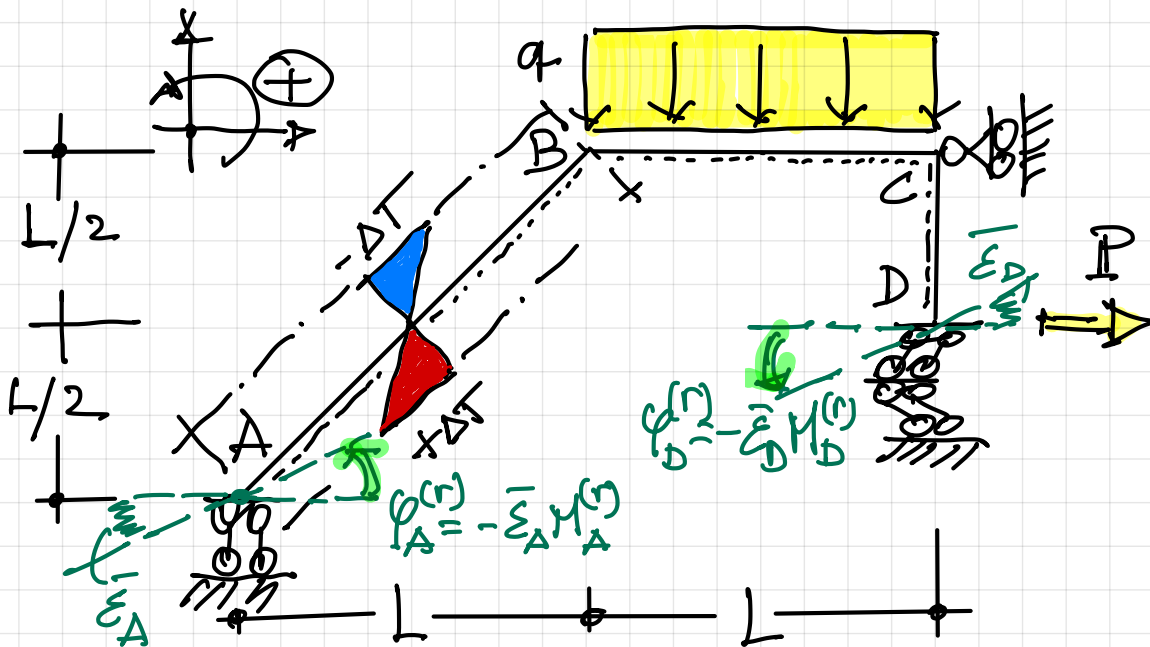


MECCANICA delle STRUTTURE - P. FUSCHI

TEST in ITINERE del 8 GENNAIO 2025

TIPO
1

ES. #1 RISOLVERE LA STRUTTURA UNA VOLTA
IPERSTANCA SEQUENTE DETERMINANDO
IL DIAGRAMMA DEI MOMENTI.



Posizioni:

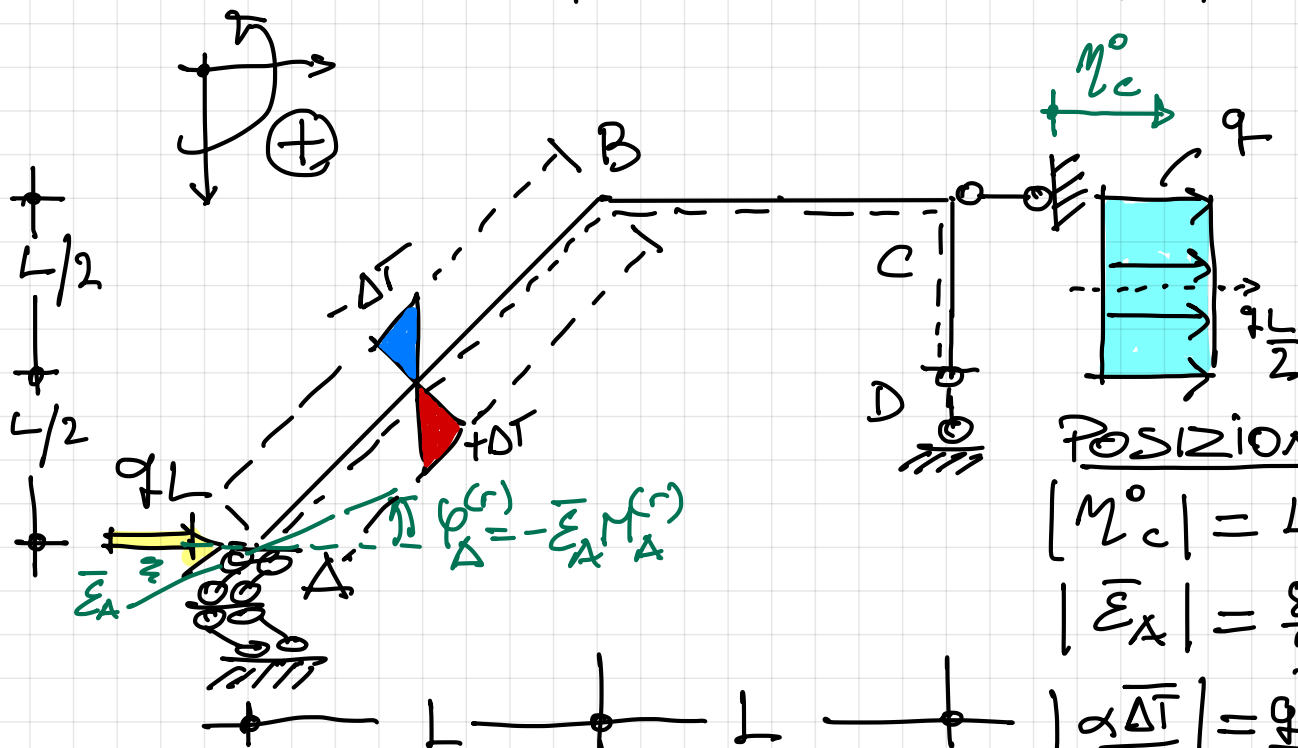
$$|P| = qL$$

$$|\bar{\epsilon}_A| = \frac{11}{24} \frac{L}{EI}$$

$$|\bar{\epsilon}_D| = \frac{1}{24} \frac{L}{EI}$$

$$\left| \alpha \frac{\Delta T}{h} \right| = \frac{qL^2}{2EI}$$

ES. #2 DETERMINARE LA ROTAZIONE DELLA SEZ. D DELLA
STRUTT. SEQUENTE CON IL METODO DELLA FORZA UNITARIA.



Posizioni:

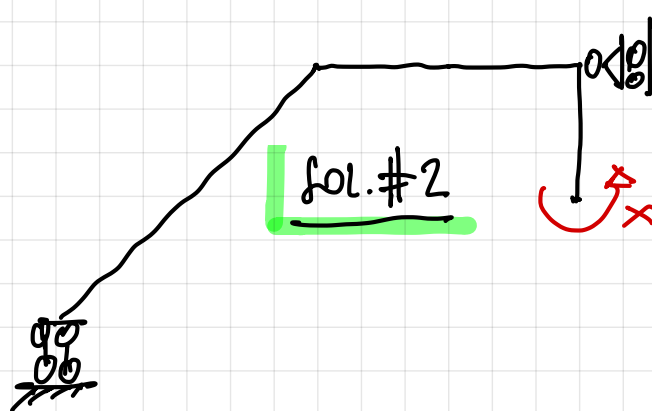
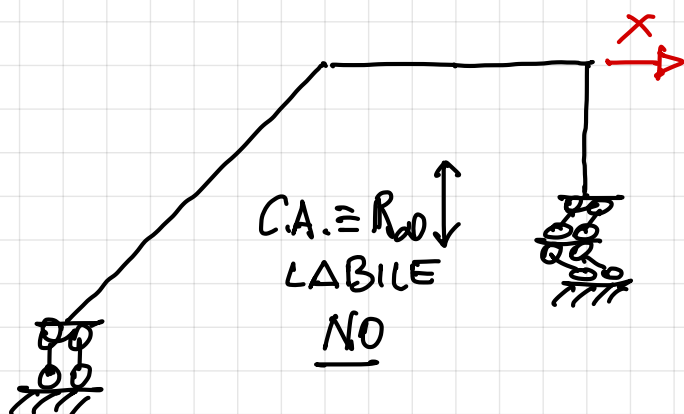
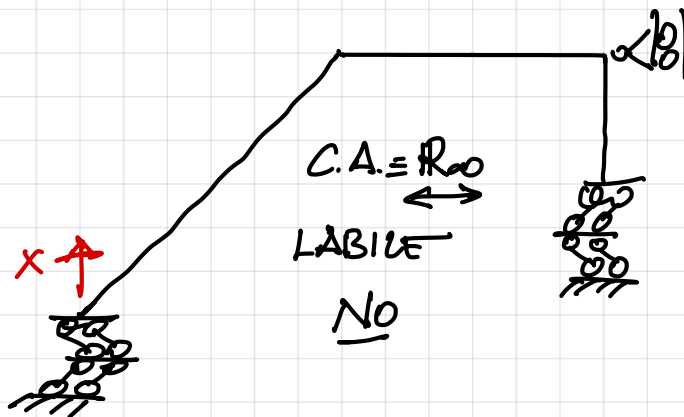
$$|M_c^0| = L^3/EI$$

$$|\bar{\epsilon}_A| = \frac{8}{9} \frac{L}{EI}$$

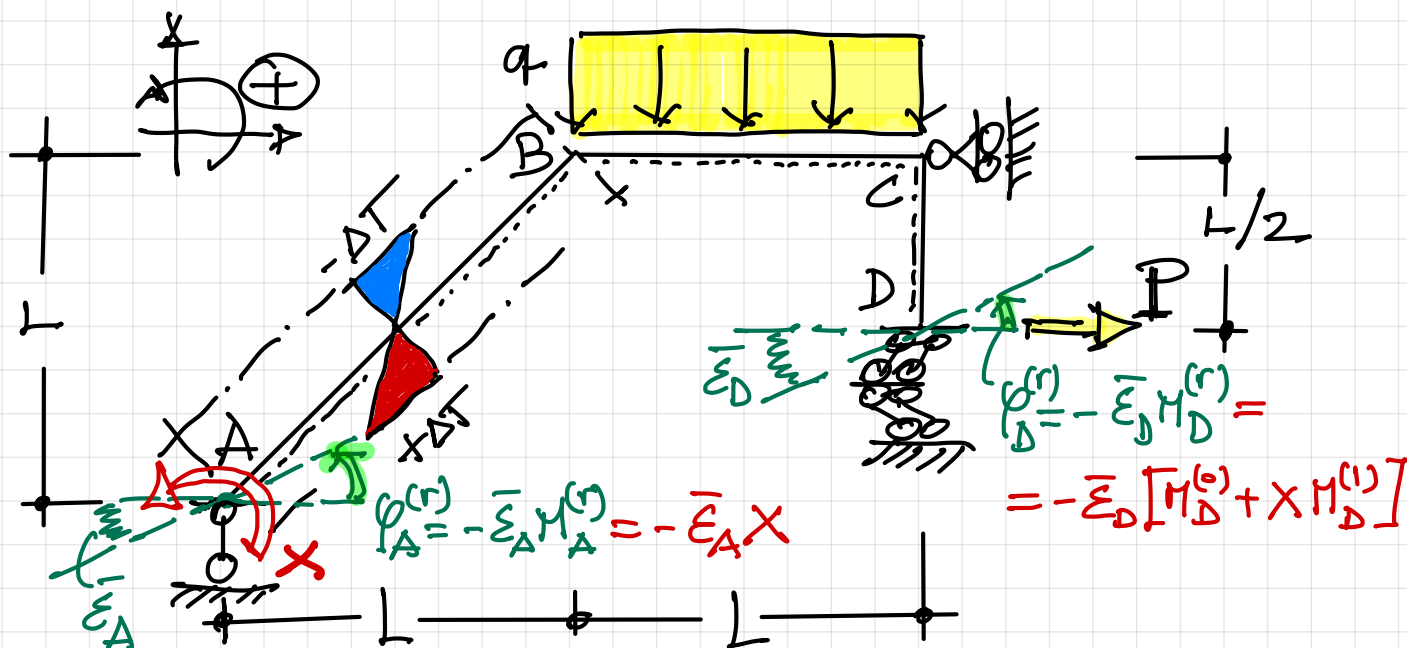
$$\left| \alpha \frac{\Delta T}{h} \right| = \frac{qL^2}{EI} \left[\frac{3}{8} + \frac{41}{48\sqrt{2}} \right]$$

SOLUZIONE

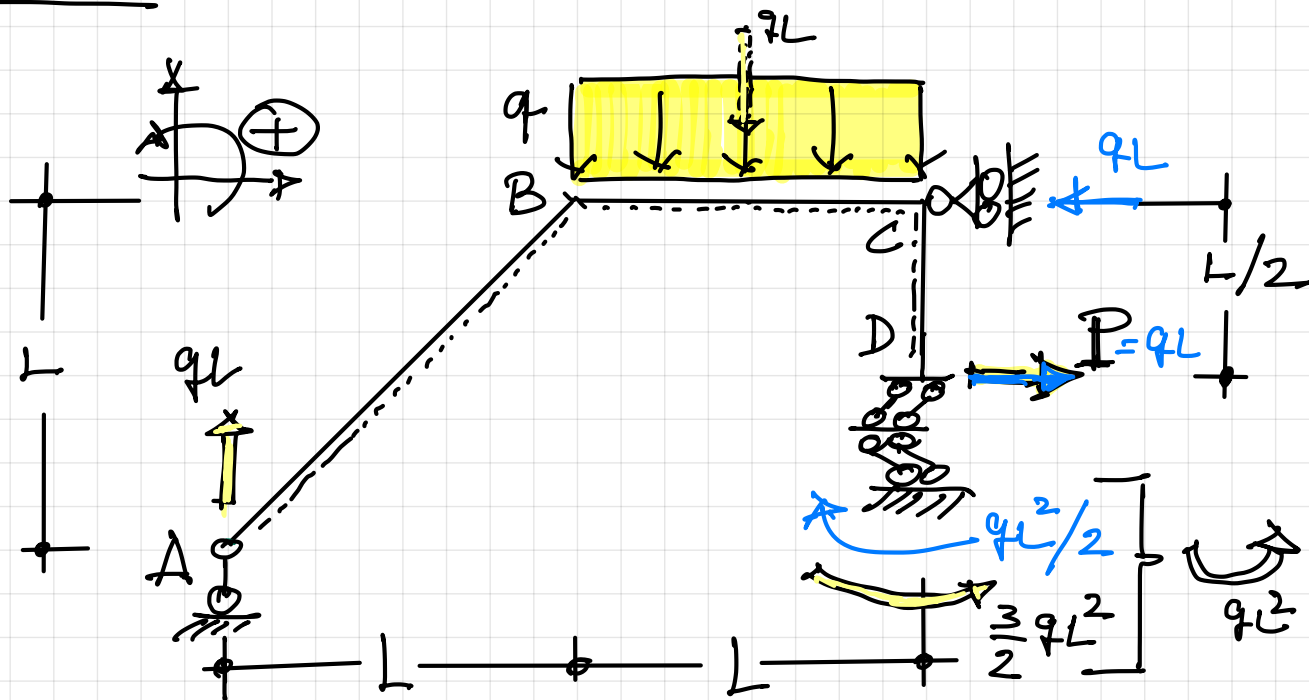
The diagram illustrates a solution path for a puzzle. It features a series of states connected by arrows. The starting state is marked with two red 'X's. A green box highlights the first move, labeled "Sol. # 1". The path continues through several intermediate states, eventually leading to a goal state marked with a large 'X'.



