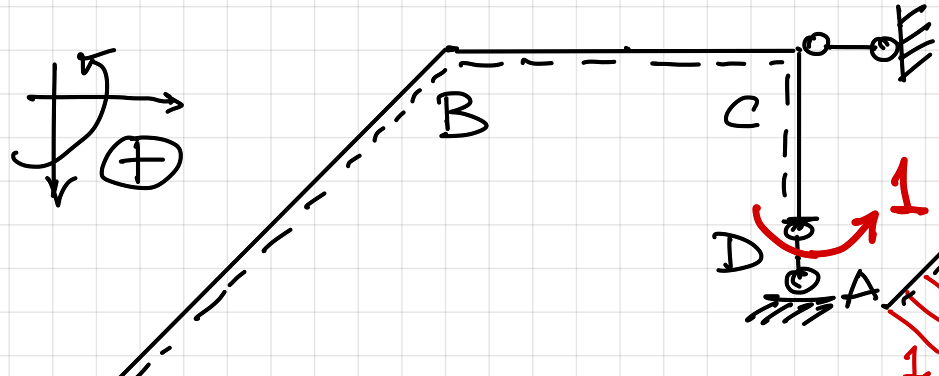
TRATTO CD $0 \le z \le \frac{L}{2}$

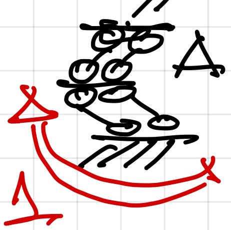


$$\boxed{M^{(r)}(z) = \frac{q\left(\frac{L}{2} - z\right)^2}{2}}$$

$$\begin{cases} M_C = \frac{qL^2}{8} \\ M_D = \phi \end{cases}$$

STRUTTURA FITTIZIA PER IL CALCOLO DI φ_D





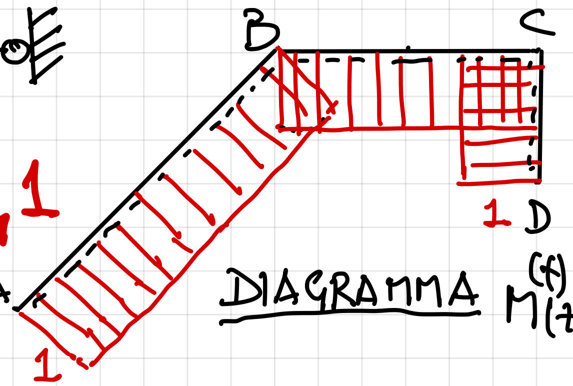
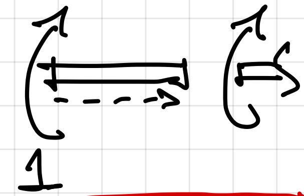


DIAGRAMMA $M^{(f)}_{(z)}$

TRATTO AB $\quad 0 \le z \le L\sqrt{2}$

TRATTO BC $\quad 0 \le z \le L$

TRATTO CD $\quad 0 \le z \le \frac{L}{2}$



$$\boxed{M^{(f)}(z) = 1} \quad \text{cost}$$

vale in tutti i tratti

$$L_{ve} = 1 \cdot \varphi_D^{(r)} + \sum_j R_j^{(f)} \eta_j^{(r)} =$$

$$-\bar{\varepsilon}_A M_A^{(r)} = -\bar{\varepsilon}_A \left[-\frac{9}{8} qL^2 \right]$$

$$= \varphi_D^{(r)} + \underbrace{R_{x_C}^{(f)}}_{=\emptyset} \eta_C^{0} + \underbrace{M_A^{(f)}}_{-1} \varphi_A^{(r)} = \varphi_D^{(r)} - \bar{\varepsilon}_A \frac{9}{8} qL^2$$

$$L_{vi} = \int_{str} M^{(f)} \frac{M^{(r)}}{EI} \, d_{str} + \int_{str} M^{(f)} \frac{\alpha \Delta T}{h} \, d_{str} =$$

$$= \frac{1}{EI} \left[\int_0^{L\sqrt{2}} \left[\frac{9}{8} qL^2 - \frac{qL\sqrt{2}}{2} \cdot z \right] dz + \int_0^{L} \frac{qL^2}{8} \, dz + \right.$$

$$\left. + \int_0^{L/2} \frac{q}{2} \underbrace{\left(\frac{L}{2} - z \right)^2}_{\frac{L^2}{4} + z^2 - Lz} \right] + \frac{\alpha \Delta T}{h} \int_0^{L\sqrt{2}} dz =$$

$$= \frac{1}{EI}\left[\frac{9}{8}qL^2 \cdot L\sqrt{2} - \frac{qL\sqrt{2}}{2}\cdot\frac{1}{\cancel{2}}\cdot L^2\cancel{2} + \frac{qL^2}{8}\cdot L + \frac{qL^2}{8}\cdot\frac{L}{2} + \frac{qL^3}{48} - \frac{qL^3}{16}\right] +$$

$$+ \alpha\frac{\Delta T}{h}\cdot L\sqrt{2} =$$

$$= \frac{qL^3}{EI}\left[\underbrace{\frac{9}{8}\sqrt{2} - \frac{\sqrt{2}}{2}}_{\frac{5}{8}\sqrt{2}} + \underbrace{\frac{1}{8} + \cancel{\frac{1}{16}} + \frac{1}{48} - \cancel{\frac{1}{16}}}_{\frac{7}{48}}\right] + \alpha\frac{\Delta T}{h}L\sqrt{2} =$$

$$= \underbrace{\frac{qL^3}{EI}\left[\frac{5}{8}\sqrt{2} + \frac{7}{48}\right] + \alpha\frac{\Delta T}{h}L\sqrt{2}}$$

⇨ $L_{ve} = L_{vi}$ fornisce

$$\left[\frac{3}{8} + \frac{41}{48\sqrt{2}}\right]\frac{qL^2}{EI}$$

$$\varphi_D^{(r)} - \underbrace{\varepsilon_A}_{\frac{8}{9}\frac{L}{EI}}\cdot\frac{9}{8}qL^2 = \underbrace{\frac{qL^3}{EI}\left[\frac{5}{8}\sqrt{2} + \frac{7}{48}\right] + \alpha\frac{\Delta T}{h}L\sqrt{2}}_{\frac{qL^3}{EI}\left[\sqrt{2}+1\right]}$$

e, in definitiva:

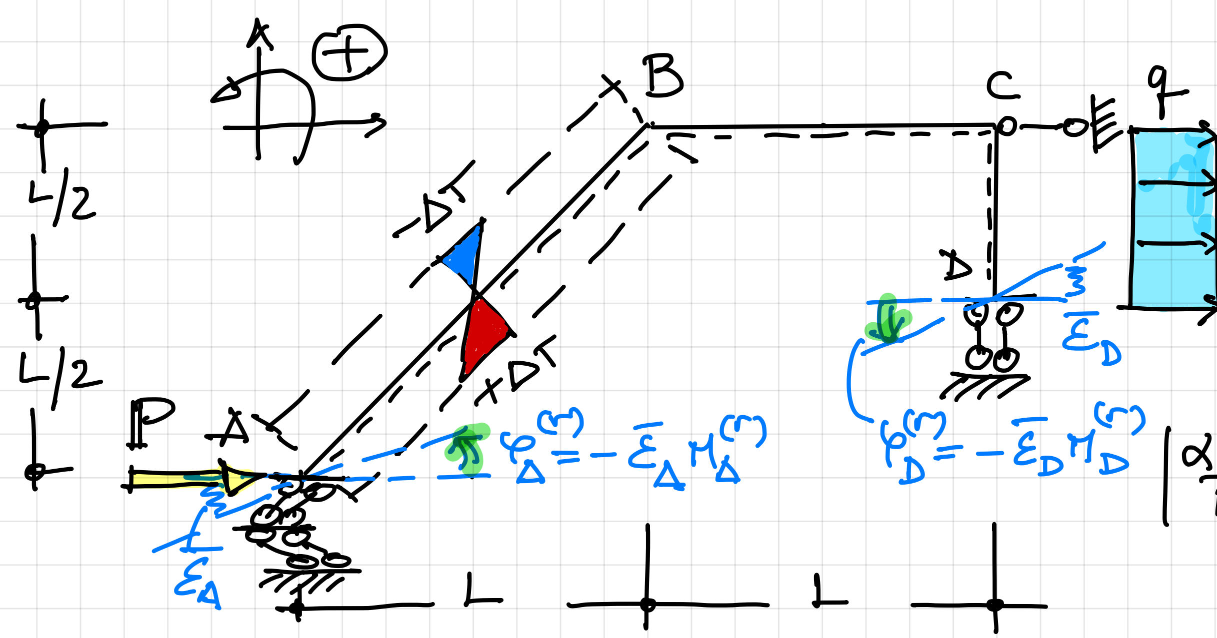
$$\boxed{\varphi_D = \frac{qL^3}{EI}\left[\sqrt{2}+2\right]}$$

⇨ POSITIVA ⇨ ANTIORARIA

MECCANICA delle STRUTTURE – P. FUSCHI
TEST in ITINERE del 8 GENNAIO 2025

TIPO 2

ES. #1 RISOLVERE LA STRUTTURA UNA VOLTA IPERSTATICA SEGUENTE DETERMINANDO IL DIAGRAMMA DEI MOMENTI_



POSIZIONI:

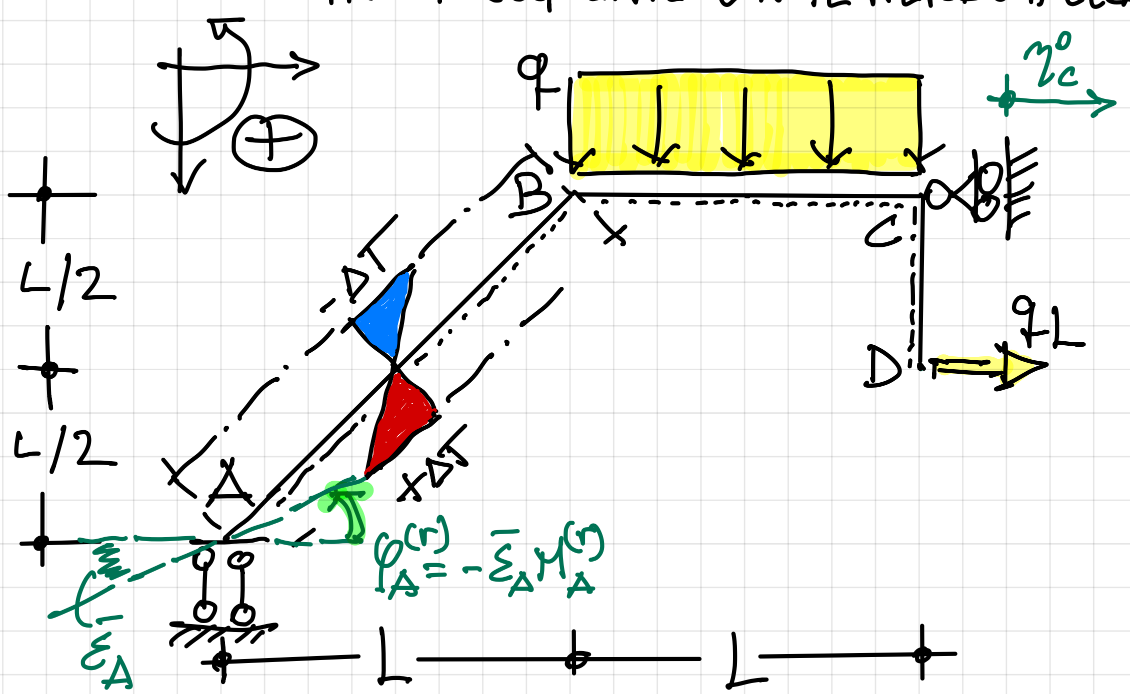
$$|\mathbb{P}| = qL$$

$$|\bar{\varepsilon}_A| = \frac{L}{EI}\left[\frac{\sqrt{2}}{9} - \frac{3}{2}\right]$$

$$|\bar{\varepsilon}_D| = \frac{8}{9}\frac{L}{EI}\sqrt{2}$$

$$\left|\frac{\alpha\Delta T}{h}\right| = \frac{qL^2}{EI}\left[\frac{37}{24\sqrt{2}} + \frac{1}{2}\right]$$

ES. #2 DETERMINARE LA ROTAZIONE DELLA SEZ. D DELLA STRUTT. SEGUENTE CON IL METODO DELLA FORZA UNITARIA_



POSIZIONI:

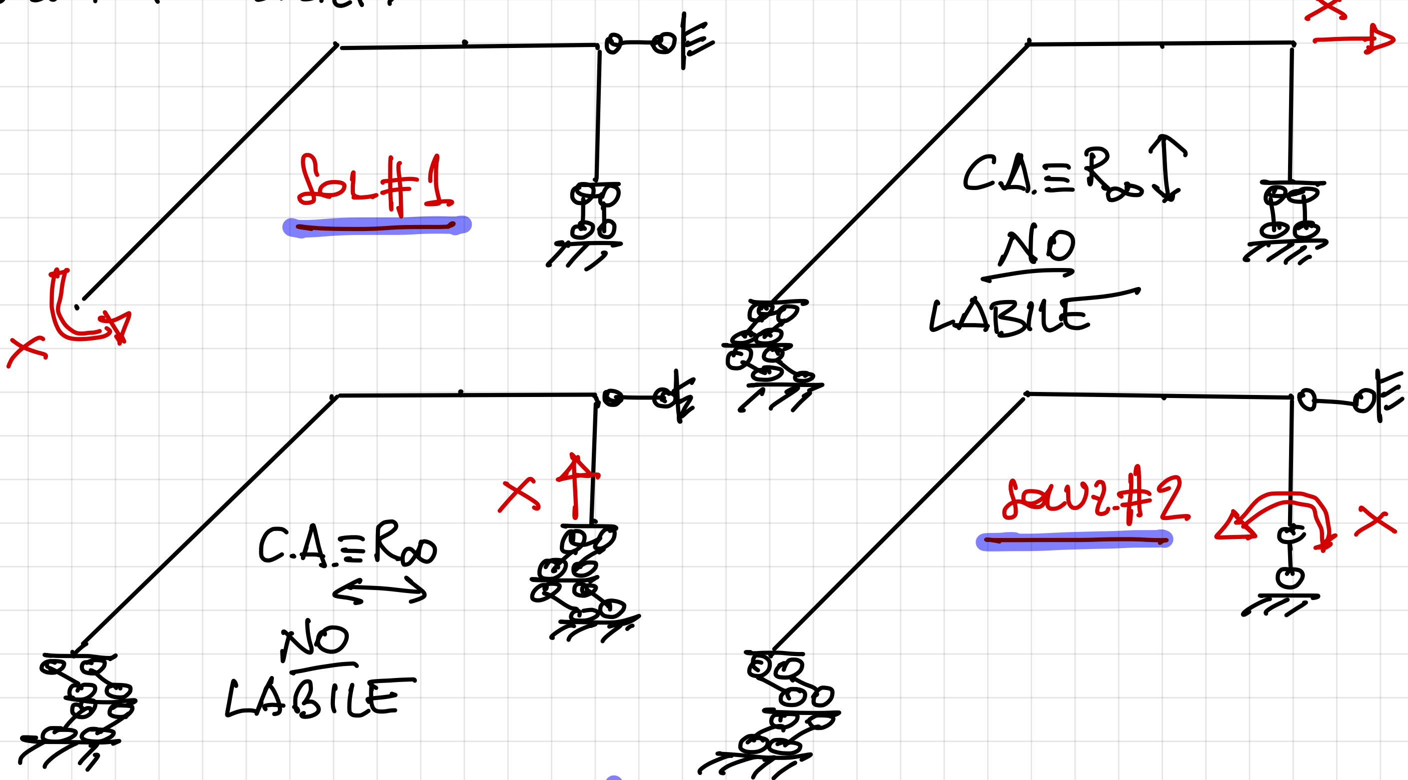
$$|\eta_c^0| = \frac{L^3}{EI}$$

$$|\bar{\varepsilon}_A| = \frac{L}{8EI}$$

$$\left|\frac{\alpha\Delta T}{h}\right| = \frac{qL^2}{2EI}$$

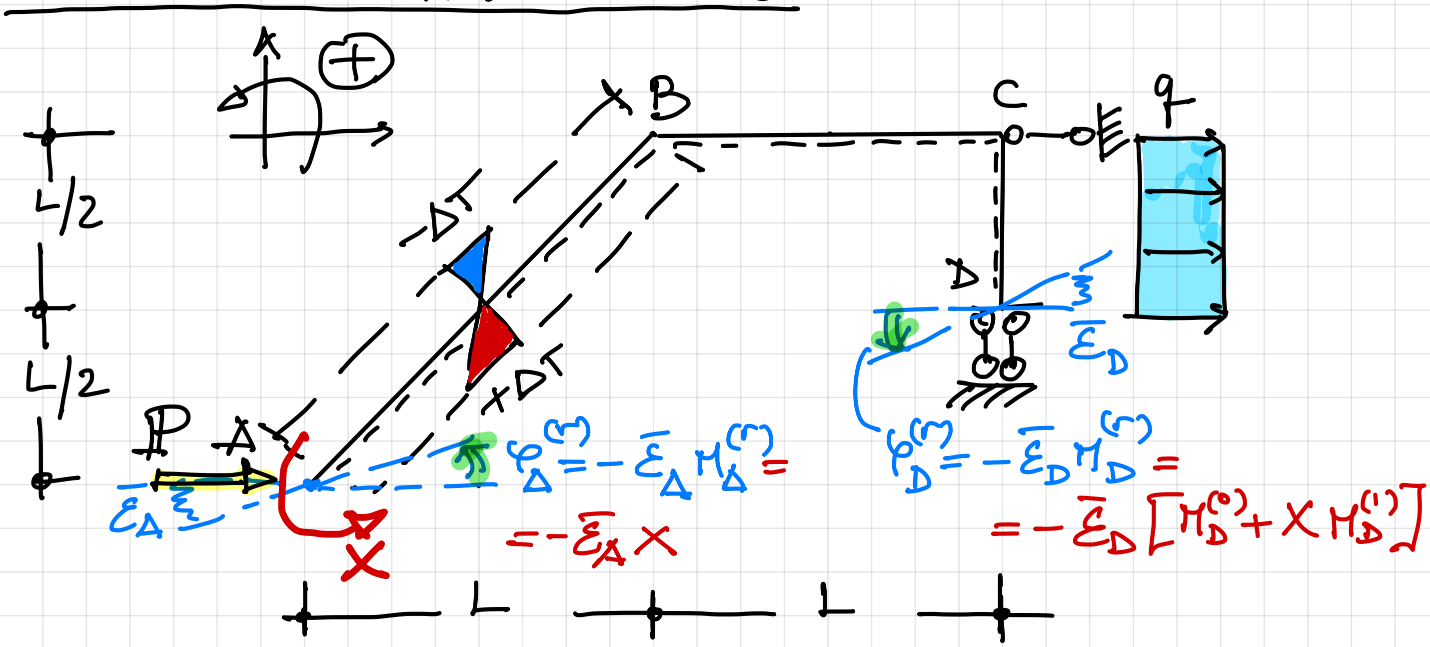
ES. #1 - tipo 2 — SOLUZIONI

SOLUZIONI POSSIBILI:



SOLUZIONE #1

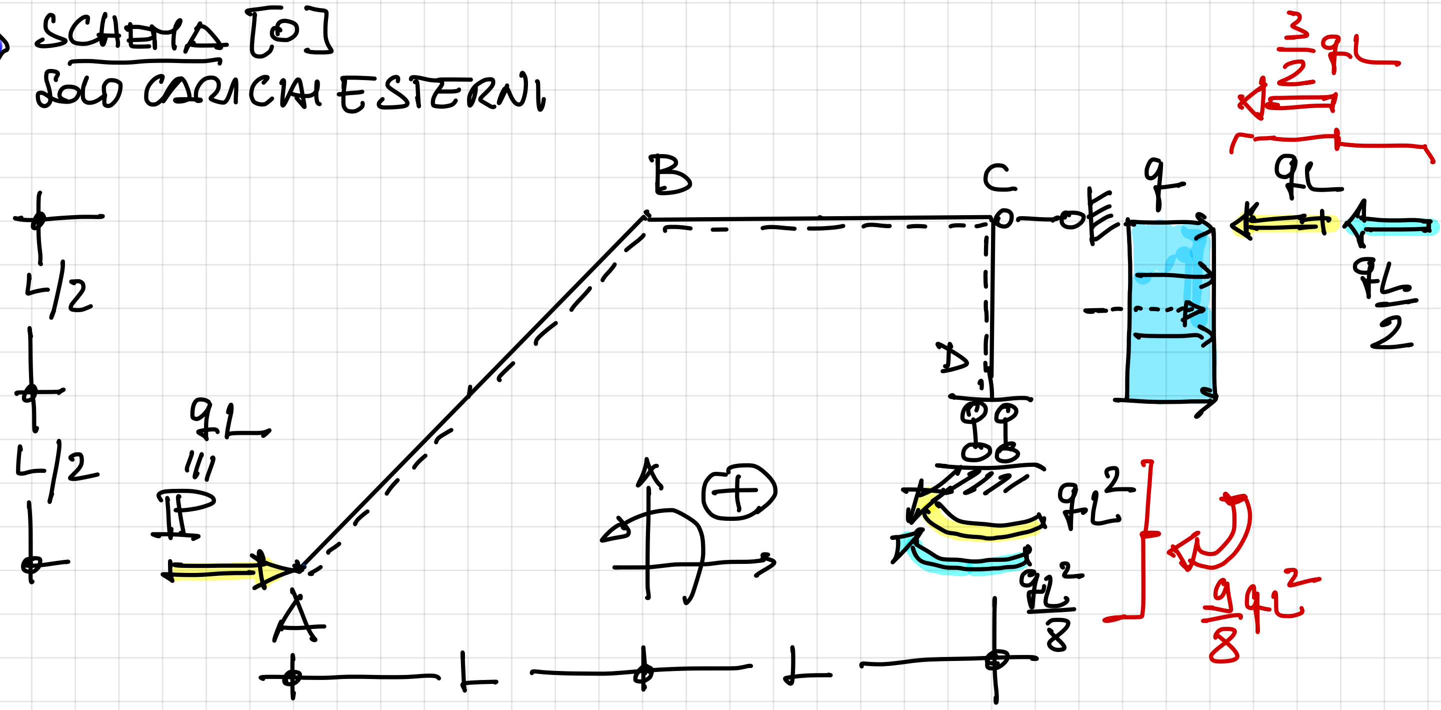
SISTEMA PRINCIPALE ISOSTATICO



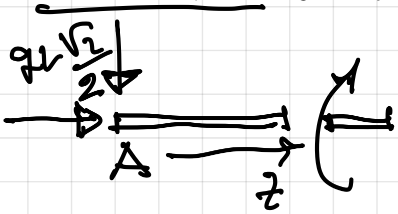
$$\varphi_A^{(r)} = -\bar{\varepsilon}_A M_A^{(r)} = -\bar{\varepsilon}_A X$$

$$\varphi_D^{(r)} = -\bar{\varepsilon}_D M_D^{(r)} = -\bar{\varepsilon}_D \left[M_D^{(0)} + X M_D^{(1)} \right]$$

SCHEMA [0]
SOLO CARICHI ESTERNI



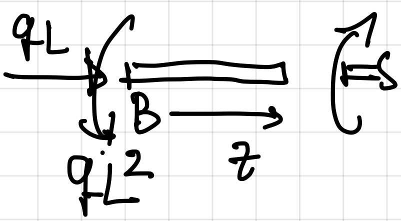
TRATTO AB $0 \le z \le L\sqrt{2}$



$$\boxed{M^{(0)}(z) = -qL\frac{\sqrt{2}}{2}\cdot z}$$

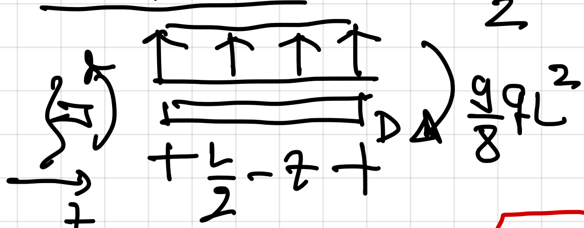
$M_A = 0$
$M_B = -qL^2$

TRATTO BC $0 \le z \le L$



$$\boxed{M^{(0)}(z) = -qL^2} \text{ cost.}$$

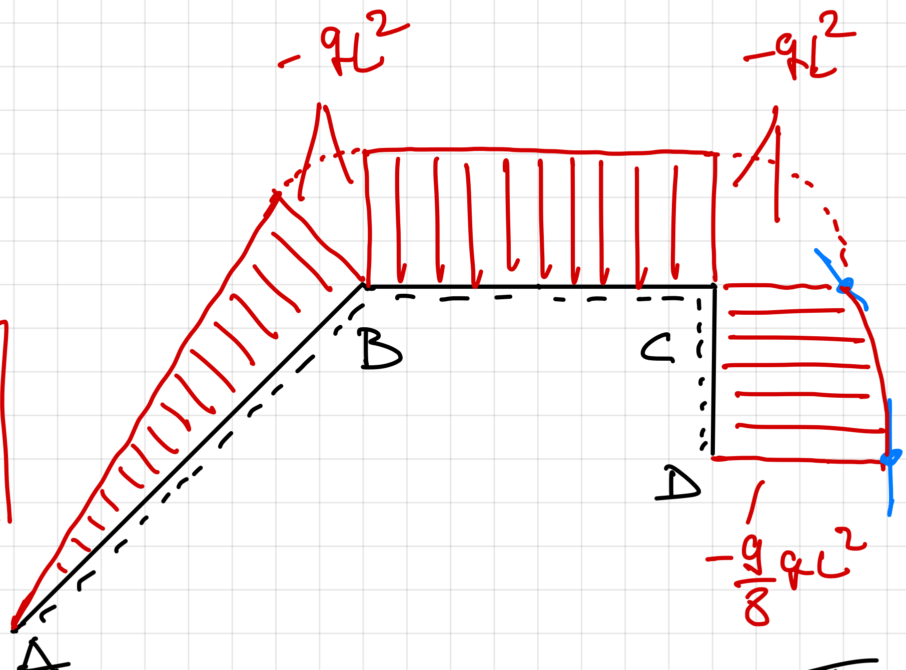
TRATTO CD $0 \le z \le \frac{L}{2}$



$$\boxed{M^{(0)}(z) = -\frac{9}{8}qL^2 + \frac{q\left(\frac{L}{2}-z\right)^2}{2}}$$

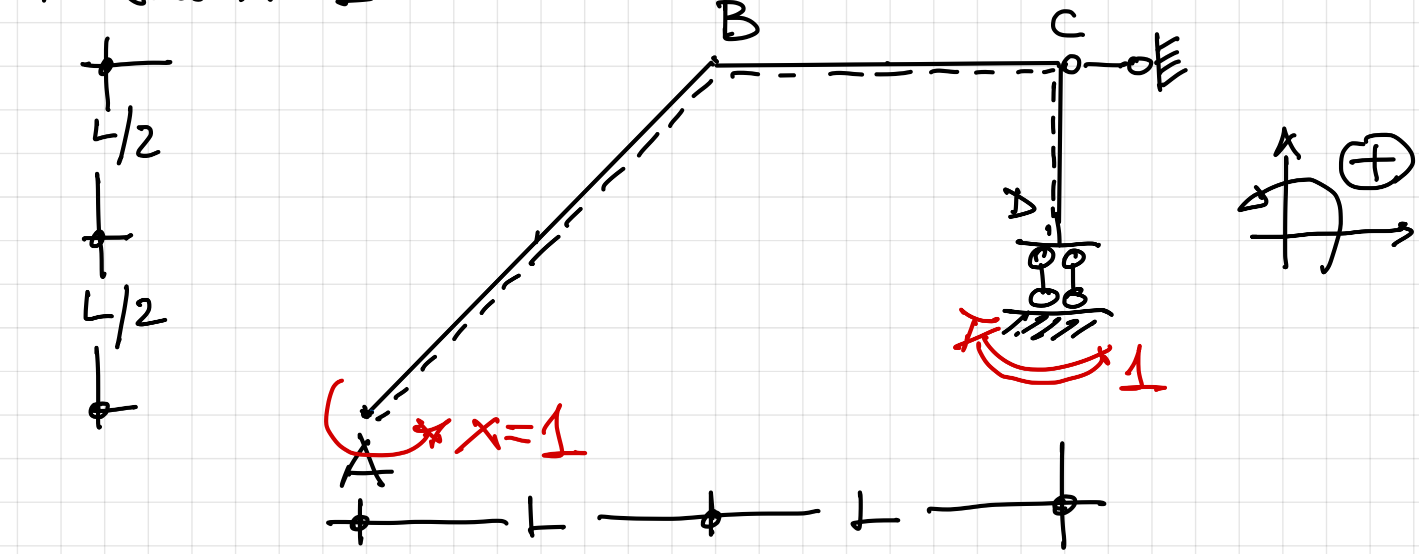
$\hookrightarrow \begin{cases} M_C = -qL^2 \\ M_D = -\frac{9}{8}qL^2 \end{cases}$

DIAGRAMMA $M^{(0)}(z)$

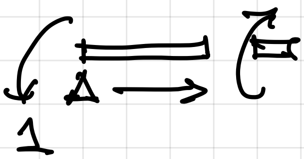


SCHEMA [1]

