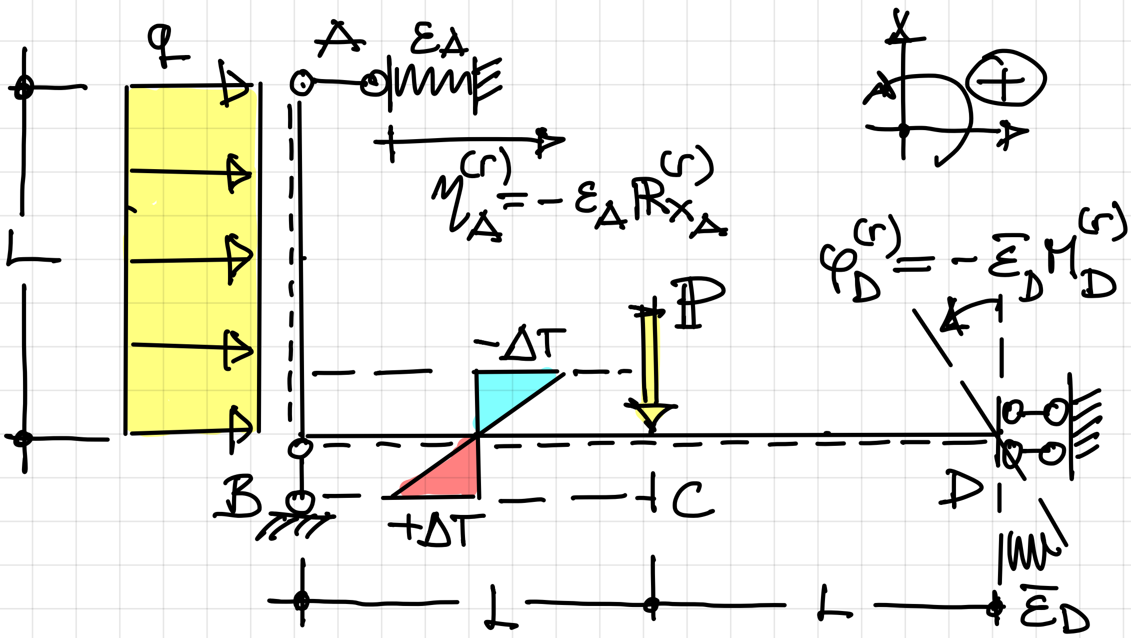
MECCANICA delle STRUTTURE – P. FUSCHI
PROVA SCRITTA del 29 GENNAIO 2025

TIPO 1

ES. #1

RISOLVERE LA STRUTTURA UNA VOLTA IPERSTATICA SEGUENTE DETERMINANDO IL DIAGRAMMA DEI MOMENTI



Posizioni:

$|\mathbb{P}| = qL$

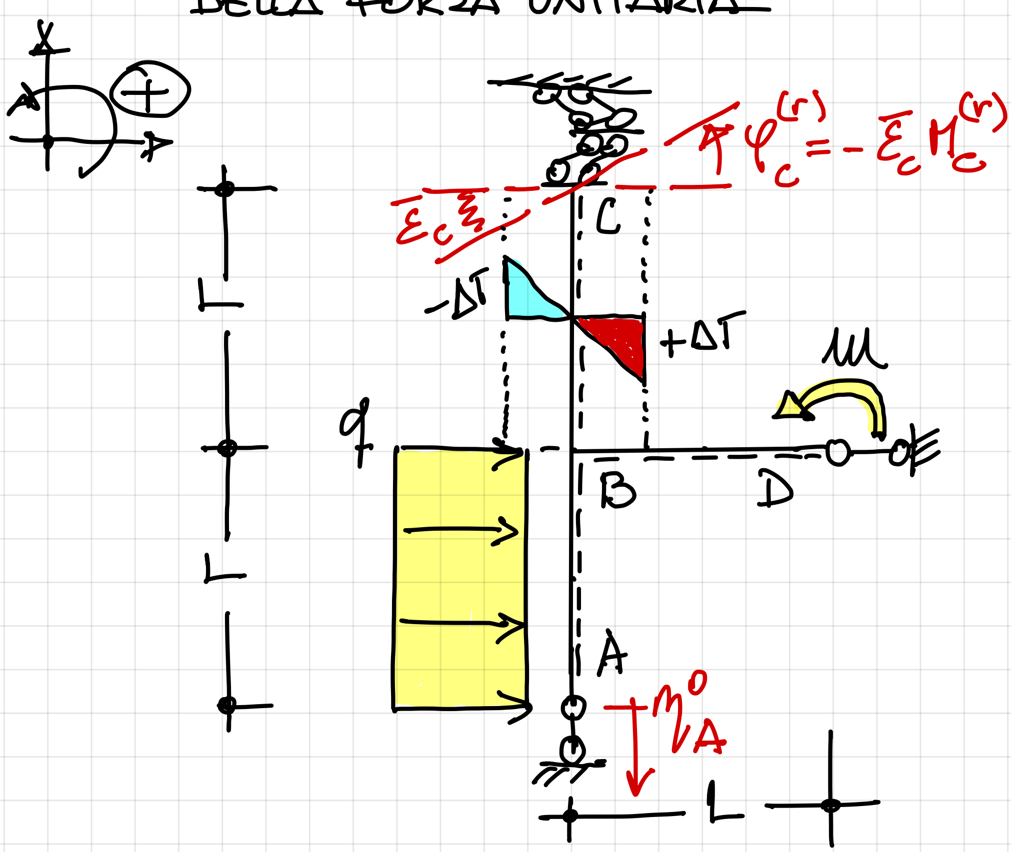
$|\varepsilon_A| = \frac{5}{3}\frac{L^3}{EI}$