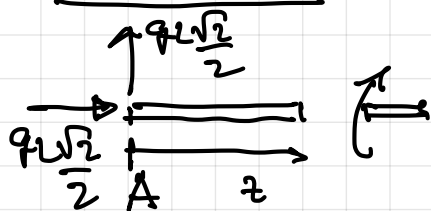
→ SISTEMA PRINCIPAL E ISOSTÁTICO



➡ SCHEMA [0]: SOLO CARICHI ESTERNI

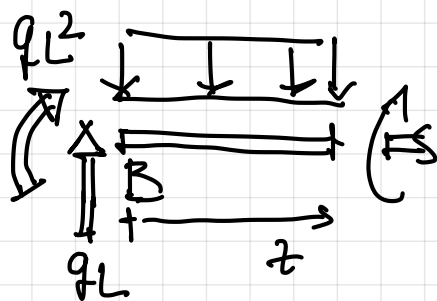


TRATTO AB $0 \leq z \leq L\sqrt{2}$



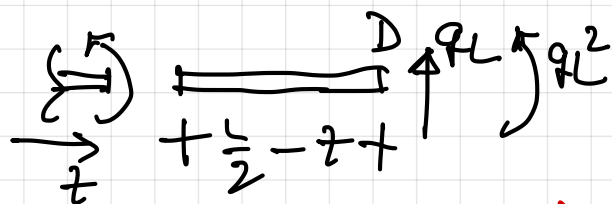
$$M^{(0)}(z) = \frac{qL\sqrt{2}}{2} \cdot z \quad \left\{ \begin{array}{l} M_A = 0 \\ M_B = qL\frac{\sqrt{2}}{2} \cdot L\sqrt{2} = qL^2 \end{array} \right.$$

TRATTO BC $0 \leq z \leq L$



$$M^{(0)}(z) = qL^2 + qL \cdot z - \frac{qz^2}{2} \quad \left\{ \begin{array}{l} M_B = qL^2 \\ M_C = 2qL^2 - \frac{qL^2}{2} = \frac{3}{2}qL^2 \end{array} \right.$$

TRATTO CD $0 \leq z \leq \frac{L}{2}$



$$M^{(0)}(z) = qL \left(\frac{L}{2} - z \right) + qL^2 \quad \left\{ \begin{array}{l} M_C = \frac{3}{2}qL^2 \\ M_D = qL^2 \end{array} \right.$$

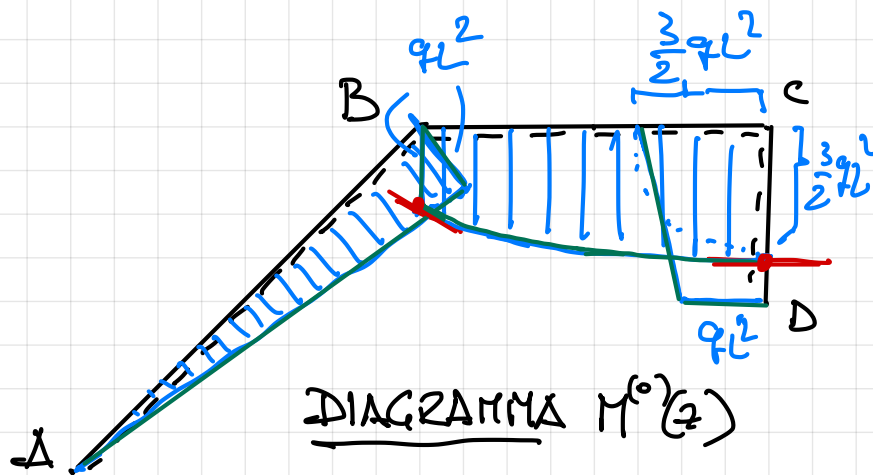
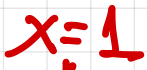


DIAGRAMMA $M^{(0)}(z)$



TRAIPO AB $0 \leq t \leq 4\sqrt{2}$

A diagram of a mechanical system with two masses, 1 and 2, on a horizontal surface. Mass 1 is on the left, and mass 2 is on the right. A spring with stiffness k connects the two masses. A damper with coefficient c is connected to mass 2, with the other end fixed to a wall on the right. Arrows indicate the positive directions for displacement x and z to the right.

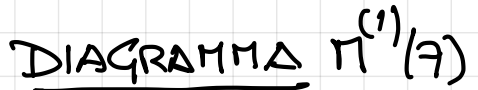
$$M^{(1)}(z) = -1 \text{ const}$$

TRAI TO Be $0 \leq z \leq L$

$\mathbb{B} \rightarrow \mathbb{A} \quad M^{(1)}(f) = -1 \text{ cost}$

TRATTO CD $0 \leq t \leq \frac{L}{2}$

$$\left(\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix} \right) \xrightarrow{D} \left(\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix} \right)_1 \quad \boxed{\psi^{(1)}(z) = -1 \text{ const}}$$



$$\Rightarrow L_{ve} = \sum_i X_i \cdot \eta_i^{(r)} + \sum_j R_j^{(l)} \eta_j^{(r)} = \underbrace{1 \cdot \varphi_A^{(r)}}_{-\bar{\varepsilon}_A X} + \underbrace{M_D^{(l)} \cdot \varphi_D^{(r)}}_{-\bar{\varepsilon}_D [\underbrace{M_D^{(0)}}_{qL^2} + X \underbrace{M_D^{(1)}}_{-1}]} =$$

$$\begin{aligned}
 \Rightarrow \underline{L_{vi}} &= \int_{SR} M^{(1)} \frac{M^{(0)}}{EI} dSR + \int_{SR} M^{(0)} \frac{\alpha \Delta T}{h} dSR = \\
 &= \frac{1}{EI} \int_{SR} M^{(1)} M^{(0)} dSR + \frac{\alpha}{EI} \int_{SR} [M^{(0)}]^2 dSR + \frac{\alpha \Delta T}{h} \int_{SR} M^{(1)} dSR = \\
 &= \frac{1}{EI} \left\{ \int_0^{L\sqrt{2}} -\frac{qL\sqrt{2}}{2} z dz + \int_0^L [-qL^2 - qLz + \frac{qz^2}{2}] dz + \right. \\
 &\quad \left. + \int_0^{\frac{L}{2}} [-qL(\frac{L}{2}-z) - qL^2] dz \right\} + \\
 &\quad + \frac{\alpha}{EI} \left\{ \int_0^{L\sqrt{2}} dz + \int_0^L dz + \int_0^{\frac{L}{2}} dz \right\} + \frac{\alpha \Delta T}{h} \int_0^{L\sqrt{2}} -1 \cdot dz = \\
 &= \frac{1}{EI} \left\{ -\frac{qL\sqrt{2}}{2} \left[\frac{z^2}{2} \right]_0^{L\sqrt{2}} - qL^2 \left[z \right]_0^L - qL \left[\frac{z^2}{2} \right]_0^L + \frac{q}{2} \left[\frac{z^3}{3} \right]_0^L + \right. \\
 &\quad \left. - \frac{qL^2}{2} \left[z \right]_0^{\frac{L}{2}} + qL \left[\frac{z^2}{2} \right]_0^{\frac{L}{2}} - qL^2 \left[z \right]_0^{\frac{L}{2}} \right\} + \\
 &\quad + \frac{\alpha}{EI} \left\{ L\sqrt{2} + L + \frac{L}{2} \right\} - \frac{\alpha \Delta T}{h} \cdot L\sqrt{2} = \\
 &= \frac{1}{EI} \left\{ -\frac{qL\sqrt{2}}{2} \frac{L^2}{2} - qL^3 - \frac{qL^3}{2} + \frac{qL^3}{6} - \frac{qL^3}{4} + \frac{qL^3}{8} - \frac{qL^3}{2} \right\} + \\
 &\quad + \frac{\alpha L}{EI} \left[\sqrt{2} + \frac{3}{2} \right] - \frac{\alpha \Delta T}{h} \cdot L\sqrt{2} = \\
 &= -\frac{qL^3}{EI} \left[\frac{\sqrt{2}}{2} + 1 + \frac{1}{2} - \frac{1}{6} + \frac{1}{4} - \frac{1}{8} + \frac{1}{2} \right] + \frac{\alpha L}{EI} \left[\sqrt{2} + \frac{3}{2} \right] - \frac{\alpha \Delta T}{h} L\sqrt{2} \\
 &\quad \underline{\underline{24 + 12 - 4 + 6 - 3 + 12 = 47/24}}
 \end{aligned}$$



$L_{ve} = L_{vi}$ formisco

$$-\bar{\varepsilon}_A X + \bar{\varepsilon}_D [qL^2 - X] = -\frac{qL^3}{EI} \left[\frac{\sqrt{2}}{2} + \frac{47}{24} \right] +$$

$$+ \frac{XL}{EI} \left[\sqrt{2} + \frac{3}{2} \right] - \frac{\alpha \Delta T}{h} L \sqrt{2}$$

$$X \left[-\bar{\varepsilon}_A - \underbrace{\bar{\varepsilon}_D}_{\frac{1}{24} \frac{L}{EI}} - \frac{L}{EI} \left(\sqrt{2} + \frac{3}{2} \right) \right] =$$

$$\frac{1}{24} \frac{L}{EI}$$

$$= -\frac{qL^3}{EI} \left[\frac{\sqrt{2}}{2} + \frac{47}{24} \right] - \underbrace{\frac{qL^2}{2EI}}_{\frac{1}{24} \frac{L}{EI}} L \sqrt{2} - \bar{\varepsilon}_D qL^2$$

$$- X \frac{L}{EI} \left[\frac{11}{24} + \frac{1}{24} + \sqrt{2} + \frac{3}{2} \right] = -\frac{qL^3}{EI} \left[\frac{\sqrt{2}}{2} + \frac{47}{24} + \frac{\sqrt{2}}{2} + \frac{1}{24} \right]$$

$\underbrace{\hspace{10em}}_{\sqrt{2}+2} \qquad \qquad \qquad \underbrace{\hspace{10em}}_{\sqrt{2}+2}$

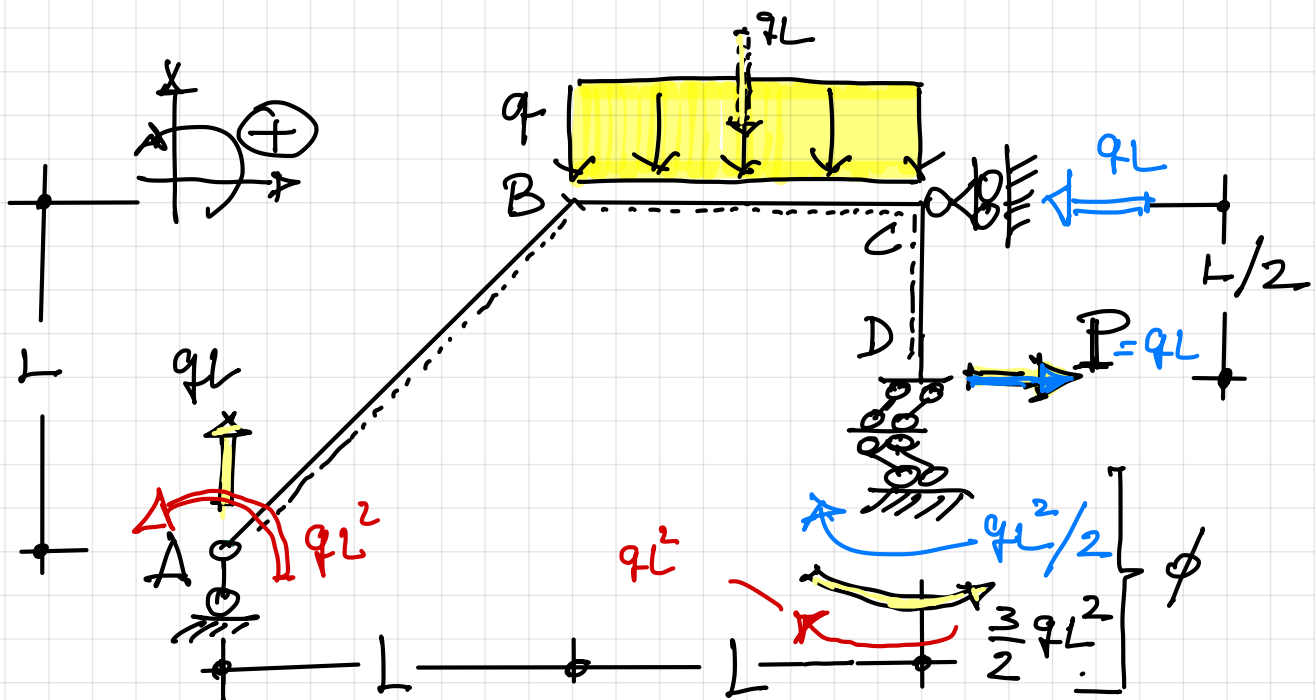
da cui

$$X = qL^2$$



POSITIVO!
verso ipotizzato
corretto

SOLUZIONE SISTEMA PRINCIPALE ISOSTATICO



TRATTO AB $0 \leq z \leq L\sqrt{2}$

$$qL^2 \left(\begin{array}{c} \uparrow qL\frac{\sqrt{2}}{2} \\ \rightarrow A \end{array} \right) \left\{ \begin{array}{l} M^{(r)}(z) = -qL^2 + qL\frac{\sqrt{2}}{2} \cdot z \\ M_A = -qL^2 \\ M_B = -qL^2 + qL\frac{\sqrt{2}}{2} \cdot L\sqrt{2} = \phi \end{array} \right.$$

TRATTO BC $0 \leq z \leq L$

$$qL^2 \left(\begin{array}{c} \downarrow q \\ \rightarrow B \end{array} \right) \left\{ \begin{array}{l} M^{(s)}(z) = qL \cdot z - \frac{qz^2}{2} \\ M_B = \phi \\ M_C = \frac{qL^2}{2} \end{array} \right.$$

TRATTO CD $0 \leq z \leq \frac{L}{2}$

$$\left(\begin{array}{c} \uparrow qL \\ \rightarrow D \end{array} \right) \left\{ \begin{array}{l} M^{(s)}(z) = qL\left(\frac{L}{2} - z\right) \\ M_C = qL^2/2 \\ M_D = \phi \end{array} \right.$$