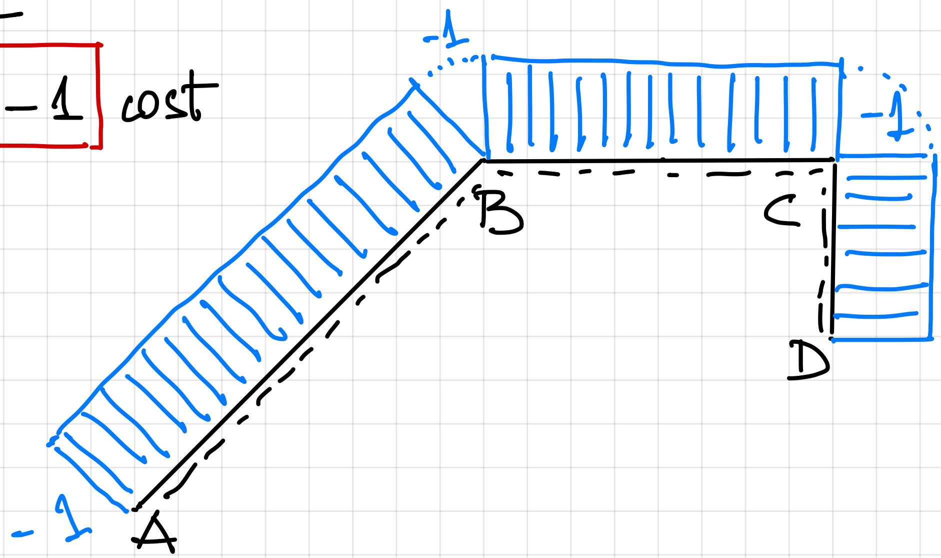
Solo $X = 1$



TRATTO AB $0 \le z \le L\sqrt{2}$



$$\boxed{M^{(1)}(z) = -1} \quad \text{cost}$$



TRATTO BC $0 \le z \le L$

&

TRATTO CD $0 \le z \le \frac{L}{2}$

... come sopra

$$\boxed{M^{(1)}(z) = -1} \quad \text{cost}$$

$$L_{ve} = \underset{\underset{1}{\parallel}}{X_i}\,\eta_i^{(r)} + \sum_j R_j^{(1)} \eta_j^{(r)} = 1 \cdot \varphi_A^{(r)} + M_D^{(1)} \varphi_D^{(r)} =$$

$$= -\bar{\varepsilon}_A X + \underbrace{M_D^{(1)}}_{-1} (-\bar{\varepsilon}_D)\left[\underbrace{M_D^{(0)}}_{-\frac{9}{8}qL^2} + X \underbrace{M_D^{(1)}}_{-1}\right] =$$

$$= \underbrace{-\bar{\varepsilon}_A X - \bar{\varepsilon}_D \left[\frac{9}{8}qL^2 + X\right]}$$

$$\mathcal{L}_{vi} = \int_{str} M^{(1)} \frac{\cancel{M^{(r)}}}{EI} ds_{tr} + \int_{str} M^{(1)} \frac{\alpha \overline{\Delta T}}{h} ds_{tr} =$$

$M^{0} + X M^{1}$

$$= \frac{1}{EI}\int_{str} M^{(1)} M^{(0)} ds_{tr} + \frac{X}{EI}\int_{str} \left[M^{(1)}\right]^2 ds_{tr} + \frac{\alpha\overline{\Delta T}}{h}\int_{str} M^{(1)} ds_{tr} =$$

$$= \frac{1}{EI}\left[\int_0^{L\sqrt{2}} \frac{qL\sqrt{2}}{2} z\, dz + \int_0^L qL^2 dz + \right.$$

$$\left. + \int_0^{\frac{L}{2}} \left[\frac{9}{8} qL^2 - \frac{q}{2}\left(\frac{L^2}{4} + z^2 - Lz\right)\right] dz \right] +$$

$$+ \frac{X}{EI}\left[\int_0^{L\sqrt{2}} dz + \int_0^L dz + \int_0^{\frac{L}{2}} dz\right] + \alpha\frac{\overline{\Delta T}}{h}\int_0 (-1)\, dz =$$

$-\frac{q}{6}\cdot\frac{L^3}{8}$

$$= \frac{1}{EI}\left[\frac{qL\sqrt{2}}{2}\cdot\left[\frac{z^2}{2}\right]_0^{L\sqrt{2}} + qL^2\cdot L + \frac{9}{8}qL^2\cdot\frac{L}{2} - \frac{qL^2}{8}\cdot\frac{L}{2} - \frac{q}{2}\left[\frac{z^3}{3}\right]_0^{\frac{L}{2}} + \right.$$

$$\left. + \frac{qL}{2}\left[\frac{z^2}{2}\right]_0^{\frac{L}{2}}\right] +$$

$\frac{q L}{4}\cdot\frac{L^2}{4}$

$\frac{\cancel{L^2 2}}{\cancel{2}}$

$$+ \frac{X}{EI}\left\{L\sqrt{2} + L + \frac{L}{2}\right\} - \alpha\frac{\overline{\Delta T}}{h}\cdot L\sqrt{2} =$$

$$= \frac{qL^3}{EI}\left[\frac{\sqrt{2}}{2} + \overbrace{1 + \frac{9}{16} - \cancel{\frac{1}{16}} - \frac{1}{48} + \cancel{\frac{1}{16}}}^{\frac{37}{24}}\right] +$$

$$+ \frac{XL}{EI}\left[\sqrt{2} + \frac{3}{2}\right] - \alpha\frac{\overline{\Delta T}}{h}\cdot L\sqrt{2} =$$

$$= \underbrace{\frac{qL^3}{EI}\left[\frac{\sqrt{2}}{2} + \frac{37}{24}\right]}\underbrace{+ \frac{XL}{EI}\left[\sqrt{2} + \frac{3}{2}\right] - \frac{\alpha\overline{\Delta T}}{h}\cdot L\sqrt{2}}$$

$L_{ve} = L_{vi}$ fornisce:

$$-\bar{\varepsilon}_A X - \bar{\varepsilon}_D \left[\frac{9}{8} qL^2 + X \right] =$$

$$= \frac{qL^3}{EI}\left[\frac{\sqrt{2}}{2} + \frac{37}{24}\right] + \frac{XL}{EI}\left[\sqrt{2} + \frac{3}{2}\right] - \frac{\alpha \Delta T}{h} \cdot L\sqrt{2}$$

$$-X\left[\bar{\varepsilon}_A + \bar{\varepsilon}_D + \frac{L}{EI}\sqrt{2} + \frac{3}{2}\frac{L}{EI}\right] =$$

$$\bar{\varepsilon}_A = \frac{L}{EI}\left[\frac{\sqrt{2}}{9} - \frac{3}{2}\right] \qquad \bar{\varepsilon}_D = \frac{8}{9}\frac{L\sqrt{2}}{EI}$$

$$= \cancel{\frac{qL^3}{EI}\left[\frac{\sqrt{2}}{2} + \frac{37}{24}\right] - \frac{\alpha \Delta T}{h} L\sqrt{2}} + \bar{\varepsilon}_D \cdot \frac{9}{8} qL^2$$

$$\frac{\alpha \Delta T}{h} = \frac{qL^2}{EI}\left[\frac{37}{24\sqrt{2}} + \frac{1}{2}\right] \qquad \bar{\varepsilon}_D = \frac{8}{9}\frac{L\sqrt{2}}{EI}$$

$$-X \frac{\cancel{L}}{\cancel{EI}} \underbrace{\left[\frac{\sqrt{2}}{9} - \cancel{\frac{3}{2}} + \frac{8}{9}\sqrt{2} + \sqrt{2} + \cancel{\frac{3}{2}}\right]}_{2\cancel{\sqrt{2}}} = \frac{qL^{\cancel{3}\,2}}{\cancel{EI}}\cancel{\sqrt{2}}$$