$|\bar{\varepsilon}_D| = \frac{2L}{EI}$

$\left|\frac{\alpha \Delta T}{h}\right| = \frac{3}{8}\frac{qL^2}{EI}$

ES. #2

CALCOLARE LO SPOST. VERTICALE DELLA SEZ. D DELLA STRUTTURA SEGUENTE CON IL METODO DELLA FORZA UNITARIA



POSIZIONI:

$|\mathcal{M}| = qL^2$

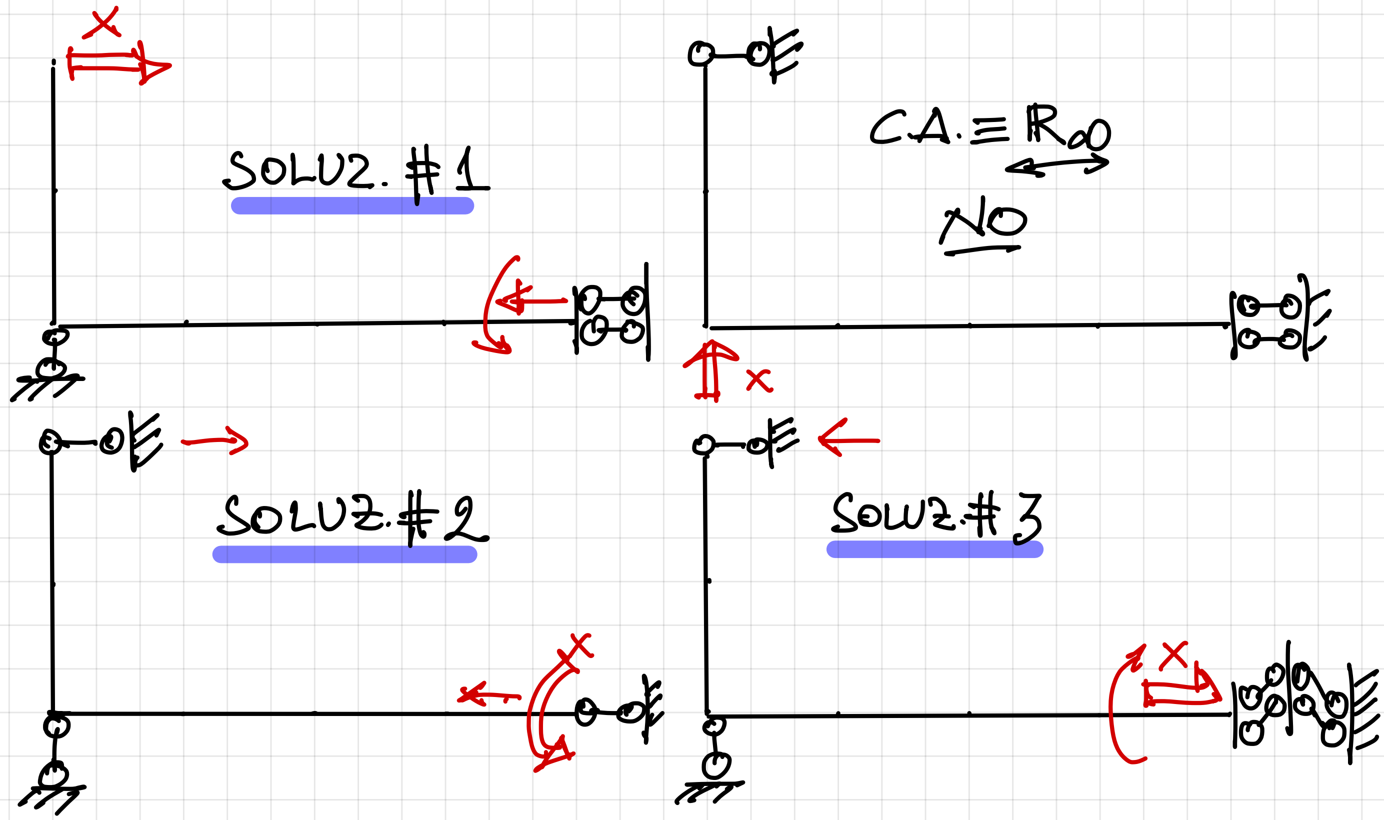
$|\bar{\varepsilon}_C| = \frac{L}{EI}$