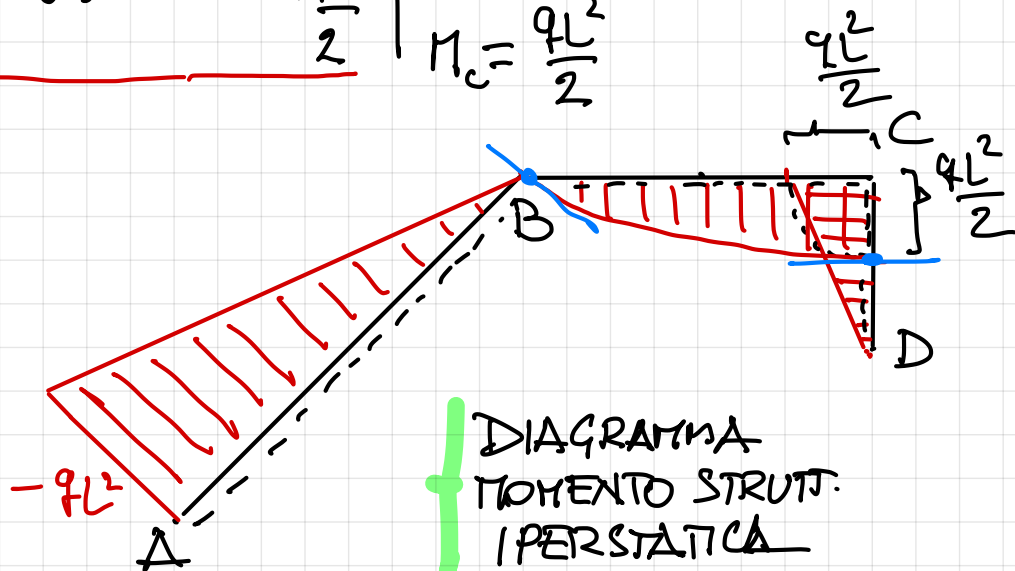
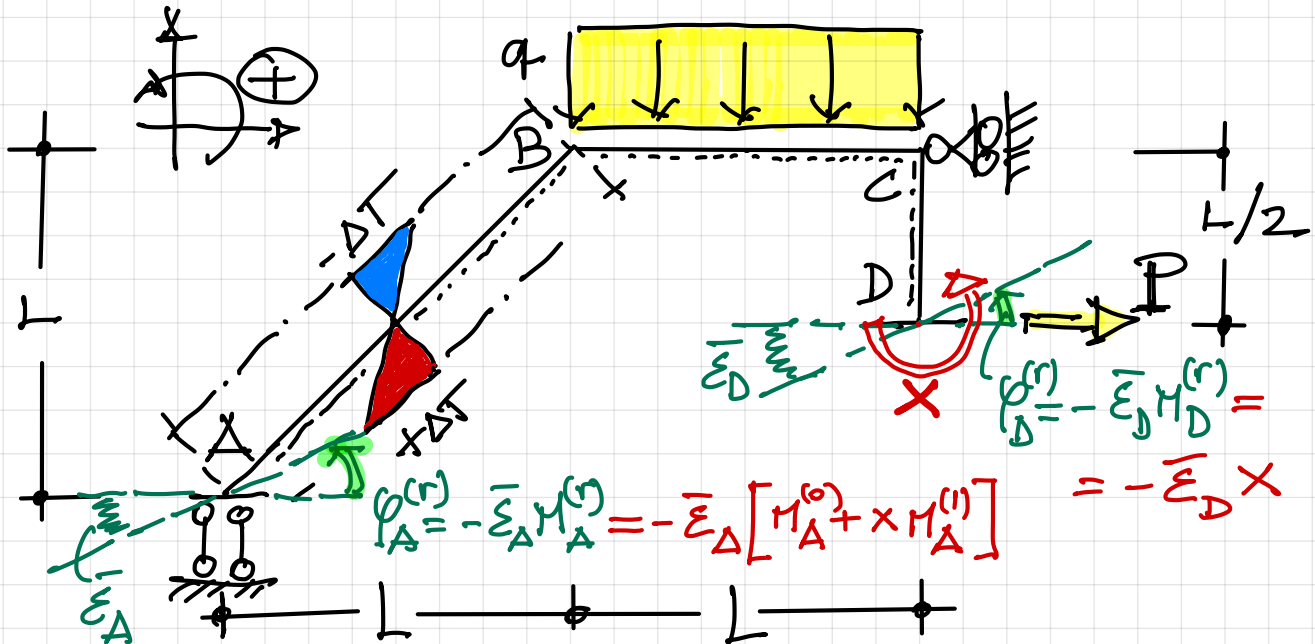


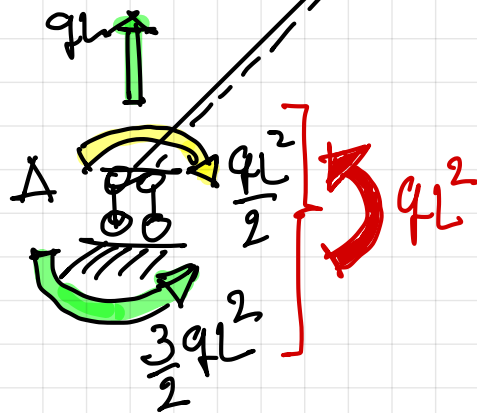
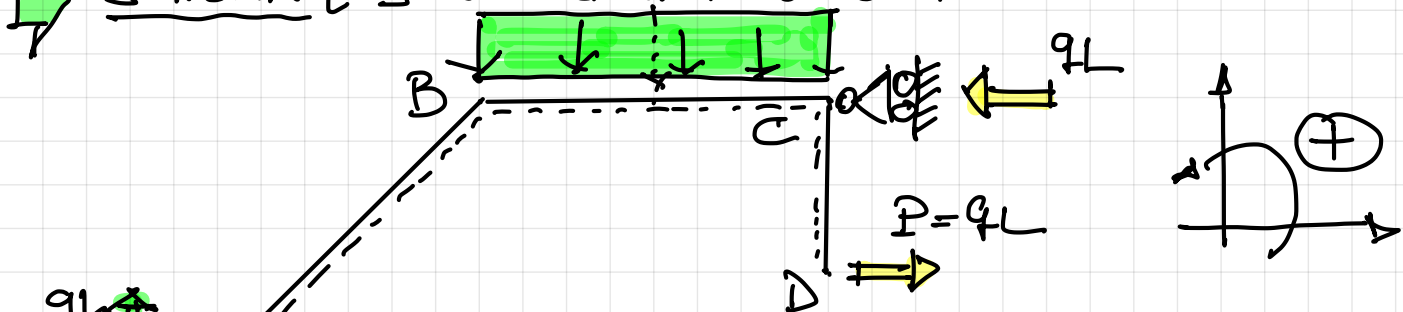
DIAGRAMMA
MOMENTO STRUT.
IPERSTATICA

SOLUZIONE #2

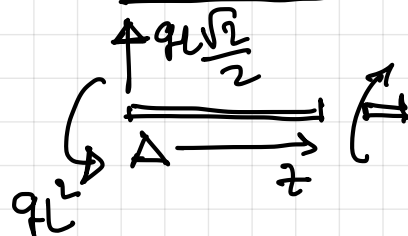
→ SISTEMA PRINCIPALE ISOSTATICO



→ SCHEMA [0] SOLO CARICHI ESTERNI



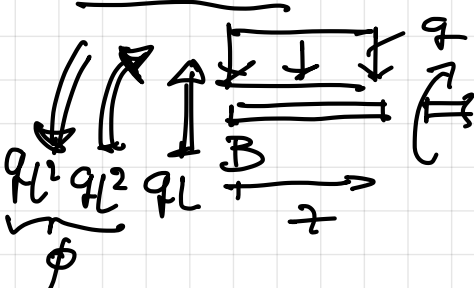
TRATTO AB $0 \leq z \leq L\sqrt{2}$



$$M^{(0)}(z) = -qL^2 + qL\frac{L}{2} \cdot z$$

$$\begin{cases} M_A = -qL^2 \\ M_B = 0 \end{cases}$$

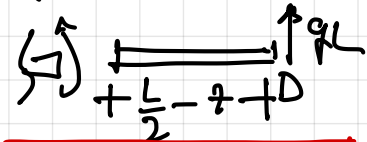
TRATTO BC $0 \leq z \leq L$



$$M^{(0)}(z) = qL \cdot z - \frac{qz^2}{2}$$

$$\begin{cases} M_B = 0 \\ M_C = \frac{qL^2}{2} \end{cases}$$

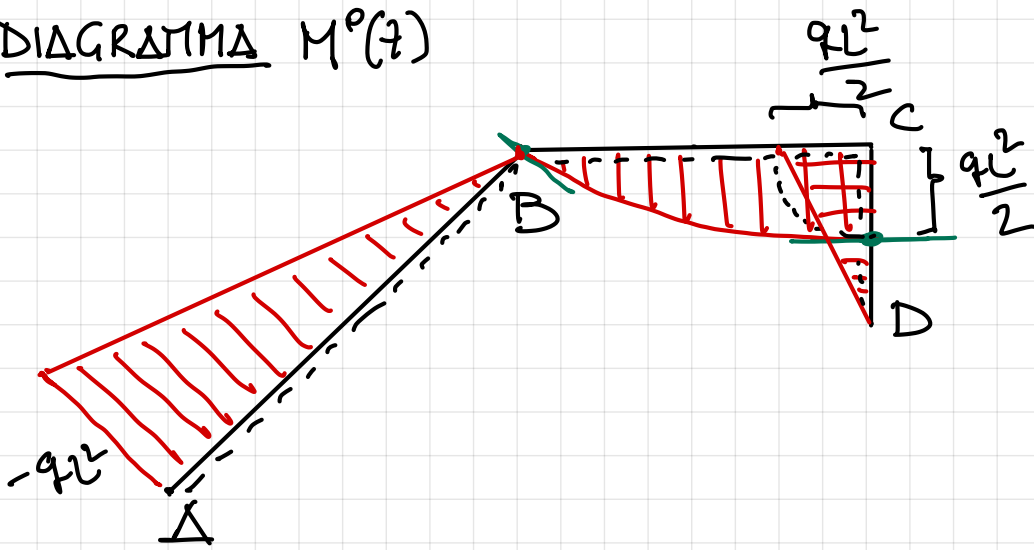
TRATTO CD $0 \leq z \leq \frac{L}{2}$



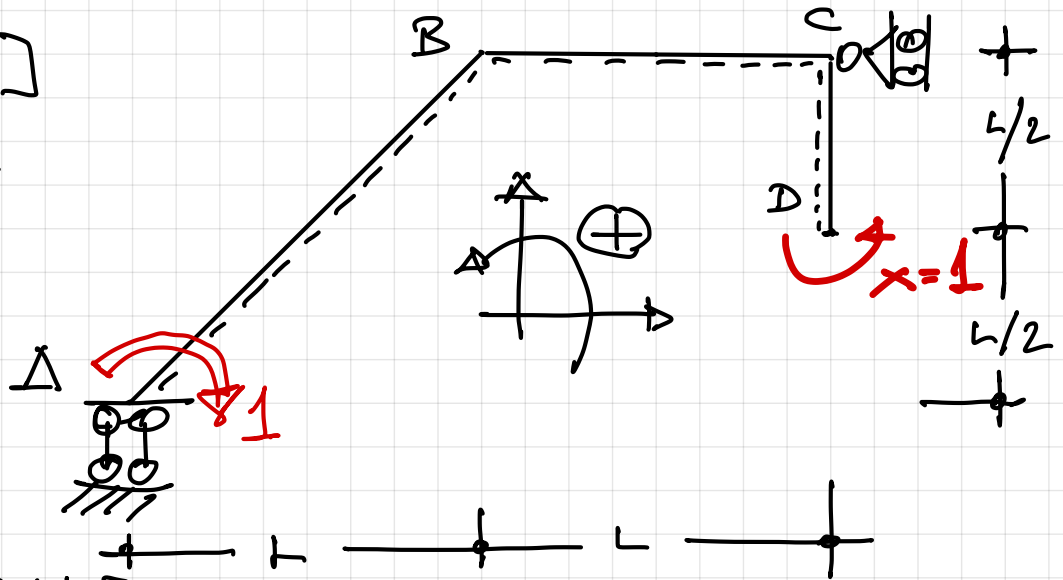
$$M^{(0)}(z) = qL\left(\frac{L}{2} - z\right)$$

$$\begin{cases} M_C = \frac{qL^2}{2} \\ M_D = 0 \end{cases}$$

DIAGRAMMA $M^0(z)$



➡ SCHEMA [1]
Solo $X=1$



TRATTO AB $0 \leq z \leq L\sqrt{2}$

$\begin{matrix} \curvearrowright A \\ \hline \end{matrix} \Rightarrow \begin{matrix} \curvearrowright \\ \hline \end{matrix} \Rightarrow \boxed{M^{(1)}(z) = 1 \cos t}$

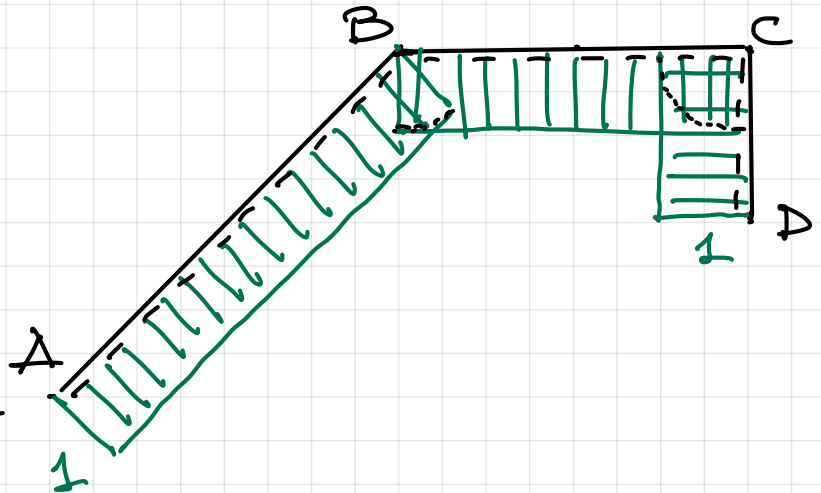
DIAGRAMMA $M^{(1)}(z)$

TRATTO BC $0 \leq z \leq L$

$\begin{matrix} \curvearrowright \\ \hline \end{matrix} \Rightarrow \begin{matrix} \curvearrowright B \\ \hline \end{matrix} \Rightarrow \boxed{M^{(1)}(z) = 1 \cos t}$

TRATTO CD $0 \leq z \leq \frac{L}{2}$

$\begin{matrix} \curvearrowright \\ \hline \end{matrix} \Rightarrow \begin{matrix} \curvearrowright C \\ \hline \end{matrix} \Rightarrow \boxed{M^{(1)}(z) = 1 \cos t}$



$$\begin{aligned}
 \Rightarrow \underline{L_{ve}} &= \sum_i X_i \eta_i^{(r)} + \sum_j R_j^{(1)} \eta_j^{(r)} = \underbrace{1 \cdot \varphi_D^{(r)}}_{-\bar{E}_D \cdot X} + \underbrace{M_A^{(1)} \varphi_A^{(r)}}_{-1} = -\bar{E}_D X + \bar{E}_A \left[\underbrace{M_A^{(10)}}_{qL^2} + X \underbrace{M_A^{(1)}}_{-1} \right] \\
 &= -\bar{E}_D X + \bar{E}_A [qL^2 - X]
 \end{aligned}$$

$$\begin{aligned}
 \Rightarrow \underline{L_{vi}} &= \int_{str} M^{(1)} \overbrace{\frac{M^{(r)}}{EI}}^{\eta^0 + \eta' \cdot X} dstr + \int_{str} M^{(1)} \frac{\alpha \bar{\Delta T}}{h} dstr = \\
 &= \frac{1}{EI} \int_{str} M^{(1)} M^{(0)} dstr + \frac{X}{EI} \int_{str} [M^{(1)}]^2 dstr + \frac{\alpha \bar{\Delta T}}{h} \int_{str} M^{(1)} dstr = \\
 &= \frac{1}{EI} \left\{ \int_0^{L\sqrt{2}} \left[-qL^2 + qL \frac{L}{2} \cdot z \right] dz + \int_0^L \left[qL \cdot z - \frac{qz^2}{2} \right] dz + \right. \\
 &\quad \left. + \int_0^{L/2} qL \left(\frac{L}{2} - z \right) dz \right\} + \\
 &\quad + \frac{X}{EI} \left\{ \int_0^{L\sqrt{2}} dz + \int_0^L dz + \int_0^{L/2} dz \right\} + \frac{\alpha \bar{\Delta T}}{h} \int_0^{L\sqrt{2}} dz = \\
 &= \frac{1}{EI} \left\{ -qL^2 \cdot L\sqrt{2} + qL \frac{L}{2} \left[\frac{z^2}{2} \right]_0^{L\sqrt{2}} + qL \cdot \frac{L^2}{2} - \frac{q}{2} \frac{L^3}{3} + \right. \\
 &\quad \left. + \frac{qL^2}{2} \cdot \frac{L}{2} - qL \left[\frac{z^2}{2} \right]_0^{\frac{L}{2}} \right\} + \\
 &\quad + \frac{X}{EI} \left\{ L\sqrt{2} + L + \frac{L}{2} \right\} + \frac{\alpha \bar{\Delta T}}{h} \cdot L\sqrt{2} =
 \end{aligned}$$

$$\begin{aligned}
&= \frac{1}{EI} \left[-qL^3\sqrt{2} + \frac{qL^3\sqrt{2}}{2} + \frac{qL^3}{2} - \frac{qL^3}{6} + \frac{qL^3}{4} - \frac{qL^3}{8} \right] + \\
&\quad + \frac{XL}{EI} \left[\sqrt{2} + \frac{3}{2} \right] + \frac{\alpha \Delta T}{h} L \sqrt{2} \\
&= \frac{qL^3}{EI} \left[-\sqrt{2} + \frac{\sqrt{2}}{2} + \frac{1}{2} - \frac{1}{6} + \frac{1}{4} - \frac{1}{8} \right] + \frac{XL}{EI} \left[\sqrt{2} + \frac{3}{2} \right] + \frac{\alpha \Delta T}{h} L \sqrt{2} \\
&\quad \underbrace{\frac{12 - 4 + 6 - 3}{24} = \frac{11}{24}}
\end{aligned}$$