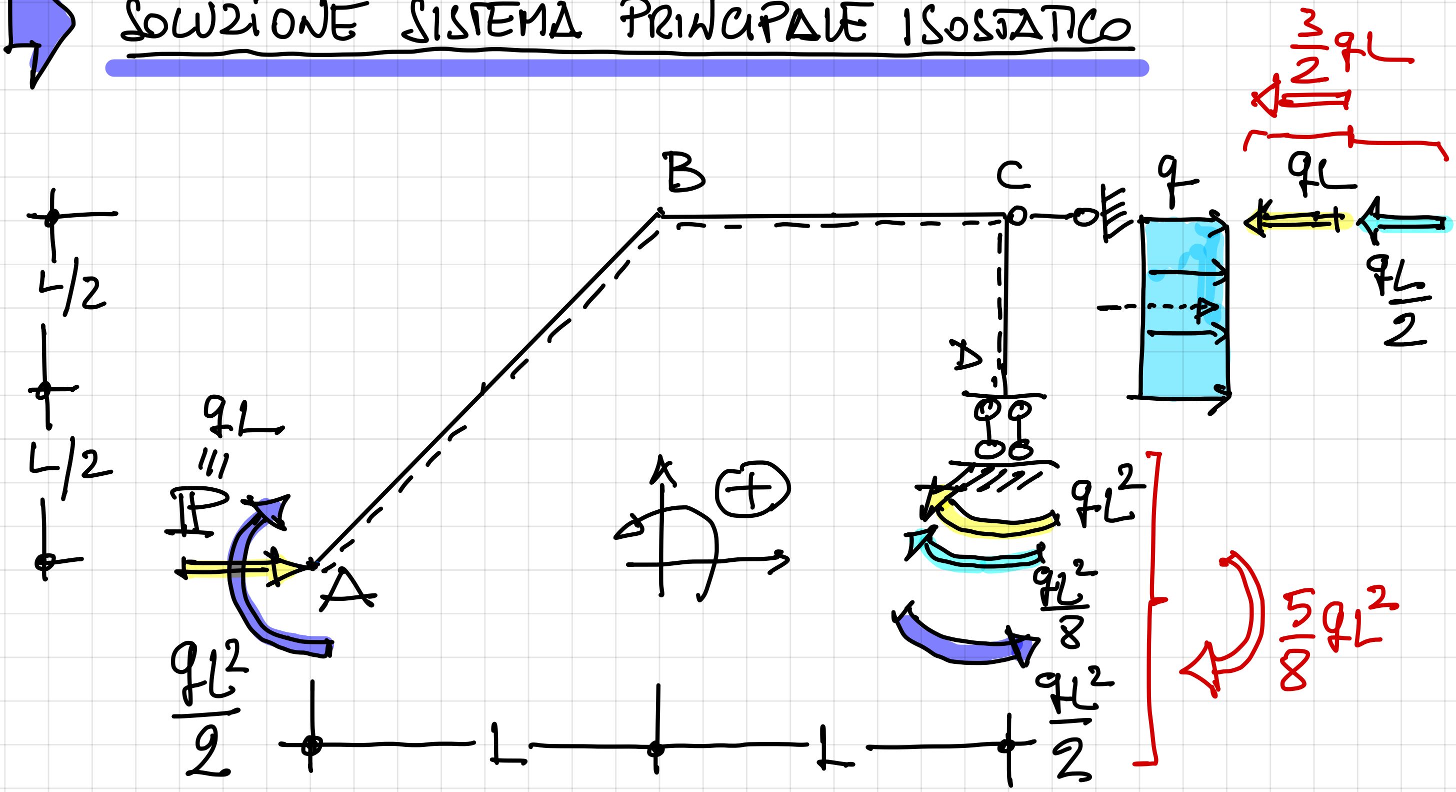
da cui

$$\boxed{X = -\frac{qL^2}{2}}$$

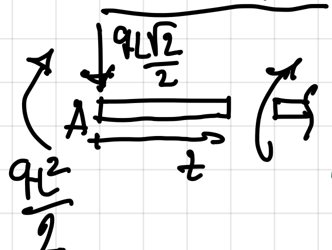
NEGATIVO!

verso opposto a quello ipotizzato

SOLUZIONE SISTEMA PRINCIPALE ISOSTATICO

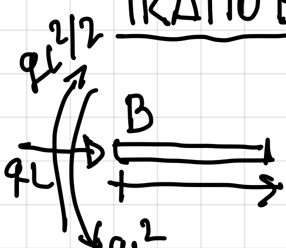


TRATTO AB $0 \le z \le L\sqrt{2}$



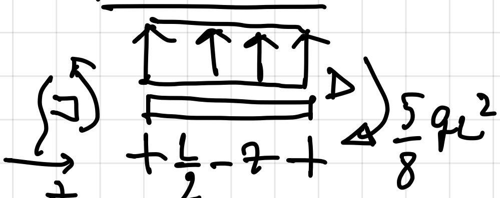
$$M^{(r)}(z) = \frac{qL^2}{2} - \frac{qL\sqrt{2}}{2} \cdot z \qquad \begin{cases} M_A = \frac{qL^2}{2} \\ M_B = -\frac{qL^2}{2} \end{cases}$$

TRATTO BC $0 \le z \le L$



$$M(z) = -\frac{qL^2}{2} \text{ cost.}$$

TRATTO CD $0 \le z \le \frac{L}{2}$



$$M^{(r)}(z) = \frac{q}{2}\left(\frac{L}{2}-z\right)^2 - \frac{5}{8}qL^2 \qquad \begin{cases} M_C = -\frac{qL^2}{2} \\ M_D = -\frac{5}{8}qL^2 \end{cases}$$

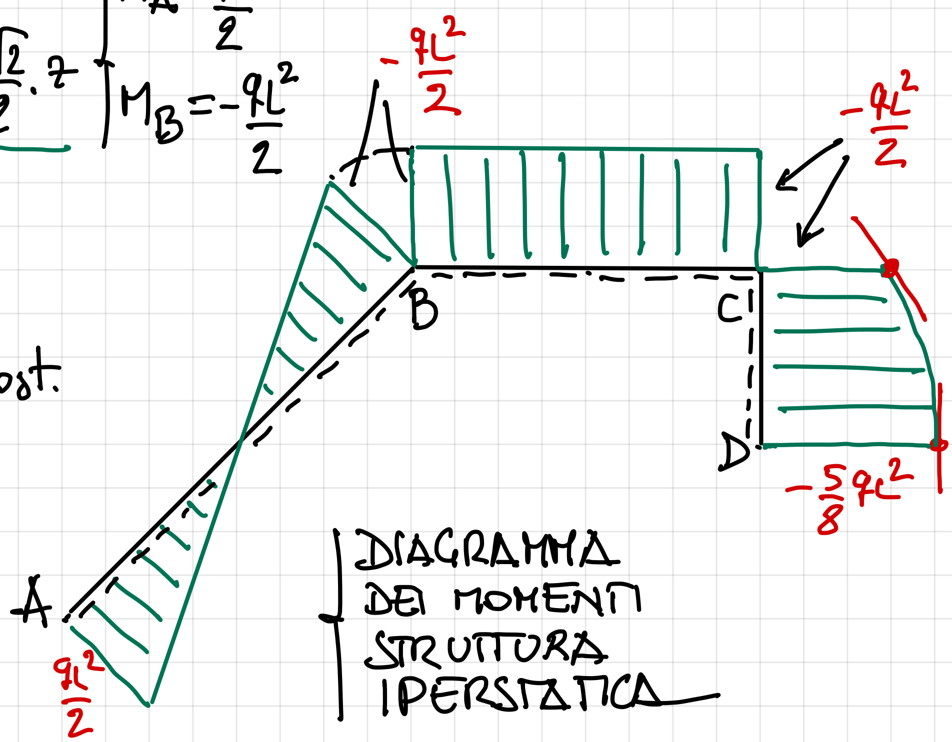
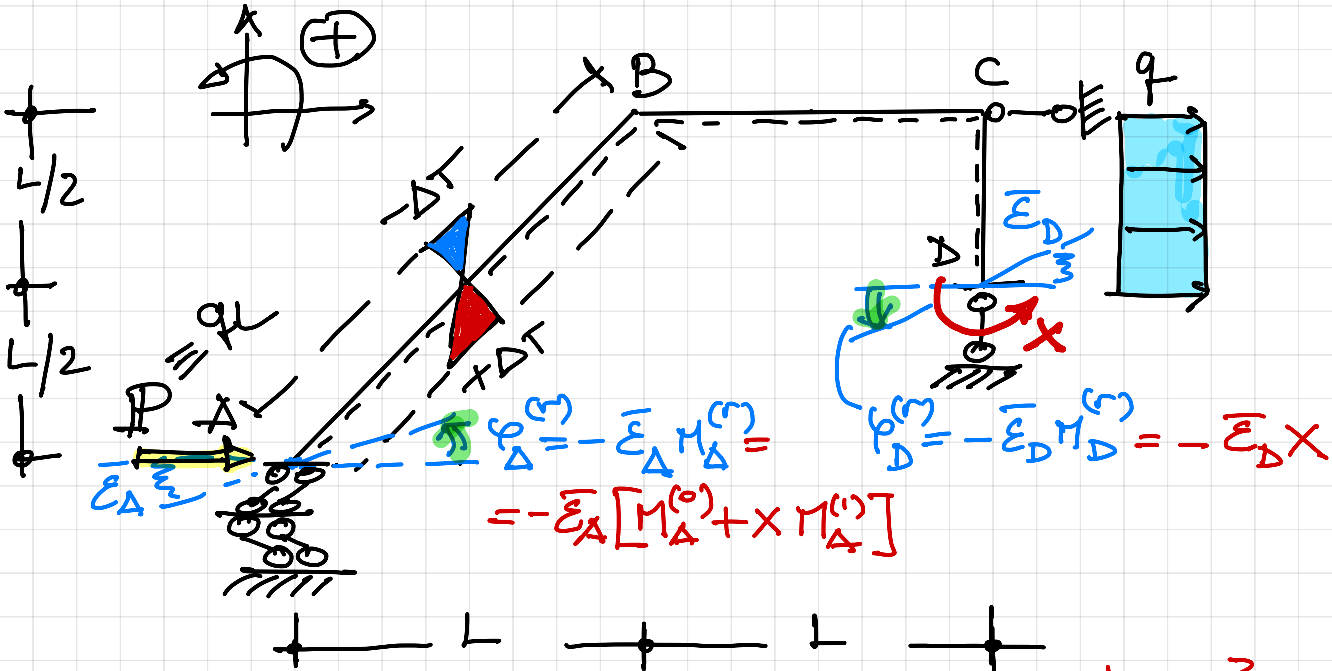