→ Lve = Lvi formula

$$-\bar{\varepsilon}_D X + \bar{\varepsilon}_A [qL^2 - X] = \frac{qL^3}{EI} \left[-\frac{\sqrt{2}}{2} + \frac{11}{24} \right] + \frac{XL}{EI} \left[\sqrt{2} + \frac{3}{2} \right] + \frac{\alpha \Delta T}{h} L \sqrt{2}$$

$$X \left[-\bar{\varepsilon}_D - \bar{\varepsilon}_A - \frac{L}{EI} \left(\sqrt{2} + \frac{3}{2} \right) \right] = \frac{qL^3}{EI} \left[-\frac{\sqrt{2}}{2} + \frac{11}{24} \right] - \bar{\varepsilon}_A qL^2 + \frac{\alpha \Delta T}{h} L \sqrt{2}$$

$\frac{11}{24} \frac{L}{EI}$
 $\frac{11}{24} \frac{L}{EI}$
 $\frac{11}{24} \frac{L}{EI}$

$$\frac{L}{EI} X \left[-\frac{1}{24} - \frac{11}{24} - \sqrt{2} - \frac{3}{2} \right] =$$

$$= \frac{qL^3}{EI} \left[-\frac{\sqrt{2}}{2} + \frac{11}{24} - \frac{11}{24} + \frac{\sqrt{2}}{2} \right]$$

0

da cui

$$X = 0$$

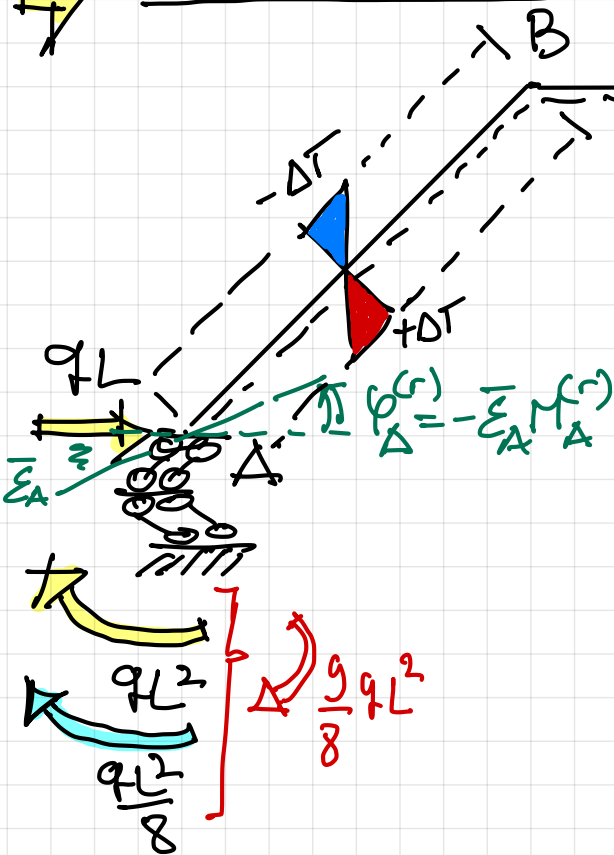
OK! CORRETTO!
c.f. RV di pag. 7

ES.#2 - tipo 1

SOLUZIONE

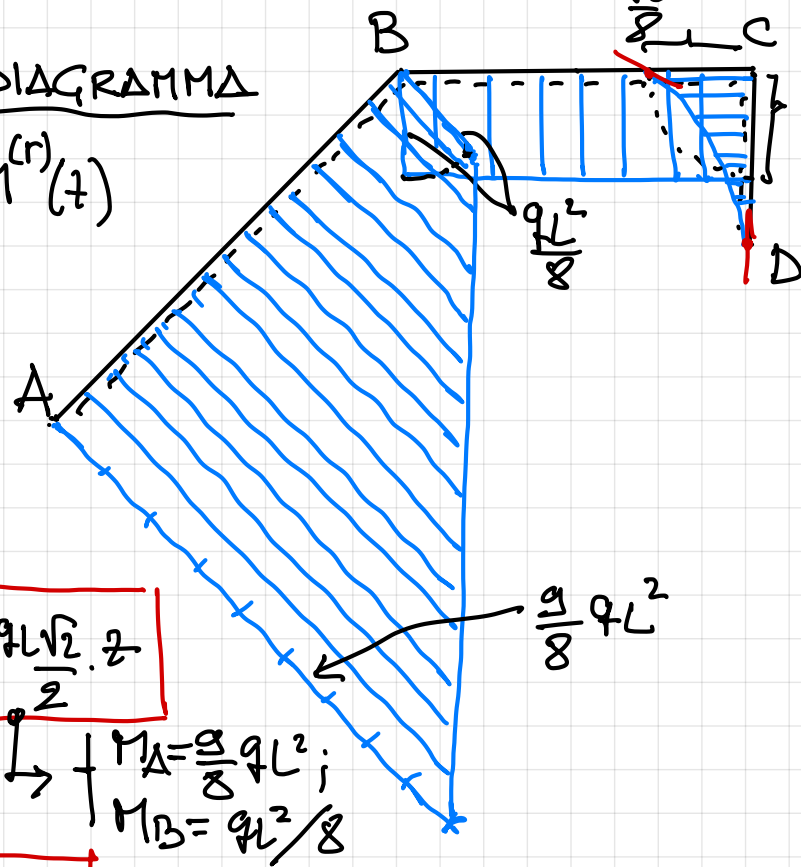


STRUTTURA REALE

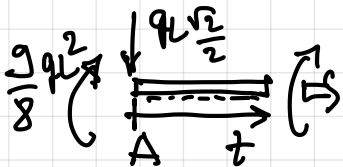


DIAGRAMMA

$M^{(r)}(z)$

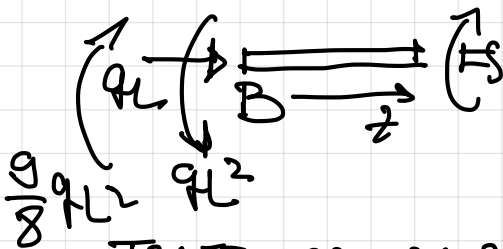


TRATTO AB $0 \leq z \leq L\sqrt{2}$



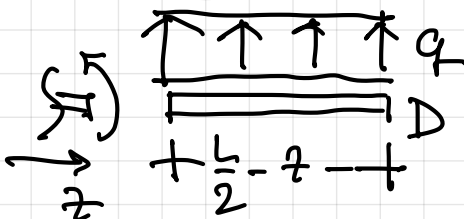
$$M^{(r)}(z) = \frac{q}{8} L^2 - \frac{qL\sqrt{2}}{2} z$$

TRATTO BC $0 \leq z \leq L$



$$M^{(r)}(z) = \frac{qL^2}{8} \cos t$$

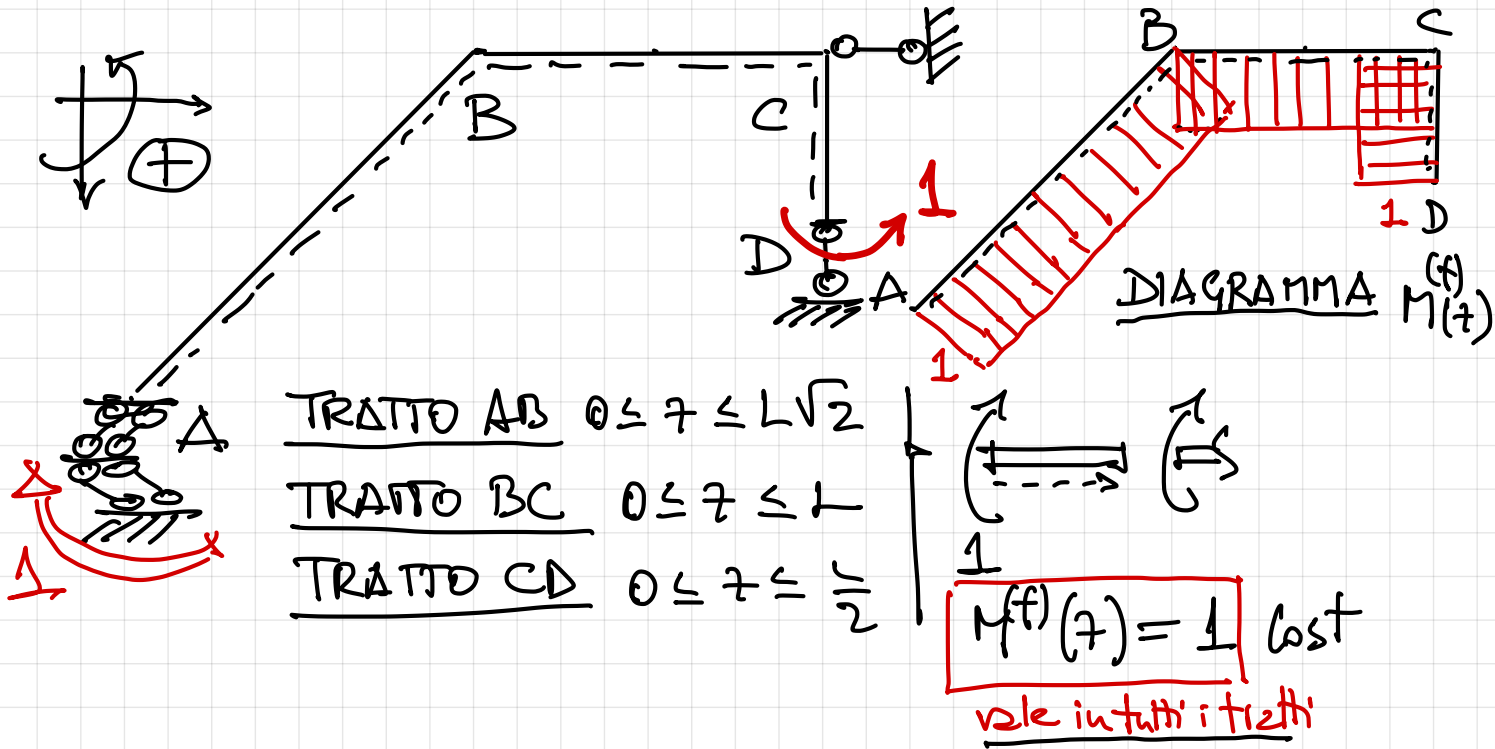
TRATTO CD $0 \leq z \leq \frac{L}{2}$



$$M^{(r)}(z) = \frac{q(\frac{L}{2} - z)^2}{2}$$



STRUTTURA FITTIZIA PER IL CALCOLO DI φ_D



$$\begin{aligned}
 L_{ve} &= 1 \cdot \varphi_D^{(r)} + \sum_j R_j^{(f)} \eta_j^{(r)} = \\
 &= \varphi_D^{(r)} + R_{x_c}^{(f)} \eta_c^{(r)} + \underbrace{M_A^{(f)}}_{-1} \varphi_A^{(r)} = \varphi_D^{(r)} - \underbrace{\bar{E}_A \frac{9}{8} q L^2}_{-\bar{E}_A M_A^{(r)} = -\bar{E}_A \left[-\frac{9}{8} q L^2 \right]}
 \end{aligned}$$

$$\begin{aligned}
 L_{vi} &= \int_{str} M^{(f)} \frac{M^{(r)}}{EI} ds + \int_{str} \eta^{(f)} \frac{\partial \Delta T}{\partial n} ds = \\
 &= \frac{1}{EI} \left[\int_0^{L\sqrt{2}} \left[\frac{9}{8} q L^2 - q L \frac{\sqrt{2}}{2} z \right] dz + \int_0^L \frac{q L^2}{8} dz + \right. \\
 &\quad \left. + \int_0^{L/2} \frac{q}{2} \left(\frac{L}{2} - z \right)^2 dz \right] + \frac{\partial \Delta T}{\partial n} \int_0^{L\sqrt{2}} dz =
 \end{aligned}$$

$$= \frac{1}{EI} \left[\frac{qL^2}{8} \cdot L\sqrt{2} - \frac{qL\sqrt{2}}{2} \cdot \frac{1}{8} \cdot L^2 + \frac{qL^2}{8} \cdot L + \frac{qL^2}{8} \cdot \frac{L}{2} + \frac{qL^3}{48} - \frac{qL^3}{16} \right] + \frac{\alpha \Delta T}{h} \cdot L\sqrt{2} =$$

$$= \frac{qL^3}{EI} \left[\underbrace{\frac{9}{8}\sqrt{2} - \frac{\sqrt{2}}{2}}_{\frac{5\sqrt{2}}{8}} + \underbrace{\frac{1}{8} + \frac{1}{16} + \frac{1}{48} - \frac{1}{16}}_{\frac{7}{48}} \right] + \frac{\alpha \Delta T}{h} L\sqrt{2} =$$

$$= \frac{qL^3}{EI} \left[\frac{5\sqrt{2}}{8} + \frac{7}{48} \right] + \frac{\alpha \Delta T}{h} L\sqrt{2}$$

⇒ $L_{ve} = L_{vi}$ fornisce

$$\varphi_D^{(r)} - \underbrace{\bar{\varepsilon}_A}_{\frac{8L}{9EI}} \cdot \frac{q}{8} L^2 = \underbrace{\frac{qL^3}{EI} \left[\frac{5\sqrt{2}}{8} + \frac{7}{48} \right] + \frac{\alpha \Delta T}{h} L\sqrt{2}}_{\frac{qL^3}{EI} [\sqrt{2} + 1]} + \underbrace{\left[\frac{3}{8} + \frac{41}{48\sqrt{2}} \right] \frac{qL^2}{EI}}$$

e, in definitiva:

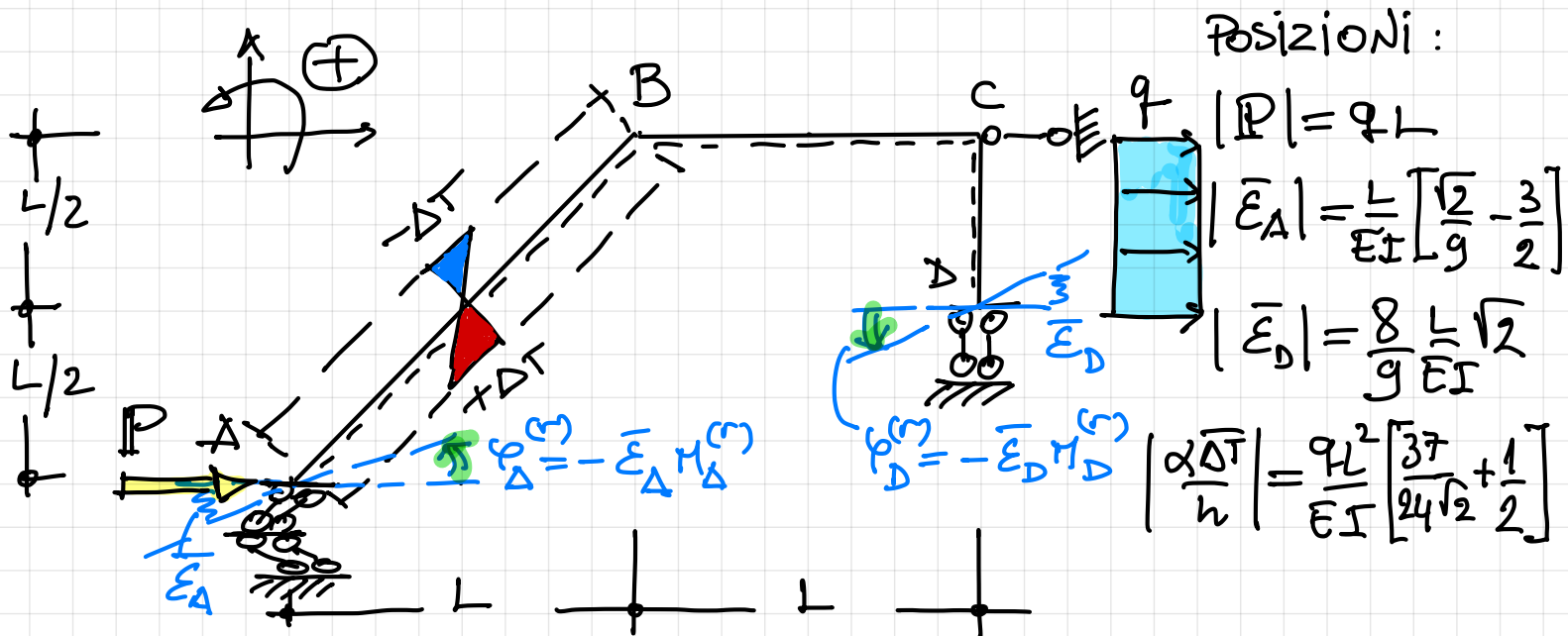
$$\varphi_D = \frac{qL^3}{EI} [\sqrt{2} + 2] \quad \Rightarrow \text{POSITIVA} \quad \Rightarrow \text{ANTIORARIA}$$

MECCANICA delle STRUTTURE - P. FUSCHI

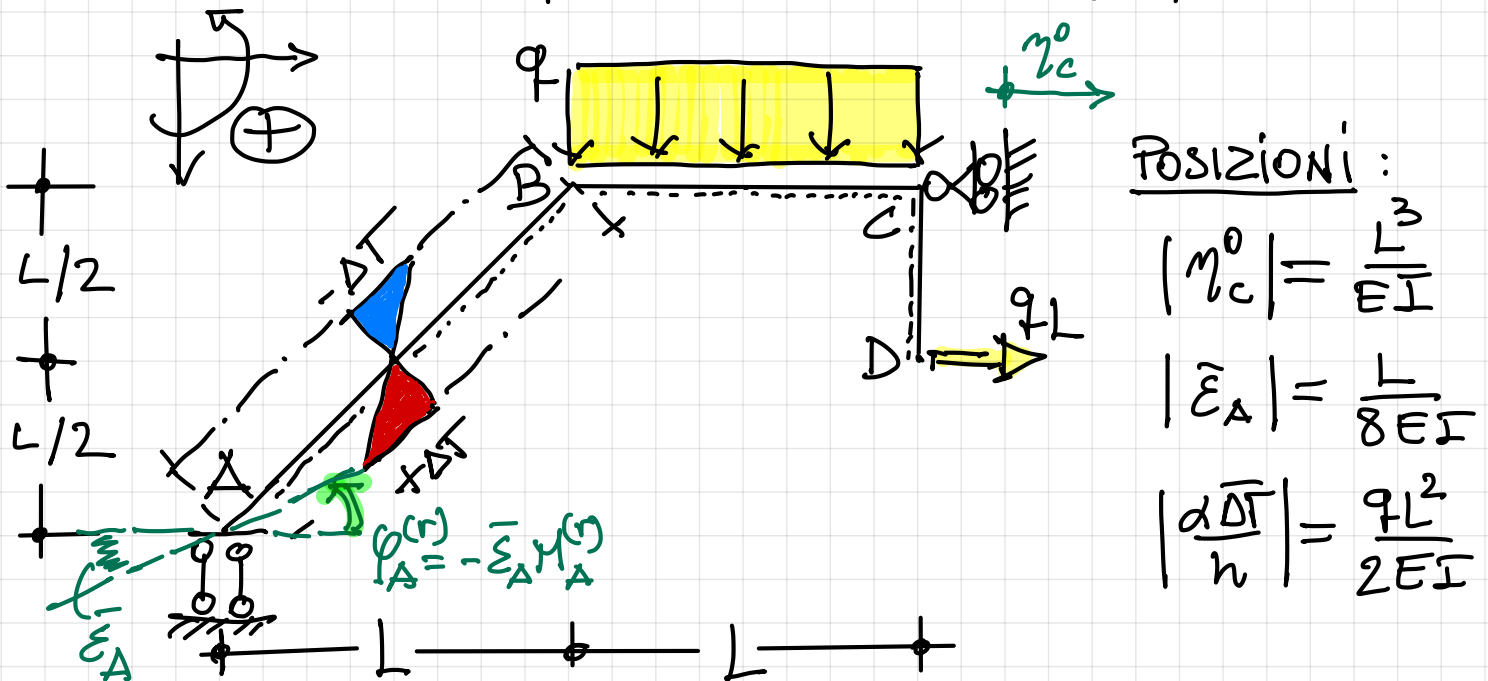
TEST in ITINERE del 8 GENNAIO 2025

TIPO
2

ES. #1 RISOLVERE LA STRUTTURA UNA VOLTA
IPERSTATICA SEGUENTE DETERMINANDO
IL DIAGRAMMA DEI MOMENTI.



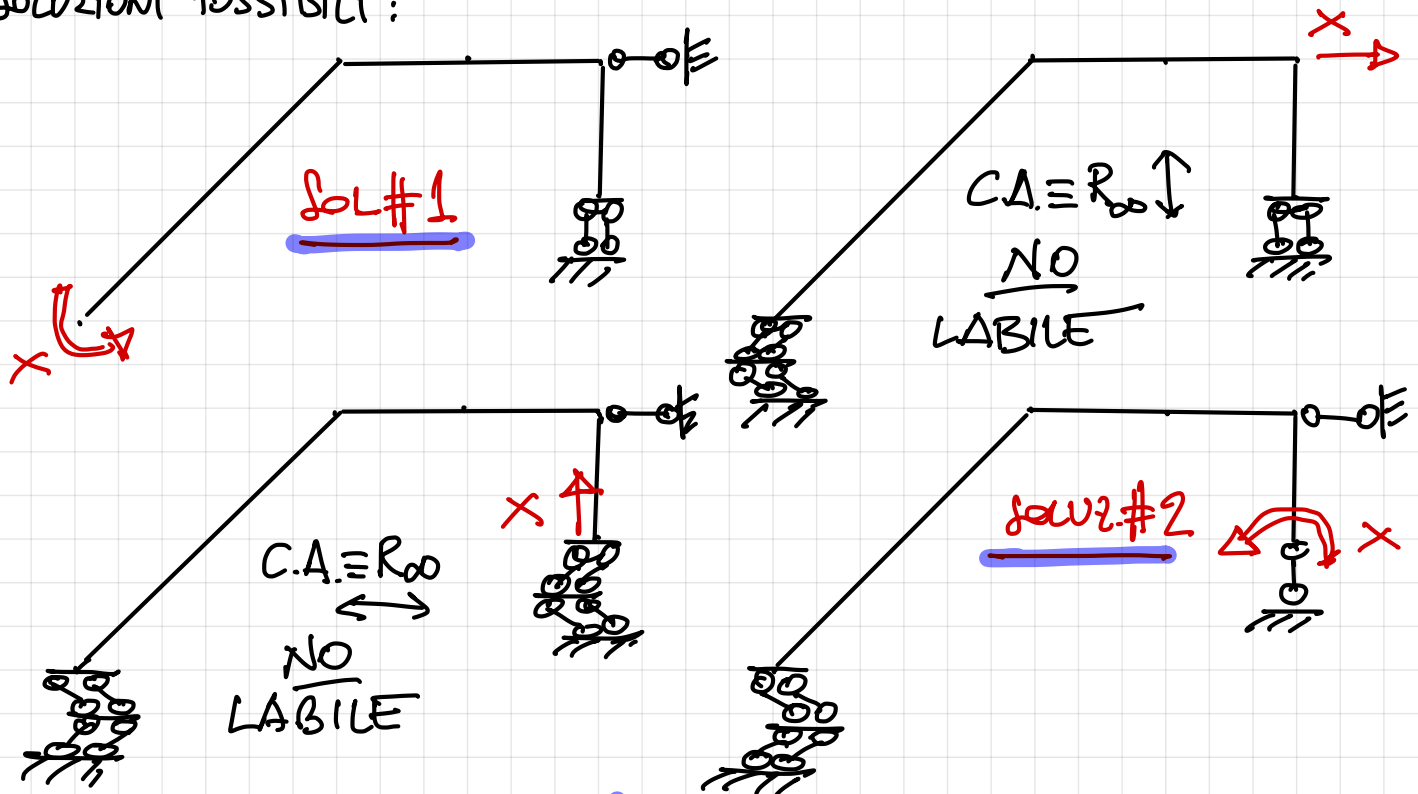
ES. #2 DETERMINARE LA ROTAZIONE DELLA SEZ. D DELLA
STRUTT. SEGUENTE CON IL METODO DELLA FORZA UNITARIA.



ES.#1 - tipo 2

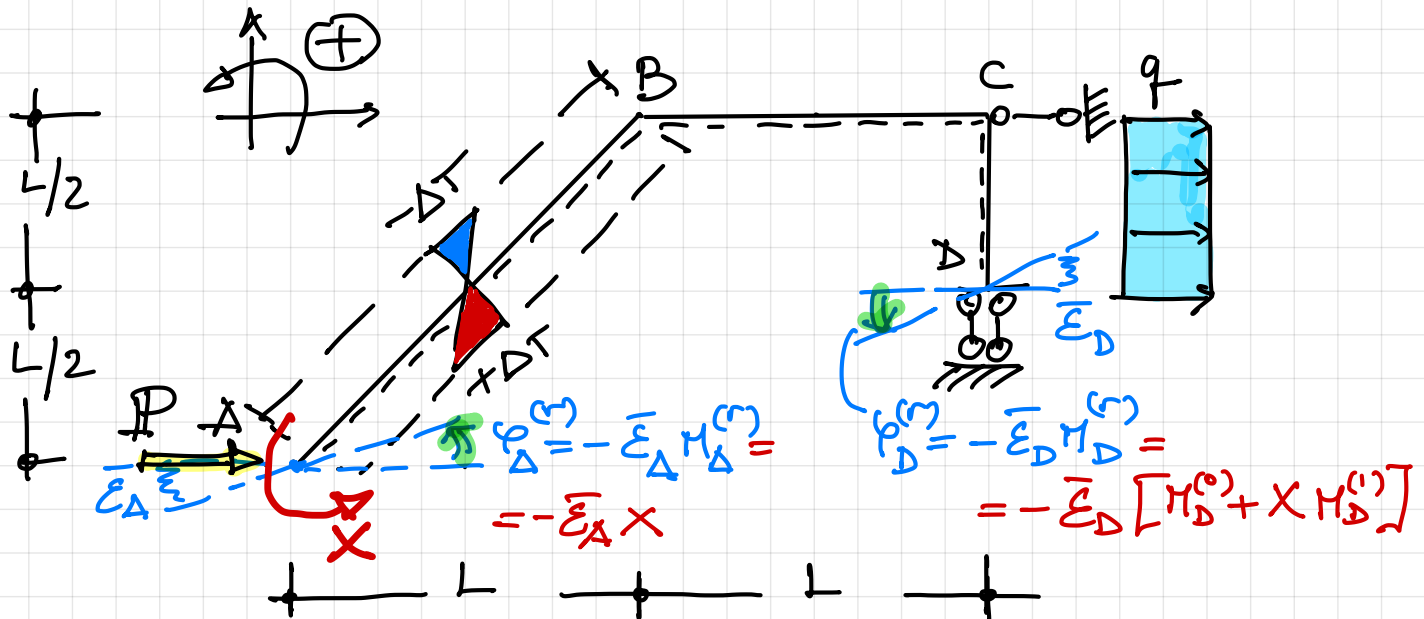
SOLUZIONI

Soluzioni possibili:

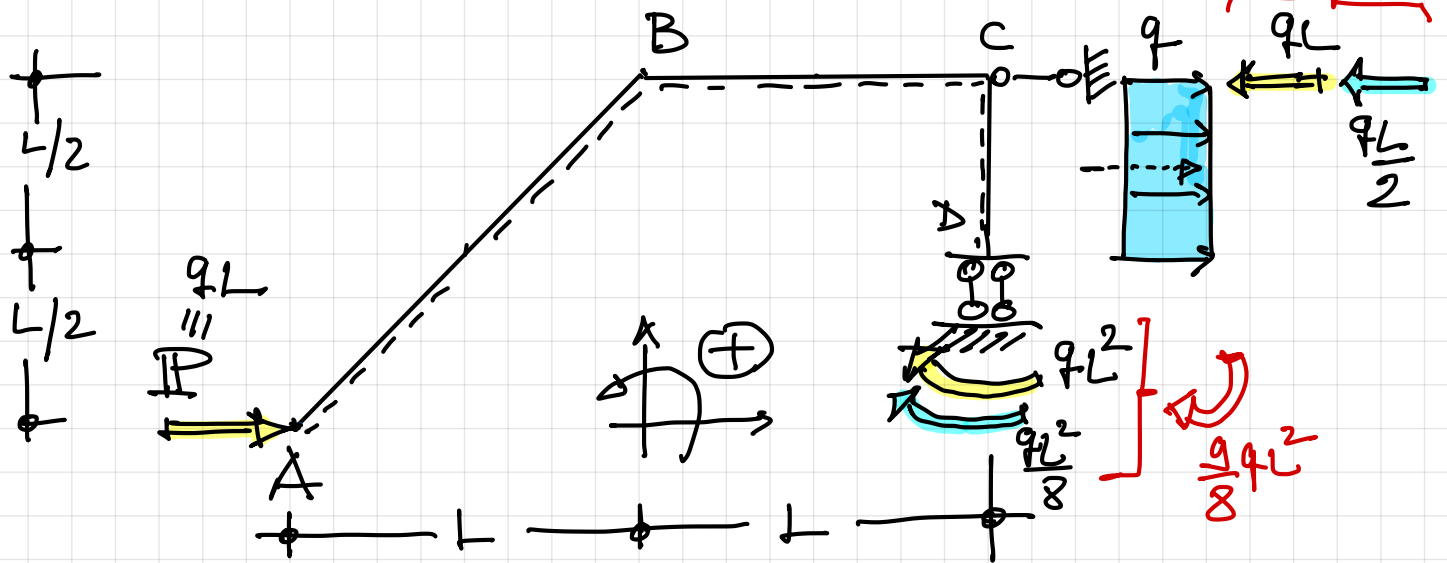


SOLUZIONE #1

SISTEMA PRINCIPALE ISOSTATICO



➔ SCHEMA [0]
SOLO CARICHI ESTERNI



TRATTO AB $0 \leq z \leq L\sqrt{2}$

$\frac{qL\sqrt{2}}{2} \downarrow$
 $\boxed{M^{(0)}(z) = -\frac{qL\sqrt{2}}{2} \cdot z}$
 $\left. \begin{array}{l} M_A = 0 \\ M_B = -qL^2 \end{array} \right\}$