DIAGRAMMA DEI MOMENTI STRUTTURA IPERSTATICA

SOLUZIONE #2

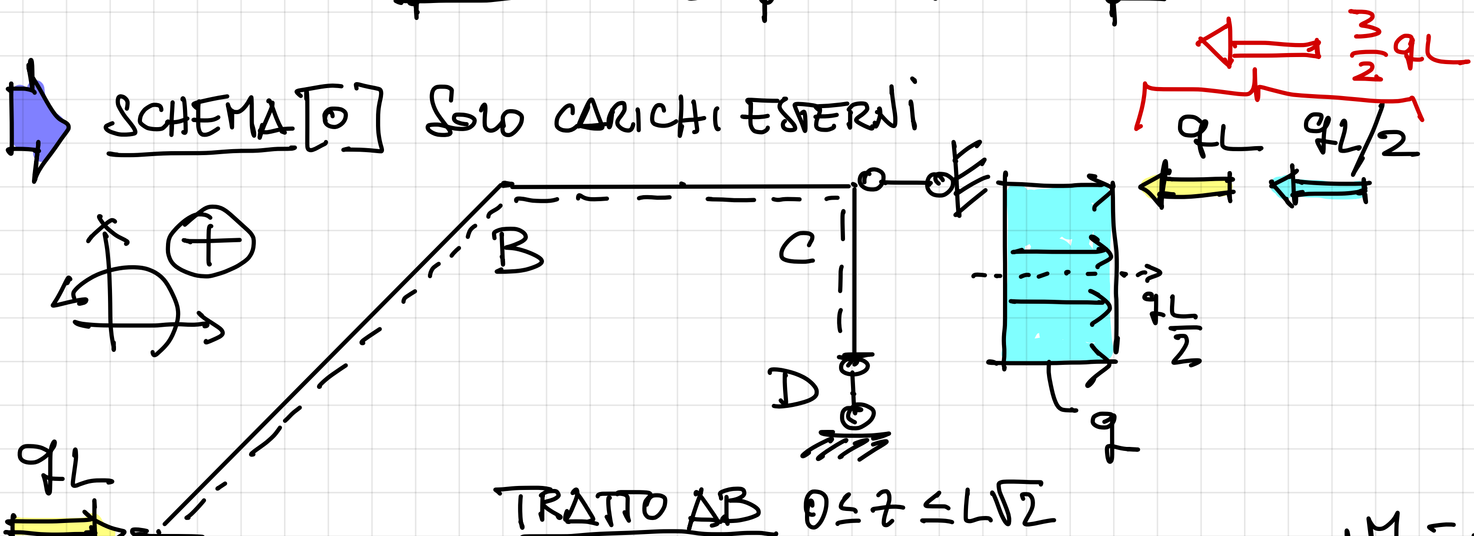
SISTEMA PRINCIPALE ISOSTATICO

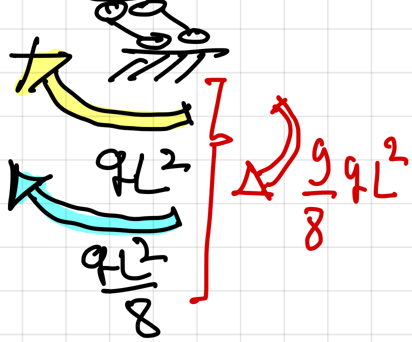


$$\varphi_A^{(m)} = -\bar{\varepsilon}_A M_A^{(m)} = -\bar{\varepsilon}_A\left[M_A^{(0)} + X M_A^{(1)}\right]$$

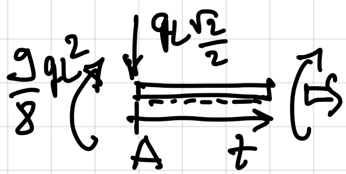
$$\varphi_D^{(m)} = -\bar{\varepsilon}_D M_D^{(m)} = -\bar{\varepsilon}_D X$$

SCHEMA [0] SOLO CARICHI ESTERNI





TRATTO AB $0 \le z \le L\sqrt{2}$

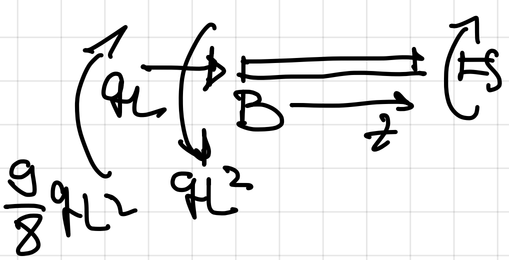


$$\boxed{M^{(0)}(z) = \frac{9}{8}qL^2 - \frac{qL\sqrt{2}}{2}\cdot z}$$

$M_A = \frac{9}{8}qL^2$

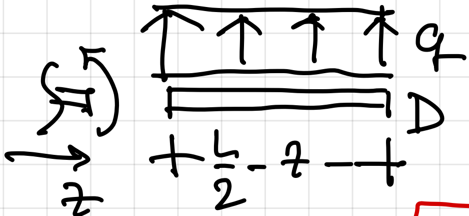
$M_B = \frac{qL^2}{8}$

TRATTO BC $0 \le z \le L$



$$\boxed{M^{(0)}(z) = \frac{qL^2}{8}} \text{ cost}$$

TRATTO CD $0 \le z \le \frac{L}{2}$

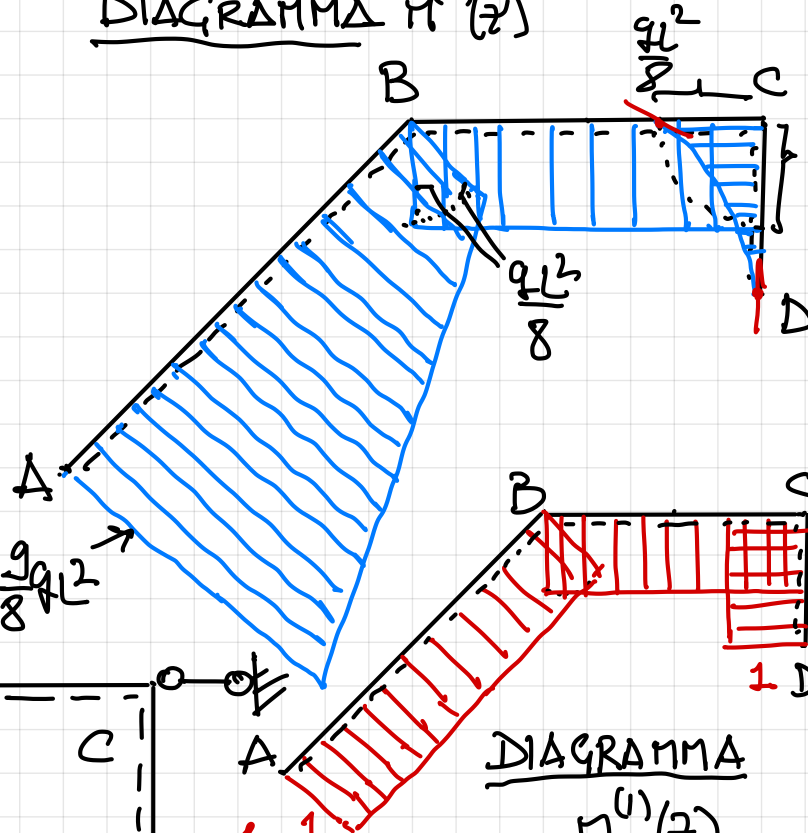


$$\boxed{M^{(0)}(z) = \frac{q\left(\frac{L}{2}-z\right)^2}{2}}$$

$M_C = \frac{qL^2}{8}$

$M_D = \phi$

DIAGRAMMA $M^{(0)}(z)$



SCHEMA [1]

SOLO $X=1$

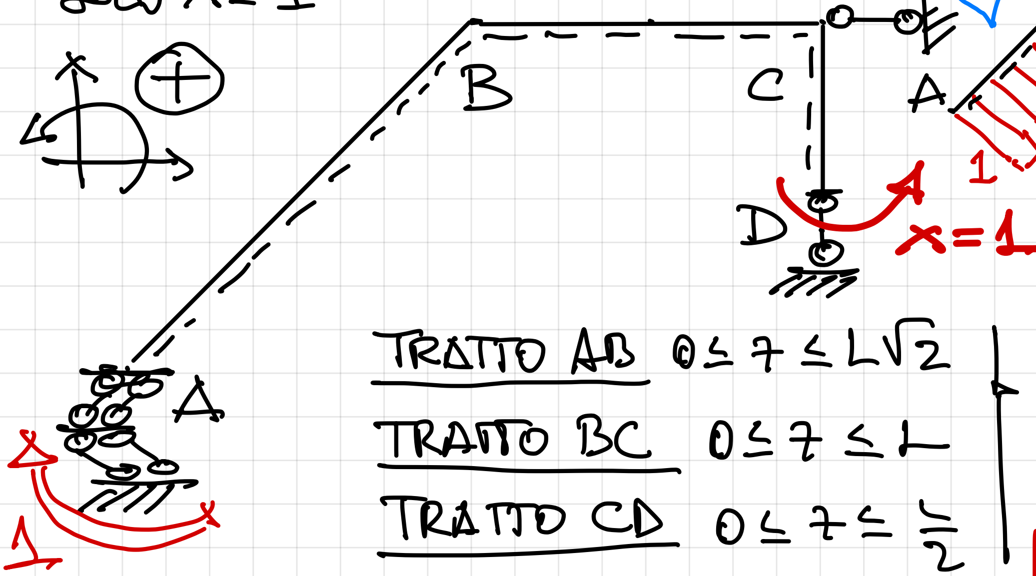
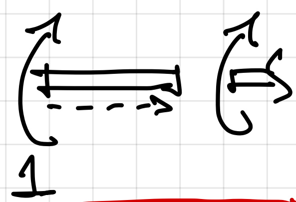


DIAGRAMMA $M^{(1)}(z)$

TRATTO AB $0 \le z \le L\sqrt{2}$

TRATTO BC $0 \le z \le L$

TRATTO CD $0 \le z \le \frac{L}{2}$



$\boxed{M^{(1)}(z) = 1}$ cost

vale in tutti i tratti

$$L_{ve} = \underbrace{X_i}_{1}\, \eta_i^{(r)} + \sum_j R_j^{(1)} \eta_j^{(r)} = 1 \cdot \varphi_D^{(r)} + M_A^{(1)} \varphi_A^{(r)} =$$

$$= -\bar{\varepsilon}_D X + \underbrace{M_A^{(1)}}_{-1}\left(-\bar{\varepsilon}_A\right)\Big[\underbrace{M_A^{(0)}}_{-\frac{9}{8}qL^2} + X \underbrace{M_A^{(1)}}_{-1}\Big] =$$

$$= -\bar{\varepsilon}_D X - \bar{\varepsilon}_A\left[\frac{9}{8}qL^2 + X\right]$$

$$L_{vi} = \int_{str} M^{(1)} \frac{M^{(T)}}{EI}\, ds_{tr} + \int_{str} M^{(1)} \frac{\alpha \overline{\Delta T}}{h}\, ds_{tr} =$$

$$= \frac{1}{EI}\int_{str} M^{(1)} M^{(0)}\, ds_{tr} + \frac{X}{EI}\int_{str} \left[M^{(1)}\right]^2 ds_{tr} + \frac{\alpha \overline{\Delta T}}{h}\int_{str} M^{(1)}\, ds_{tr} =$$

$$= \frac{1}{EI}\left[\int_0^{L\sqrt{2}} \left[\frac{9}{8} qL^2 - \frac{qL\sqrt{2}}{2}\cdot z\right] dz + \int_0^{L} \frac{qL^2}{8}\, dz + \int_0^{L/2} \frac{q}{2}\left(\frac{L}{2} - z\right)^2\right] + \qquad \frac{L^2}{4} + z^2 - Lz$$