TRATTO BC $0 \leq z \leq L$

$\frac{qL}{\downarrow}$
 $\boxed{M^{(0)}(z) = -qL^2 \text{ cost.}}$

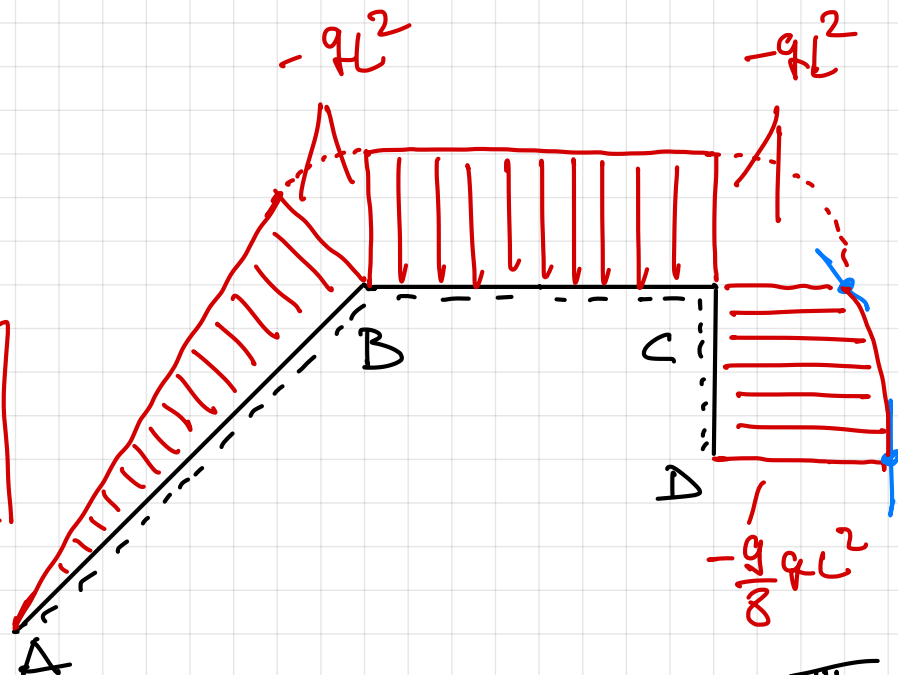
TRATTO CD $0 \leq z \leq \frac{L}{2}$

$\frac{qL}{\downarrow}$
 $\frac{qL^2}{8}$

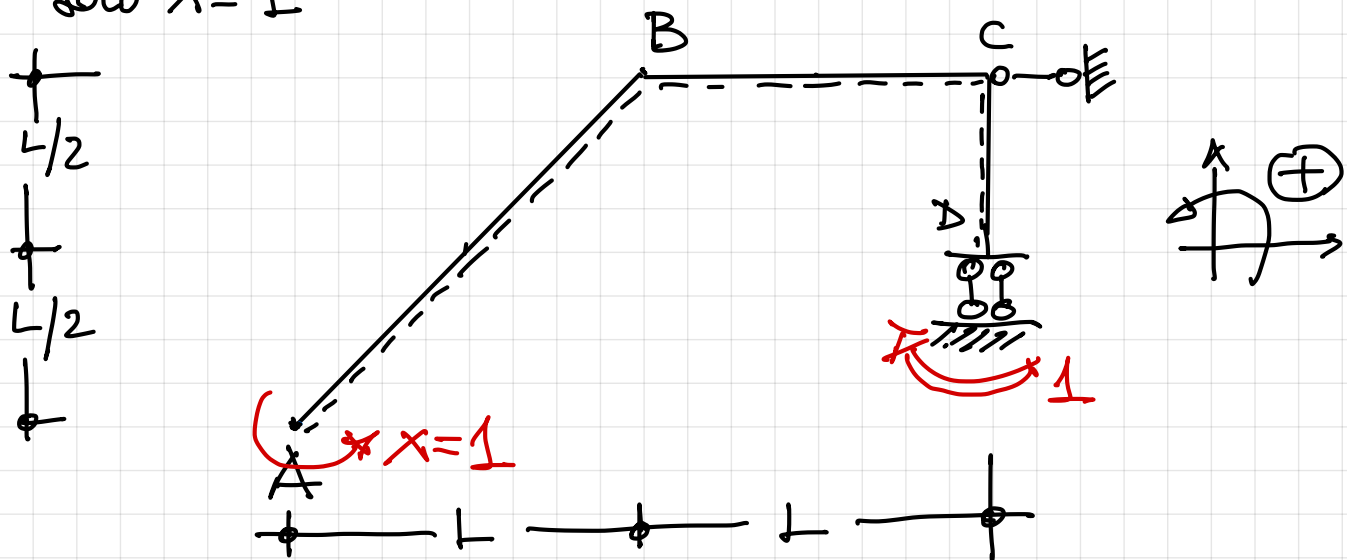
$\boxed{M^{(0)}(z) = -\frac{9}{8}qL^2 + \frac{q(\frac{L}{2}-z)^2}{2}}$

$M_C = -qL^2$
 $M_D = -\frac{9}{8}qL^2$

DIAGRAMMA $M^{(0)}(z)$



SCHEMA [1]
Solo $X=1$



TRATTO AB $0 \leq z \leq L\sqrt{2}$

$M^{(1)}(z) = -1 \cos t$

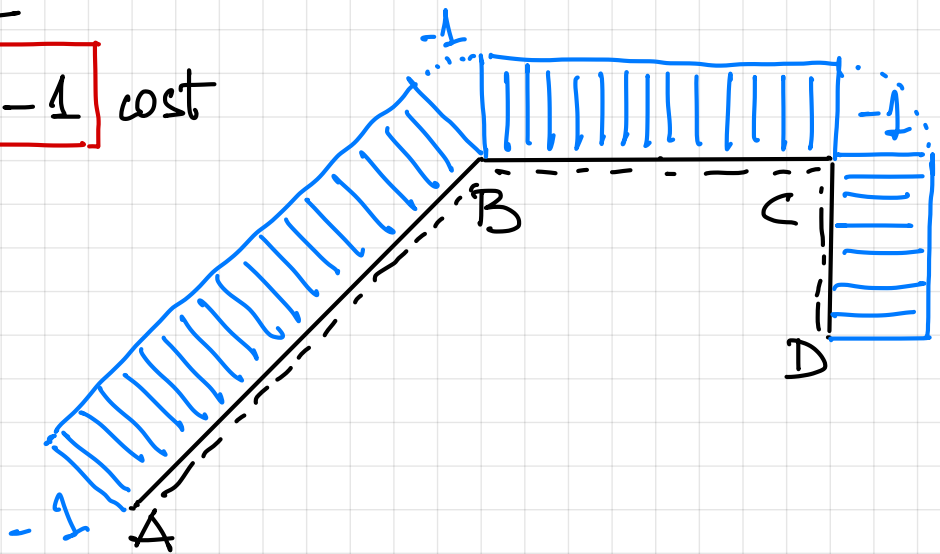
TRATTO BC $0 \leq z \leq L$

&

TRATTO CD $0 \leq z \leq \frac{L}{2}$

... Come sopra

$M^{(1)}(z) = -1 \cos t$



$$Lve = \sum_i X_i \eta_i^{(r)} + \sum_j P_j^{(1)} \eta_j^{(r)} = 1 \cdot \varphi_A^{(r)} + M_D^{(1)} \varphi_D^{(r)} =$$

$$= -\bar{E}_A X + \underset{-1}{M_D^{(1)}} (-\bar{E}_D) \left[\underset{-\frac{9}{8}qL^2}{M_D^{(0)}} + X \underset{-1}{M_D^{(1)}} \right] =$$

$$= -\bar{E}_A X - \bar{E}_D \left[\frac{9}{8}qL^2 + X \right]$$



$$L_{vi} = \int_{str} M^{(1)} \cancel{\frac{M^{(r)}}{EI}} ds + \int_{str} M^{(1)} \frac{\alpha \Delta T}{h} ds =$$

$$= \frac{1}{EI} \int_{str} M^{(1)} M^{(0)} ds + \frac{\alpha}{EI} \int_{str} [M^{(1)}]^2 ds + \frac{\alpha \Delta T}{h} \int_{str} M^{(1)} ds =$$

$$= \frac{1}{EI} \left[\int_0^{L\sqrt{2}} q \frac{L\sqrt{2}}{2} z dz + \int_0^L q L^2 dz + \int_{\frac{L}{2}}^L \left[\frac{q}{8} L^2 - \frac{q}{2} \left(\frac{L^2}{4} + z^2 - Lz \right) \right] dz \right] +$$

$$+ \frac{\alpha}{EI} \left[\int_0^{L\sqrt{2}} dz + \int_0^L dz + \int_{\frac{L}{2}}^L \frac{L}{2} dz \right] + \frac{\alpha \Delta T}{h} \int_0^{L\sqrt{2}} (-1) dz =$$

$$= \frac{1}{EI} \left[\frac{qL\sqrt{2}}{2} \cdot \left[\frac{z^2}{2} \right]_0^{L\sqrt{2}} + qL^2 \cdot L + \frac{q}{8} qL^2 \cdot \frac{L}{2} - \frac{qL^2}{8} \cdot \frac{L}{2} - \frac{q}{2} \left[\frac{z^3}{3} \right]_{\frac{L}{2}}^L + \right.$$

$$\left. + \frac{qL}{2} \left[\frac{z^2}{2} \right]_{\frac{L}{2}}^L \right] +$$

$$+ \frac{\alpha}{EI} \left[L\sqrt{2} + L + \frac{L}{2} \right] - \frac{\alpha \Delta T}{h} \cdot L\sqrt{2} =$$

$$= \frac{qL^3}{EI} \left[\frac{\sqrt{2}}{2} + 1 + \frac{9}{16} - \cancel{\frac{1}{16}} - \frac{1}{48} + \cancel{\frac{1}{16}} \right] +$$

$$+ \frac{\alpha L}{EI} \left[\sqrt{2} + \frac{3}{2} \right] - \frac{\alpha \Delta T}{h} \cdot L\sqrt{2} =$$

$$= \frac{qL^3}{EI} \left[\frac{\sqrt{2}}{2} + \frac{37}{24} \right] + \frac{\alpha L}{EI} \left[\sqrt{2} + \frac{3}{2} \right] - \frac{\alpha \Delta T}{h} \cdot L\sqrt{2}$$

➡ $L_{ve} = L_{vi}$ fornisce:

$$-\bar{\varepsilon}_A X - \bar{\varepsilon}_D \left[\frac{9}{8} q L^2 + X \right] =$$

$$= \frac{q L^3}{EI} \left[\frac{\sqrt{2}}{2} + \frac{37}{24} \right] + \frac{X L}{EI} \left[\sqrt{2} + \frac{3}{2} \right] - \frac{\alpha \Delta T}{h} \cdot L \sqrt{2}$$

$$-X \left[\bar{\varepsilon}_A + \bar{\varepsilon}_D + \frac{L}{EI} \sqrt{2} + \frac{3}{2} \frac{L}{EI} \right] =$$

$$\frac{L}{EI} \left[\frac{\sqrt{2}}{9} - \frac{3}{2} \right]$$

$$\frac{8}{9} \frac{L \sqrt{2}}{EI}$$

$$= \frac{q L^3}{EI} \left[\frac{\sqrt{2}}{2} + \frac{37}{24} \right] - \frac{\alpha \Delta T}{h} L \sqrt{2} + \bar{\varepsilon}_D \cdot \frac{9}{8} q L^2$$

$$\frac{8}{9} \frac{L \sqrt{2}}{EI}$$

$$\frac{q L^2}{EI} \left[\frac{37}{24 \sqrt{2}} + \frac{1}{2} \right]$$

$$-X \frac{L}{EI} \left[\frac{\sqrt{2}}{9} - \frac{3}{2} + \frac{8}{9} \sqrt{2} + \sqrt{2} + \frac{3}{2} \right] = \frac{q L^3}{EI} \sqrt{2}$$

$$2 \sqrt{2}$$

da cui

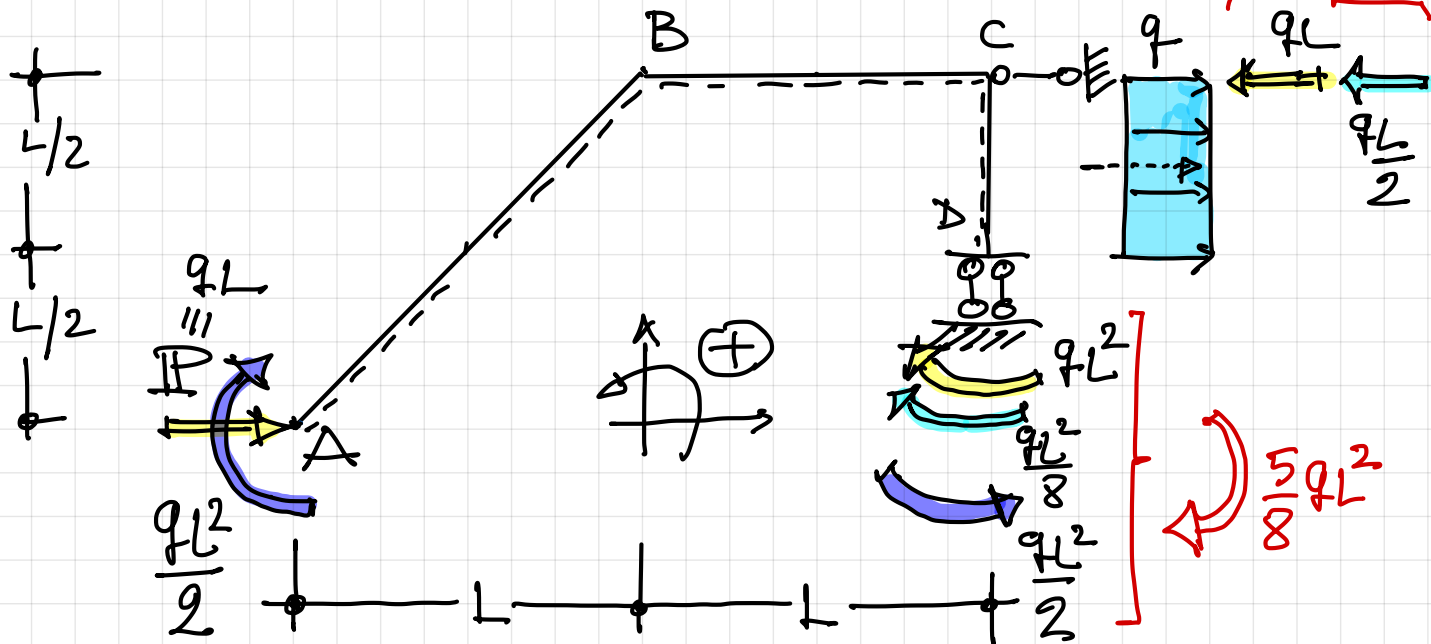
$$X = - \frac{q L^2}{2}$$

NEGATIVO!

↳ verso opposto a quello ipotizzato



SOLUZIONE SISTEMA PRINCIPALE ISOSTATICO



TRATTO AB $0 \leq z \leq L\sqrt{2}$

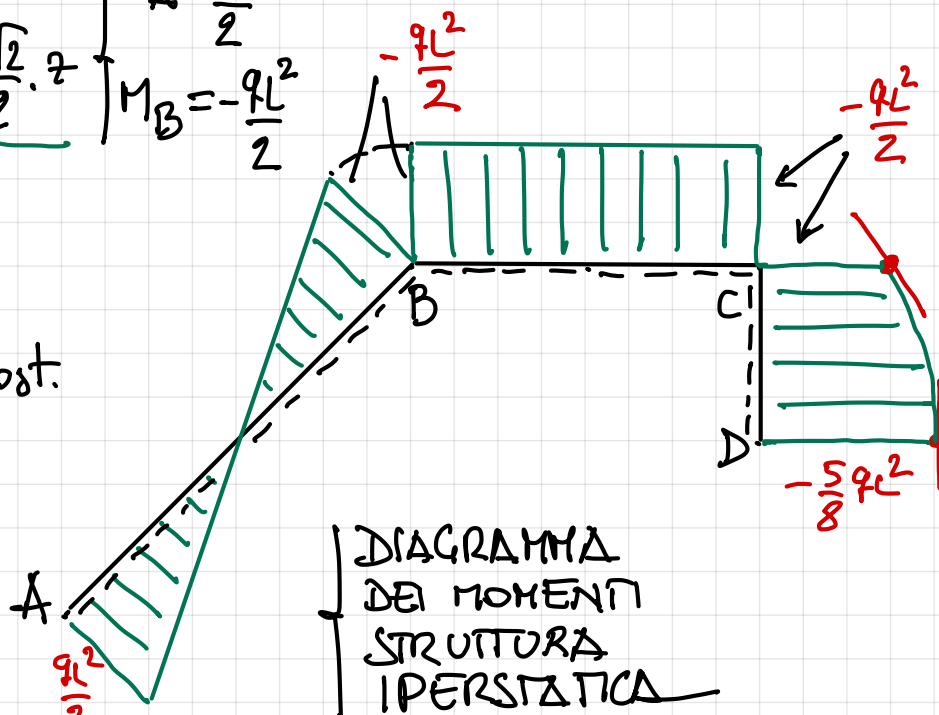
$$\left(\begin{array}{c} \downarrow qL\sqrt{2}/2 \\ \leftarrow qL/2 \end{array} \right) \left(\begin{array}{c} \rightarrow \\ \uparrow \end{array} \right) \left\{ \begin{array}{l} M^{(r)}(z) = \frac{qL^2}{2} - \frac{qL\sqrt{2}}{2} \cdot z \\ M_A = \frac{qL^2}{2} \\ M_B = -\frac{qL^2}{2} \end{array} \right.$$