$$+ \frac{X}{EI}\left[\int_0^{L\sqrt{2}} dz + \int_0^{L} dz + \int_0^{L/2} dz\right] + \frac{\alpha \overline{\Delta T}}{h}\int_0^{L\sqrt{2}} dz =$$

$$= \frac{1}{EI}\left[\frac{9}{8} qL^2 \cdot L\sqrt{2} - \frac{qL\sqrt{2}}{2}\cdot\frac{1}{\cancel{2}}\cdot L^2 \cancel{2} + \frac{qL^2}{8}\cdot L + \frac{qL^2}{8}\cdot\frac{L}{2} + \frac{qL^3}{48} - \frac{qL^3}{16}\right] +$$

$$+ \frac{X}{EI}\left[L\sqrt{2} + L + \frac{L}{2}\right] + \frac{\alpha \overline{\Delta T}}{h}\cdot L\sqrt{2} =$$

$$= \frac{qL^3}{EI}\left[\underbrace{\frac{9}{8}\sqrt{2} - \frac{\sqrt{2}}{2}}_{\frac{5}{8}\sqrt{2}} + \underbrace{\frac{1}{8} + \cancel{\frac{1}{16}} + \frac{1}{48} - \cancel{\frac{1}{16}}}_{\frac{7}{48}}\right] + \frac{XL}{EI}\left[\sqrt{2} + \frac{3}{2}\right] + \frac{\alpha \overline{\Delta T}}{h} L\sqrt{2} =$$

$$= \underbrace{\frac{qL^3}{EI}\left[\frac{5}{8}\sqrt{2} + \frac{7}{48}\right] + \frac{XL}{EI}\left[\sqrt{2} + \frac{3}{2}\right] + \frac{\alpha \overline{\Delta T}}{h} L\sqrt{2}}_{}$$

$L_{ve} = L_{vi}$ fornisce

$$-\bar{\varepsilon}_D X - \bar{\varepsilon}_A\left[\frac{9}{8}qL^2 + X\right] =$$

$$= \frac{qL^3}{EI}\left[\frac{5}{8}\sqrt{2} + \frac{7}{48}\right] + \frac{XL}{EI}\left[\sqrt{2}+\frac{3}{2}\right] + \frac{\alpha\,\Delta T}{h}L\sqrt{2}$$

$$-X\left[\underbrace{\bar{\varepsilon}_D}_{\frac{8}{9}\frac{L}{EI}\sqrt{2}} + \underbrace{\bar{\varepsilon}_A}_{\frac{L}{EI}\left[\frac{\sqrt{2}}{9}-\frac{3}{2}\right]} + \frac{L}{EI}\sqrt{2} + \frac{3}{2}\frac{L}{EI}\right] =$$

$$= \frac{qL^3}{EI}\left[\frac{5}{8}\sqrt{2} + \frac{7}{48}\right] + \underbrace{\frac{\alpha\,\Delta T}{h}}_{\frac{qL^2}{EI}\left[\frac{37}{24\sqrt{2}}+\frac{1}{2}\right]}\cdot L\sqrt{2} + \underbrace{\bar{\varepsilon}_A}_{\frac{L}{EI}\left[\frac{\sqrt{2}}{9}-\frac{3}{2}\right]}\cdot\frac{9}{8}qL^2$$

$$-X\frac{L}{EI}\left[\underbrace{\frac{8}{9}\sqrt{2}+\frac{\sqrt{2}}{9}-\cancel{\frac{3}{2}}+\sqrt{2}}_{2\sqrt{2}}+\cancel{\frac{3}{2}}\right] =$$

$$= \frac{qL^3}{EI}\left[\frac{5}{8}\sqrt{2}+\frac{7}{48}+\frac{37}{24}+\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{8}-\frac{27}{16}\right]$$

$$\frac{5\sqrt{2}}{4} \qquad \frac{7+74-81}{48} = \emptyset$$

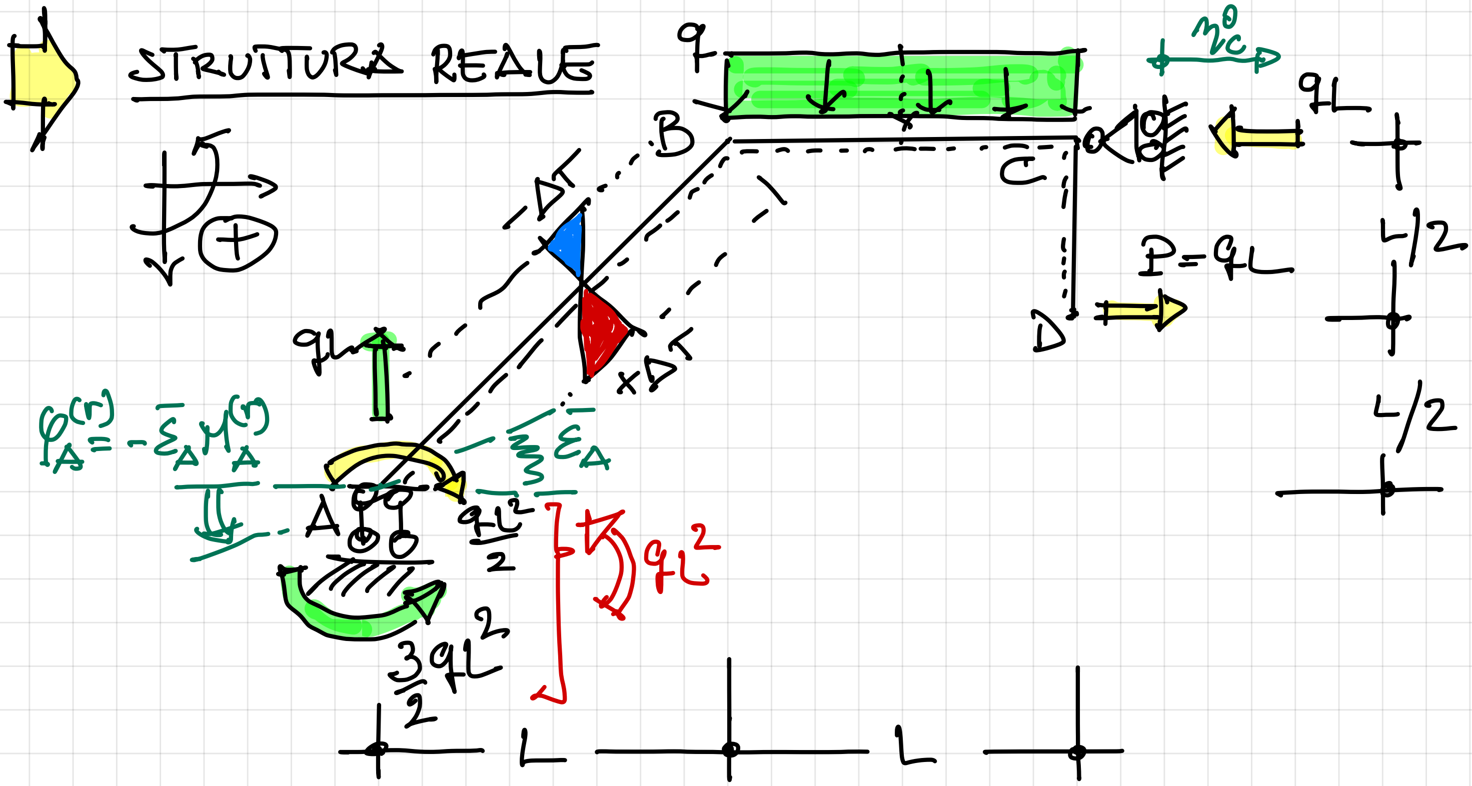
$$-X\frac{\cancel{L}}{\cancel{EI}}\,2\cancel{\sqrt{2}} = \frac{qL^{\cancel{3}\,2}}{\cancel{EI}}\cdot\frac{5}{4}\cancel{\sqrt{2}}$$

da cui $\boxed{X = -\frac{5}{8}qL^2}$

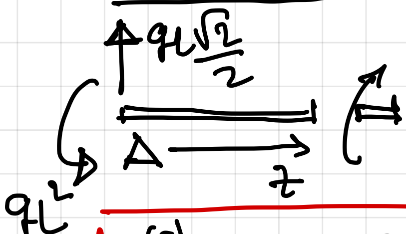
NEGATIVA
VERSO OPPOSTO
A QUELLO IPOTIZZATO!
VALORE OK! cfr. RV e pag. VII

ES. #2 - tipo 2 SOLUZIONE

STRUTTURA REALE



TRATTO AB $0 \le z \le L\sqrt{2}$



$$\boxed{M^{(r)}(z) = -qL^2 + qL\frac{\sqrt{2}}{2} \cdot z}$$

$$\begin{cases} M_A = -qL^2 \\ M_B = \emptyset \end{cases}$$

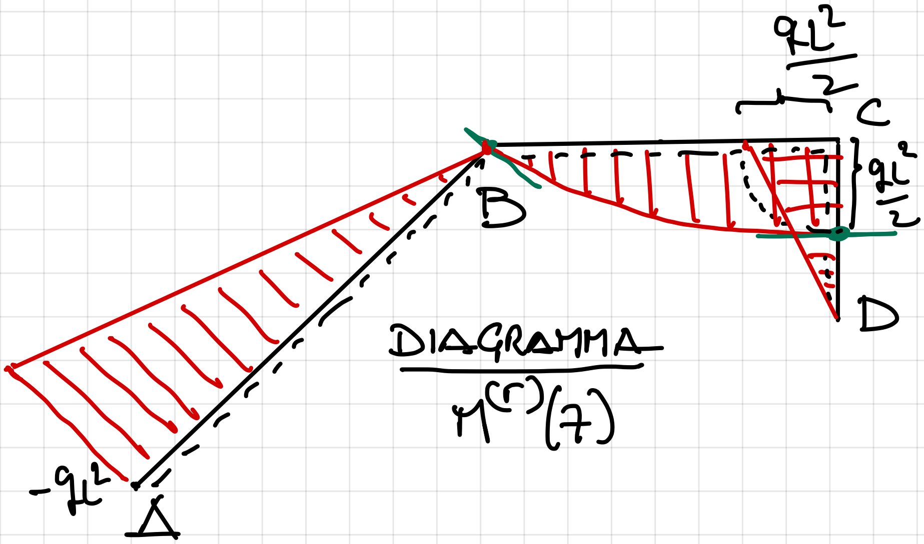
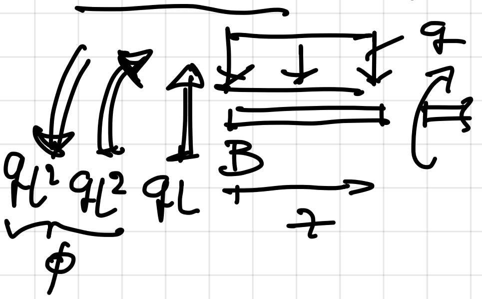