TRATTO BC $0 \leq z \leq L$

$$\left(\begin{array}{c} \downarrow qL \\ \leftarrow qL/2 \end{array} \right) \left(\begin{array}{c} \rightarrow \\ \uparrow \end{array} \right) \left\{ \begin{array}{l} M^{(r)}(z) = -\frac{qL^2}{2} \text{ cost.} \\ M_B = -\frac{qL^2}{2} \\ M_C = -\frac{qL^2}{8} \end{array} \right.$$

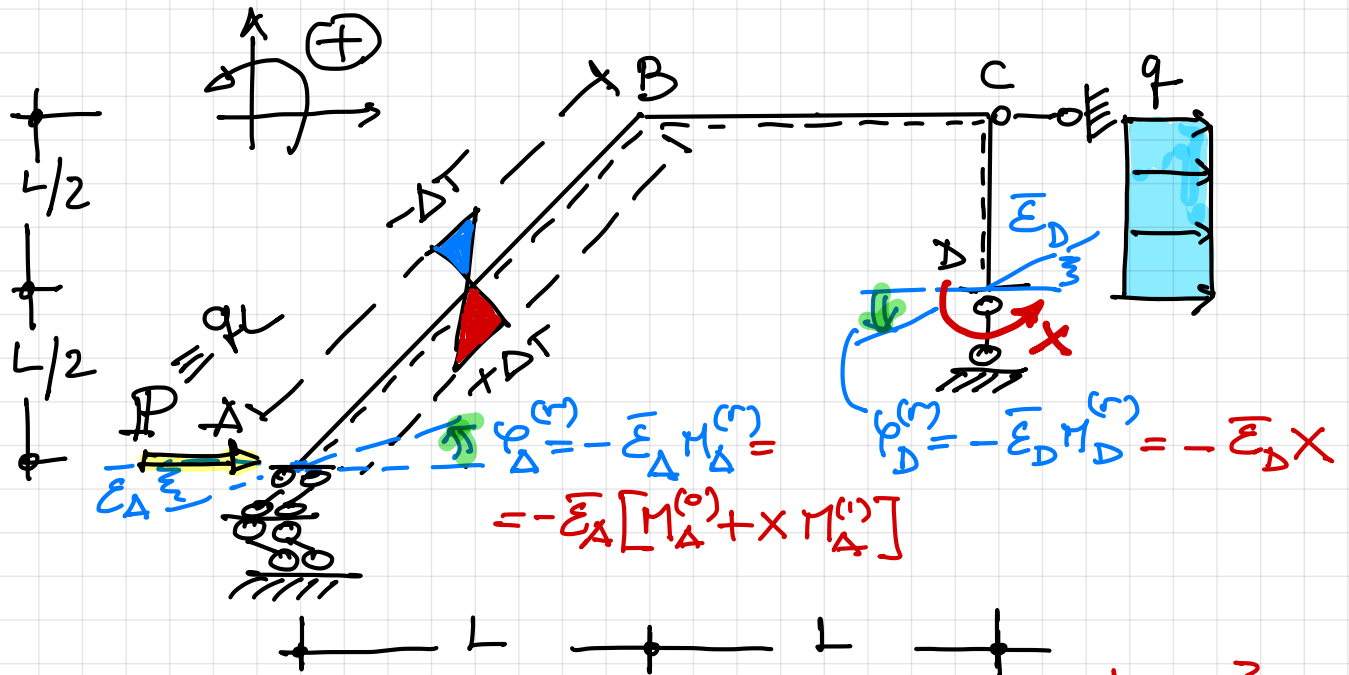
TRATTO CD $0 \leq z \leq \frac{L}{2}$

$$\left(\begin{array}{c} \uparrow \uparrow \uparrow \\ \leftarrow qL/2 \end{array} \right) \left(\begin{array}{c} \rightarrow \\ \uparrow \end{array} \right) \left\{ \begin{array}{l} M^{(r)}(z) = \frac{q(\frac{L}{2}-z)^2}{2} - \frac{5qL^2}{8} \\ M_C = -\frac{qL^2}{2} \\ M_D = -\frac{5qL^2}{8} \end{array} \right.$$

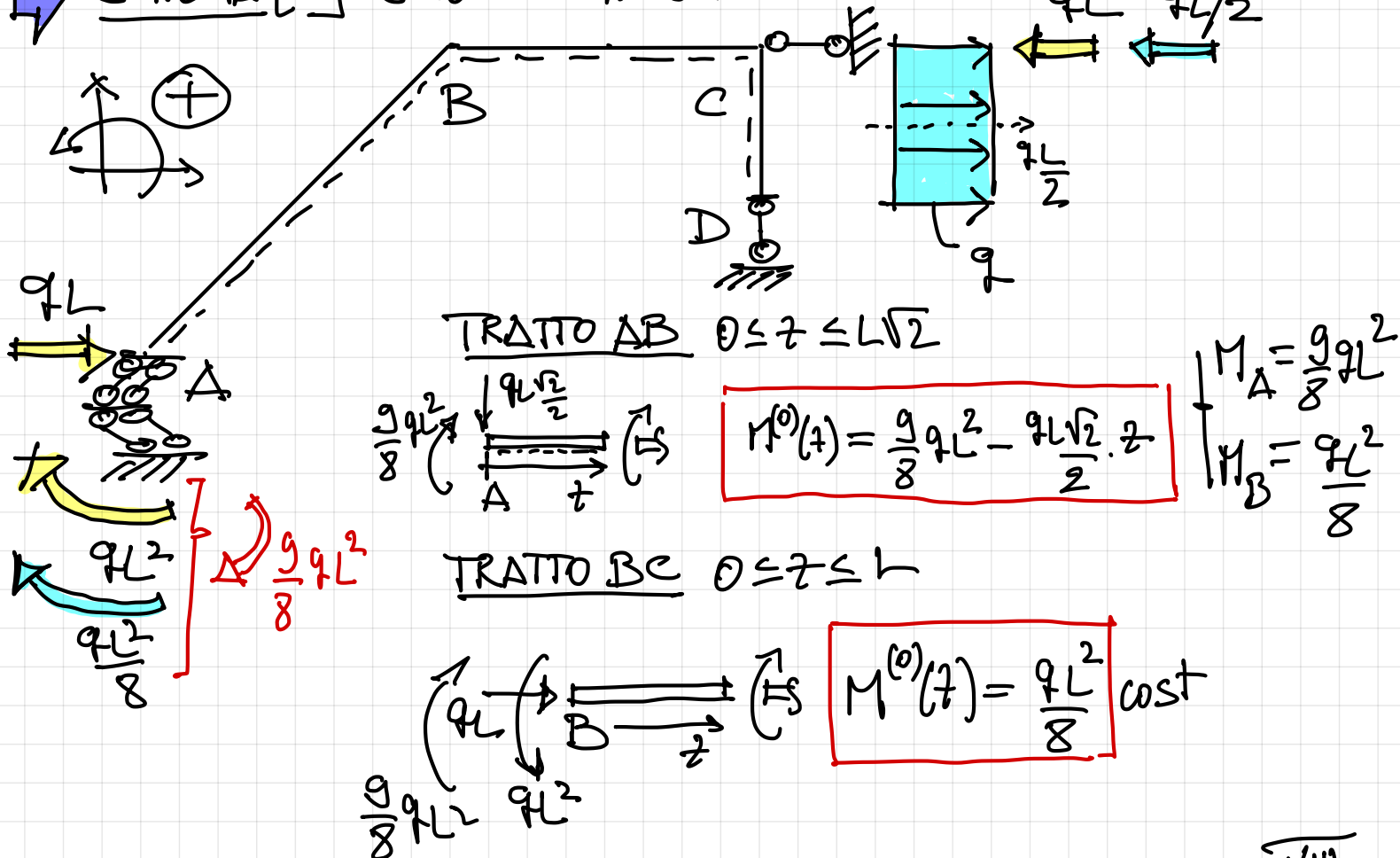


SOLUZIONE #2

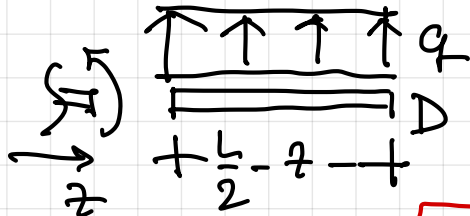
SISTEMA PRINCIPALE ISOSTATICO



SCHEMA [0] SOLO CARICHI ESTERNI



TRATTO CD $0 \leq z \leq \frac{L}{2}$



$$M^{(0)}(z) = \frac{q(\frac{L}{2} - z)^2}{2} \quad \left\{ \begin{array}{l} M_C = \frac{qL^2}{8} \\ M_D = 0 \end{array} \right.$$

SCHEMA [1]
Solo $X=1$

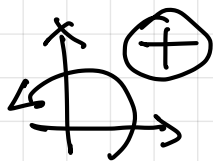


DIAGRAMMA $M^{(0)}(z)$

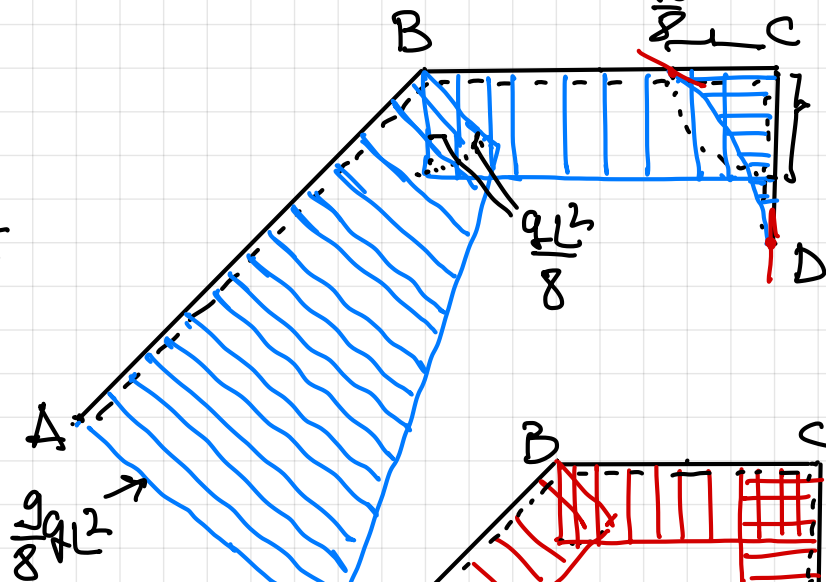


DIAGRAMMA
 $M^{(1)}(z)$

$$M^{(1)}(z) = 1 \quad \text{cost}$$

vale in tutti i tratti

TRATTO AB $0 \leq z \leq L\sqrt{2}$

TRATTO BC $0 \leq z \leq L$

TRATTO CD $0 \leq z \leq \frac{L}{2}$

$$\begin{aligned} L_{ve} &= \sum_i x_i \eta_i^{(r)} + \sum_j R_j^{(1)} \eta_j^{(r)} = 1 \cdot \varphi_D^{(r)} + M_A^{(1)} \varphi_A^{(r)} = \\ &= -\bar{E}_D x + \underbrace{M_A^{(1)}}_{-1} (-\bar{E}_A) \left[\underbrace{M_A^{(0)}}_{-\frac{9}{8}qL^2} + x \underbrace{\eta_A^{(1)}}_{-1} \right] = \\ &= -\bar{E}_D x - \bar{E}_A \left[\frac{9}{8}qL^2 + x \right] \end{aligned}$$

$$\begin{aligned}
\Rightarrow L_{vi} &= \int_{str} M^{(1)} \frac{M^{(r)}}{EI} dstr + \int_{str} m^{(1)} \frac{\alpha \Delta T}{h} dstr = \\
&= \frac{1}{EI} \int_{str} M^{(1)} M^{(0)} dstr + \frac{\chi}{EI} \int_{str} [M^{(1)}]^2 dstr + \frac{\alpha \Delta T}{h} \int_{str} m^{(1)} dstr = \\
&= \frac{1}{EI} \left[\int_0^{L\sqrt{2}} \left[\frac{q}{8} qL^2 - \frac{qL\sqrt{2}}{2} \cdot z \right] dz + \int_0^L \frac{qL^2}{8} dz + \right. \\
&\quad \left. + \int_0^{L/2} \frac{q}{2} \left(\frac{L}{2} - z \right)^2 dz \right] + \frac{L^2}{4} + z^2 - Lz \\
&\quad + \frac{\chi}{EI} \left[\int_0^{L\sqrt{2}} dz + \int_0^L dz + \int_0^{L/2} dz \right] + \frac{\alpha \Delta T}{h} \int_0^{L\sqrt{2}} dz = \\
&= \frac{1}{EI} \left[\frac{q}{8} qL^2 \cdot L\sqrt{2} - \frac{qL\sqrt{2}}{2} \cdot \frac{1}{2} \cdot L^2 + \frac{qL^2}{8} \cdot L + \frac{qL^2}{8} \cdot \frac{L}{2} + \frac{qL^3}{48} - \frac{qL^3}{16} \right] + \\
&\quad + \frac{\chi}{EI} \left[L\sqrt{2} + L + \frac{L}{2} \right] + \frac{\alpha \Delta T}{h} \cdot L\sqrt{2} = \\
&= \frac{qL^3}{EI} \left[\underbrace{\frac{9}{8}\sqrt{2} - \frac{\sqrt{2}}{2}}_{\frac{5\sqrt{2}}{8}} + \underbrace{\frac{1}{8} + \cancel{\frac{1}{16}} + \frac{1}{48} - \cancel{\frac{1}{16}}}_{\frac{7}{48}} \right] + \frac{\chi L}{EI} \left[\sqrt{2} + \frac{3}{2} \right] + \frac{\alpha \Delta T}{h} L\sqrt{2} = \\
&= \frac{qL^3}{EI} \left[\frac{5\sqrt{2}}{8} + \frac{7}{48} \right] + \frac{\chi L}{EI} \left[\sqrt{2} + \frac{3}{2} \right] + \frac{\alpha \Delta T}{h} L\sqrt{2}
\end{aligned}$$

➡ $L_{ve} = L_{vi}$ fornisce

$$-\bar{\varepsilon}_D X - \bar{\varepsilon}_A \left[\frac{9}{8} q L^2 + X \right] =$$

$$= \frac{q L^3}{EI} \left[\frac{5\sqrt{2}}{8} + \frac{7}{48} \right] + \frac{X L}{EI} \left[\sqrt{2} + \frac{3}{2} \right] + \frac{\alpha \Delta T}{h} L \sqrt{2}$$

$$-X \left[\bar{\varepsilon}_D + \bar{\varepsilon}_A + \frac{L}{EI} \sqrt{2} + \frac{3}{2} \frac{L}{EI} \right] =$$

$$\frac{8}{9} \frac{L}{EI} \sqrt{2}$$

$$\frac{L}{EI} \left[\frac{\sqrt{2}}{9} - \frac{3}{2} \right]$$

$$= \frac{q L^3}{EI} \left[\frac{5\sqrt{2}}{8} + \frac{7}{48} \right] + \frac{\alpha \Delta T}{h} L \sqrt{2} + \bar{\varepsilon}_A \cdot \frac{9}{8} q L^2$$

$$\frac{q L^2}{EI} \left[\frac{37}{24\sqrt{2}} + \frac{1}{2} \right]$$

$$\frac{L}{EI} \left[\frac{\sqrt{2}}{9} - \frac{3}{2} \right]$$

$$-X \frac{L}{EI} \left[\frac{8}{9} \sqrt{2} + \frac{\sqrt{2}}{9} - \frac{3}{2} + \sqrt{2} + \frac{3}{2} \right] =$$