DIAGRAMMA $M^{(r)}(z)$

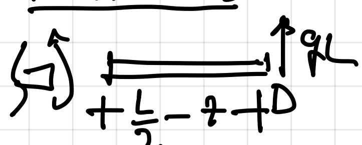
TRATTO BC $0 \le z \le L$



$$\boxed{M^{(r)}(z) = qL \cdot z - \frac{qz^2}{2}}$$

$$\begin{cases} M_B = \emptyset \\ M_C = \frac{qL^2}{2} \end{cases}$$

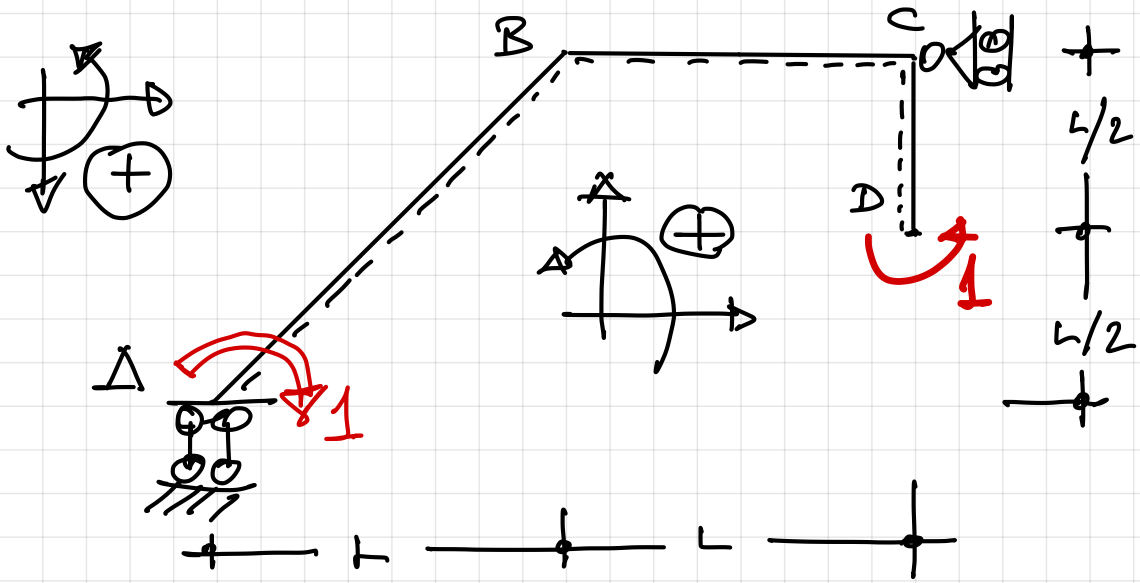
TRATTO CD $0 \le z \le \frac{L}{2}$



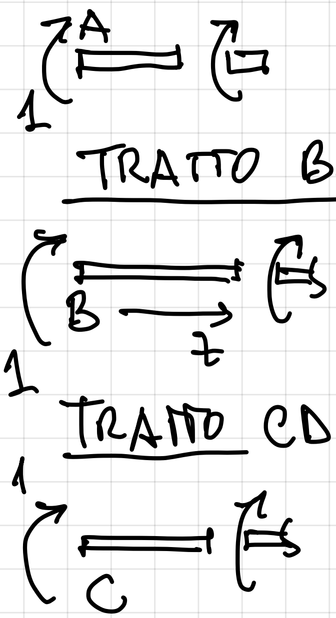
$$\boxed{M^{(r)}(z) = qL\left(\frac{L}{2} - z\right)}$$

$$\begin{cases} M_C = \frac{qL^2}{2} \\ M_D = \emptyset \end{cases}$$

STRUTTURA FITTIZIA PER IL CALCOLO DI φ_D



TRATTO AB $0 \le z \le L\sqrt{2}$



$\boxed{M^{(f)}(z) = 1}$ cost

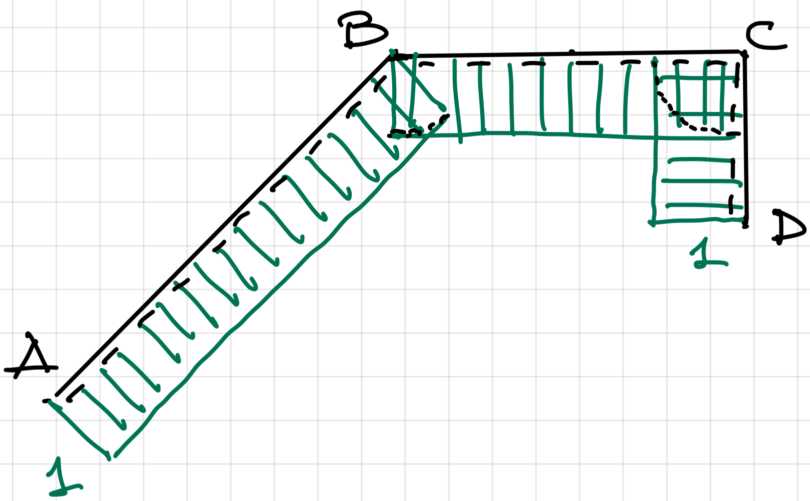
TRATTO BC $0 \le z \le L$

$\boxed{M^{(f)}(z) = 1}$ cost

TRATTO CD $0 \le z \le \frac{L}{2}$

$\boxed{M^{(f)}(z) = 1}$ cost

DIAGRAMMA $M^{(f)}(z)$



$$L_{ve} = 1 \cdot \varphi_D^{(r)} + \sum_j R_j^{(f)} \eta_j^{(r)} = \varphi_D^{(r)} + \underbrace{M_A^{(f)}}_{-1} \overbrace{\varphi_A^{(r)}}^{-\bar{\varepsilon}_A q L^2} + \underbrace{R_{x_C}^{(f)}}_{\emptyset} \eta_C^{0} =$$

$$= \varphi_D^{(r)} + \bar{\varepsilon}_A q L^2$$

$$L_{vi} = \int_{str} M^{(f)} \frac{M^{(r)}}{EI} \, ds_{tr} + \int_{str} m^{(f)} \frac{\alpha \overline{\Delta T}}{h} \, ds_{tr} =$$

$$= \frac{1}{EI} \left\{ \int_0^{L\sqrt{2}} \left[-qL^2 + qL\frac{\sqrt{2}}{2} \cdot z \right] dz + \int_0^{L} \left[qL \cdot z - \frac{qz^2}{2} \right] dz + \int_0^{L/2} qL\left(\frac{L}{2} - z\right) dz \right\} + \frac{\alpha \overline{\Delta T}}{h} \int_0^{L\sqrt{2}} dz =$$

$$= \frac{1}{EI} \left\{ -qL^2 \cdot L\sqrt{2} + qL\frac{\sqrt{2}}{2} \left[\frac{z^2}{2} \right]_0^{L\sqrt{2}} + qL \cdot \frac{L^2}{2} - \frac{q}{2}\frac{L^3}{3} + \frac{qL^2}{2} \cdot \frac{L}{2} - qL \left[\frac{z^2}{2} \right]_0^{\frac{L}{2}} \right\} + \frac{\alpha \overline{\Delta T}}{h} \cdot L\sqrt{2} =$$

$$= \frac{1}{EI} \left\{ -qL^3\sqrt{2} + \frac{qL^3\sqrt{2}}{2} + \frac{qL^3}{2} - \frac{qL^3}{6} + \frac{qL^3}{4} - \frac{qL^3}{8} \right\} + \frac{\alpha \overline{\Delta T}}{h} L\sqrt{2} =$$

$$= \frac{qL^3}{EI} \left\{ -\sqrt{2} + \frac{\sqrt{2}}{2} + \underbrace{\frac{1}{2} - \frac{1}{6} + \frac{1}{4} - \frac{1}{8}}_{\frac{12-4+6-3}{24} = \frac{11}{24}} \right\} + \frac{\alpha \overline{\Delta T}}{h} \cdot L\sqrt{2} =$$

$$= \frac{qL^3}{EI} \left[-\frac{\sqrt{2}}{2} + \frac{11}{24} \right] + \frac{\alpha \overline{\Delta T}}{h} \cdot L\sqrt{2}$$

⇒ Lve = Lvi fornisce

$$\varphi_D^{(r)} + \underbrace{\bar{\varepsilon}_A}_{\frac{L}{8EI}}\, qL^2 = \frac{qL^3}{EI}\left[\cancel{-\frac{\sqrt{2}}{2}} + \frac{11}{24}\right] + \cancel{\underbrace{\frac{\alpha\,\overline{\Delta T}}{h}}_{\frac{qL^2}{2EI}} \cdot L\sqrt{2}}$$

e, in definitiva:

$$\boxed{\varphi_D^{(r)}} = \frac{qL^3}{EI}\left[\underbrace{\frac{11}{24} - \frac{1}{8}}_{\frac{1}{3}}\right] = \boxed{\frac{qL^3}{3EI}} \Rightarrow$$

POSITIVA ANTIORARIA!