$2\sqrt{2}$

$$= \frac{q L^3}{EI} \left[\frac{5\sqrt{2}}{8} + \frac{7}{48} + \frac{37}{24} + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{8} - \frac{27}{16} \right]$$

$$\frac{7 + 74 - 81}{48} = 0$$

$$-X \frac{L}{EI} 2\sqrt{2} = \frac{q L^3}{EI} \frac{5\sqrt{2}}{4}$$

da cui $X = -\frac{5 q L^2}{8}$

NEGATIVA

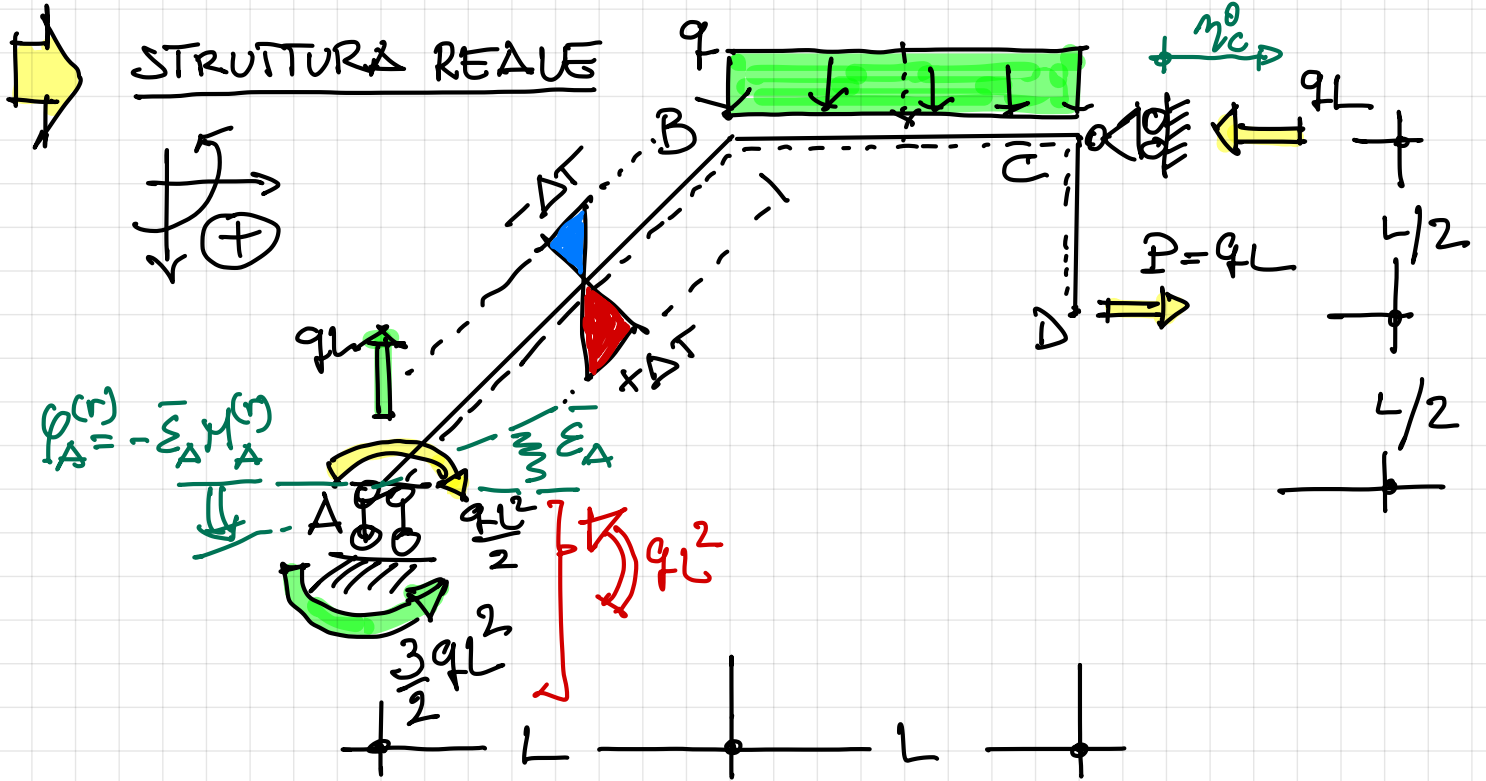
VERSO OPPOSTO

A QUELLO IPOTIZZATO!

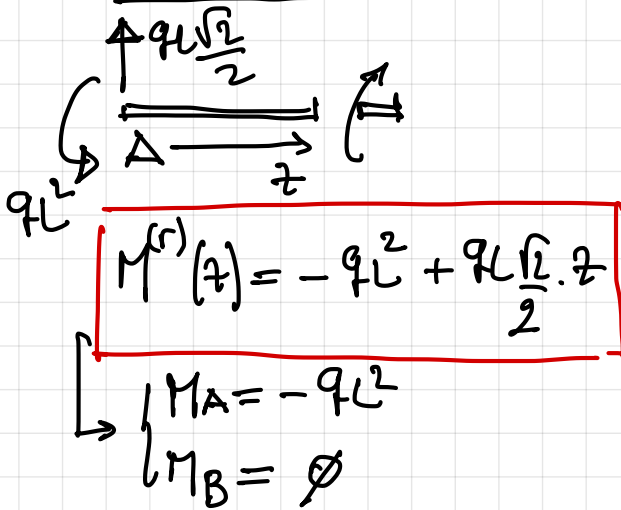
VALORE OK! cfr. RV e pag. VII

ES.#2 - tipo 2

SOLUZIONE



TRATTO AB $0 \leq z \leq L\sqrt{2}$



$$M^{(r)}(z) = -qL^2 + qL \frac{L}{2} \cdot z$$

$$\begin{cases} M_A = -qL^2 \\ M_B = 0 \end{cases}$$

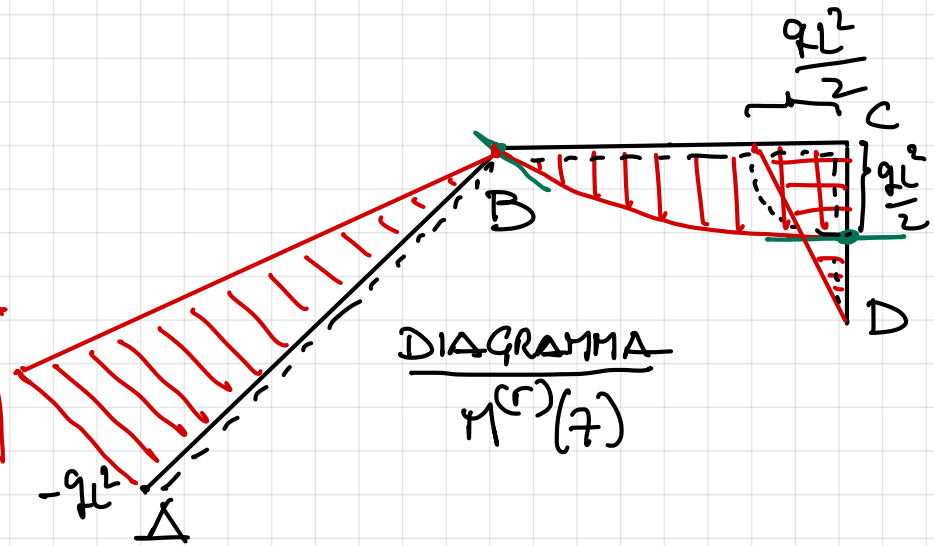
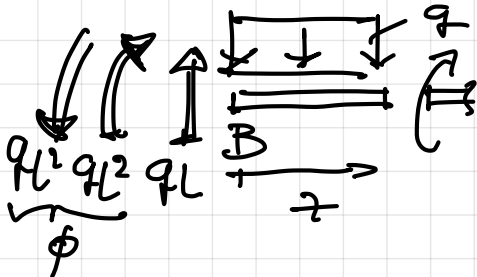


DIAGRAMMA
 $M^{(r)}(z)$

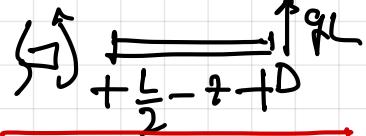
TRATTO BC $0 \leq z \leq L$



$$M^{(r)}(z) = qL \cdot z - \frac{qz^2}{2}$$

$$\begin{cases} M_B = 0 \\ M_C = \frac{qL^2}{2} \end{cases}$$

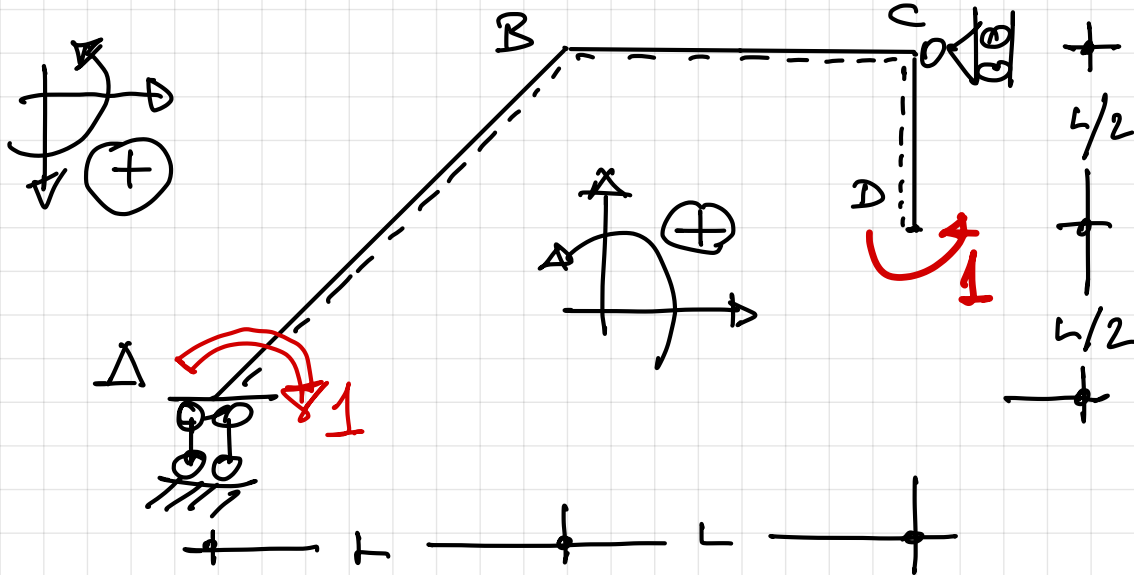
TRATTO CD $0 \leq z \leq \frac{L}{2}$



$$M^{(r)}(z) = qL \left(\frac{L}{2} - z \right)$$

$$\begin{cases} M_C = \frac{qL^2}{2} \\ M_D = 0 \end{cases}$$

➡ STRUTTURA FINITIA PER IL CALCOLO DI φ_D



TRATTO AB $0 \leq z \leq L\sqrt{2}$

1 $\begin{matrix} \curvearrowright A \\ \text{---} \\ \curvearrowright \end{matrix} \Rightarrow \boxed{M^{(f)}(z) = 1 \cos z}$

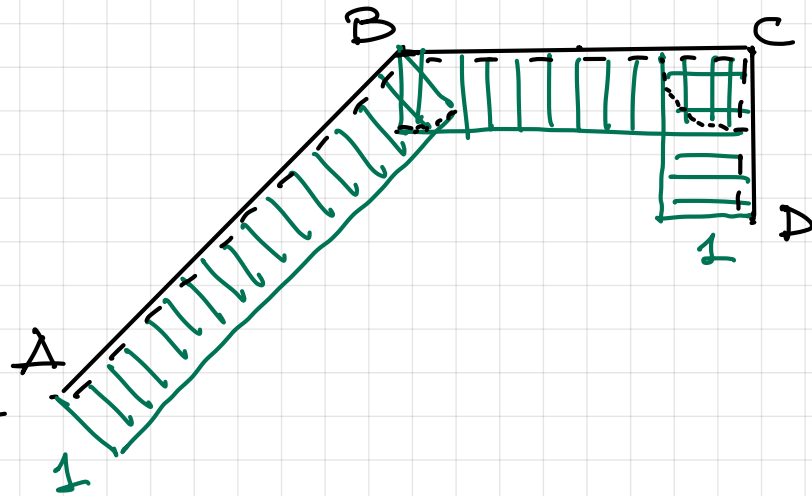
DIAGRAMMA $M^{(f)}(z)$

TRATTO BC $0 \leq z \leq L$

1 $\begin{matrix} \curvearrowright \\ \text{---} \\ \curvearrowright B \end{matrix} \Rightarrow \boxed{M^{(f)}(z) = 1 \cos z}$

TRATTO CD $0 \leq z \leq \frac{L}{2}$

1 $\begin{matrix} \curvearrowright \\ \text{---} \\ \curvearrowright C \end{matrix} \Rightarrow \boxed{M^{(f)}(z) = 1 \cos z}$



➡ L_{ve} $= 1 \cdot \varphi_D^{(r)} + \sum_j R_j^{(f)} \eta_j^{(r)} = \varphi_D^{(r)} + \underbrace{M_A^{(f)}}_{-1} \underbrace{\varphi_A^{(r)}}_{-\bar{\epsilon}_A \eta L^2} + \underbrace{R_{x_c}^{(f)}}_{\phi} \eta_c^0 =$

$= \varphi_D^{(r)} + \bar{\epsilon}_A \eta L^2$

$$\Rightarrow \underline{L_{vi}} = \int_{str} M^{(t)} \frac{M^{(r)}}{EI} dstr + \int_{str} \eta^{(t)} \frac{\alpha \Delta T}{h} dstr =$$

$$= \frac{1}{EI} \left\{ \int_0^{L\sqrt{2}} \left[-qL^2 + qL \frac{L}{2} \cdot z \right] dz + \int_0^L \left[qL \cdot z - \frac{qz^2}{2} \right] dz + \right. \\ \left. + \int_0^{L/2} qL \left(\frac{L}{2} - z \right) dz \right\} + \frac{\alpha \Delta T}{h} \int_0^{L\sqrt{2}} dz =$$

$$= \frac{1}{EI} \left\{ -qL^2 \cdot L\sqrt{2} + qL \frac{L}{2} \left[\frac{z^2}{2} \right]_0^{L\sqrt{2}} + qL \cdot \frac{L}{2} - \frac{q}{2} \frac{L^3}{3} + \right. \\ \left. + \frac{qL^2}{2} \cdot \frac{L}{2} - qL \left[\frac{z^2}{2} \right]_0^{\frac{L}{2}} \right\} + \frac{\alpha \Delta T}{h} \cdot L\sqrt{2} =$$

$$= \frac{1}{EI} \left\{ -qL^3\sqrt{2} + \frac{qL^3\sqrt{2}}{2} + \frac{qL^3}{2} - \frac{qL^3}{6} + \frac{qL^3}{4} - \frac{qL^3}{8} \right\} + \frac{\alpha \Delta T}{h} L\sqrt{2} =$$

$$= \frac{qL^3}{EI} \left\{ -\sqrt{2} + \frac{\sqrt{2}}{2} + \frac{1}{2} - \frac{1}{6} + \frac{1}{4} - \frac{1}{8} \right\} + \frac{\alpha \Delta T}{h} \cdot L\sqrt{2} =$$

$$\frac{12 - 4 + 6 - 3}{24} = \frac{11}{24}$$

$$= \frac{qL^3}{EI} \left[-\frac{\sqrt{2}}{2} + \frac{11}{24} \right] + \frac{\alpha \Delta T}{h} \cdot L\sqrt{2}$$

⇒ $L_{ve} = L_{vi}$ fornisce

$$\varphi_D^{(r)} + \underbrace{\frac{qL^2}{8EI}}_{\frac{L}{8EI}} = \frac{qL^3}{EI} \left[-\cancel{\frac{\sqrt{2}}{2}} + \frac{11}{24} \right] + \cancel{\frac{\alpha \Delta T \cdot L \sqrt{2}}{\frac{L}{2EI} + \frac{qL^2}{2EI}}}$$

e, in definitiva:

$$\varphi_D^{(r)} = \frac{qL^3}{EI} \left[\underbrace{\frac{11}{24} - \frac{1}{8}}_{\frac{1}{3}} \right] = \frac{qL^3}{3EI} \Rightarrow \text{POSITIVA ANTIORARIA!}$$