$|\eta_A^0| = \frac{qL^4}{2EI}$

$\left|\frac{\alpha \Delta T}{h}\right| = \frac{2qL^2}{EI}$

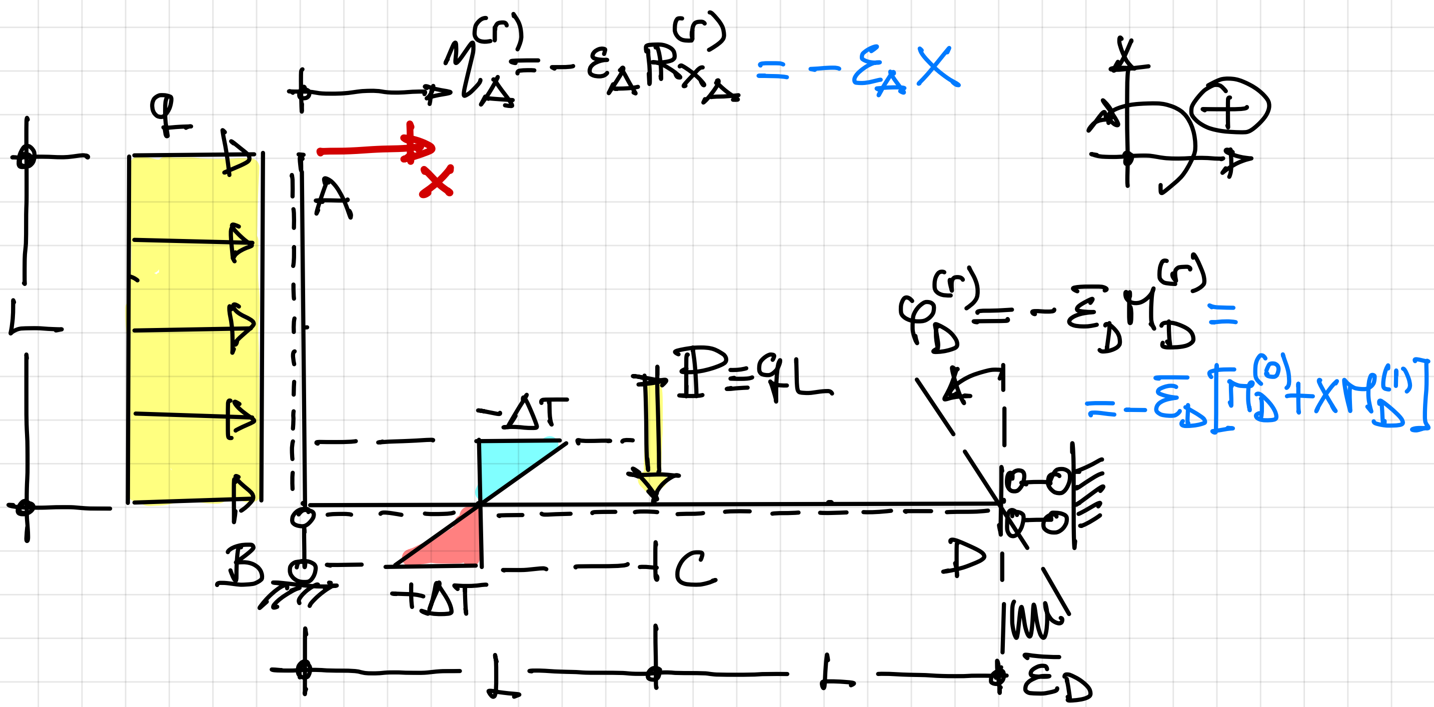
ES.#1 – TIPO 1 SOLUZIONI

SOLUZIONI POSSIBILI

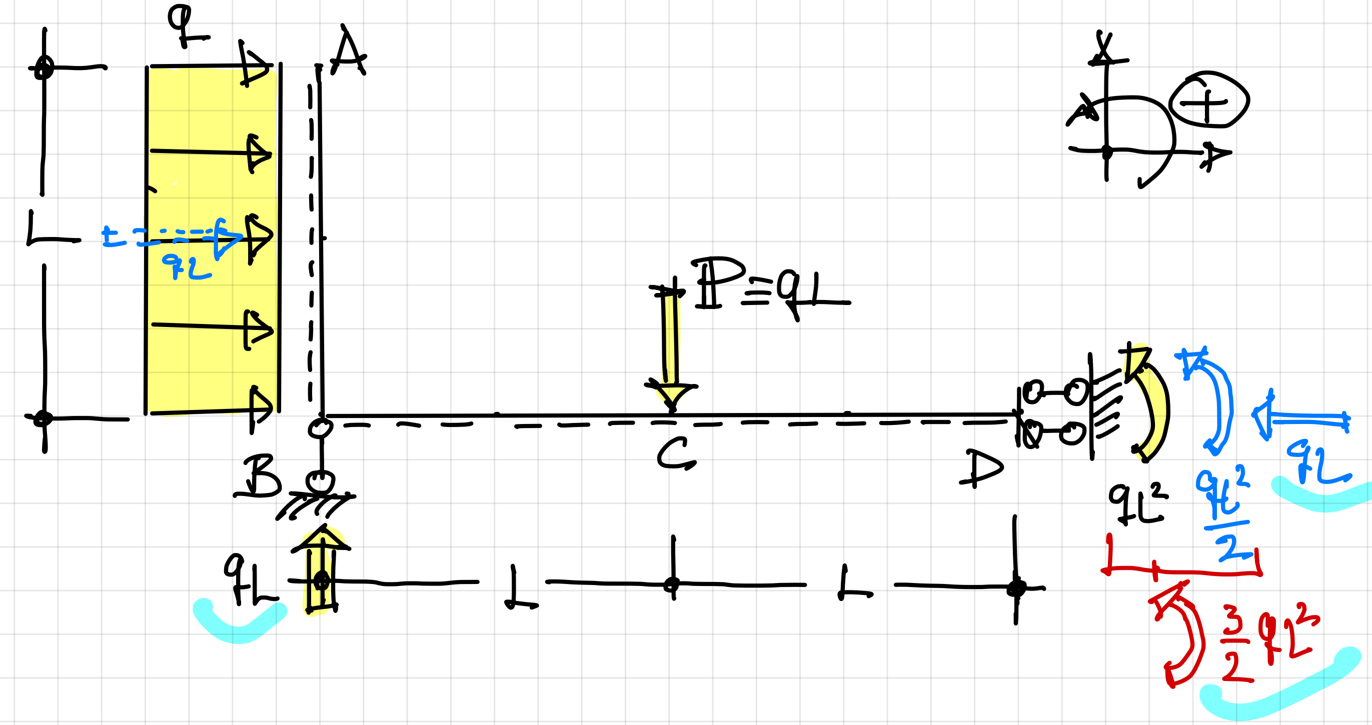


SOLUZIONE #1

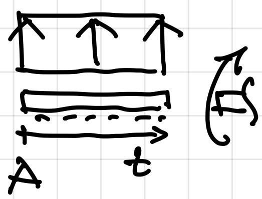
SISTEMA PRINCIPALE ISOSTATICO



SCHEMA [0] – SOLO CARICHI ESTERNI



TRATTO AB $0 \leq z \leq L$

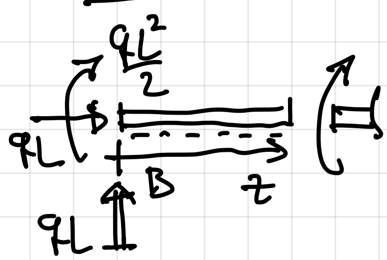


$$\boxed{M^{(0)}(z) = \frac{q z^2}{2}}$$

$M_A = \phi$

$M_B = \frac{qL^2}{2}$

TRATTO BC $0 \leq z \leq L$

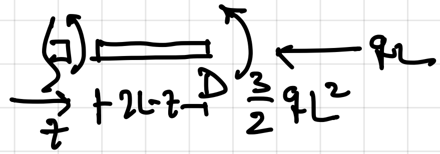


$$\boxed{M^{(0)}(z) = \frac{qL^2}{2} + qL \cdot z}$$

$M_B = \frac{qL^2}{2}$

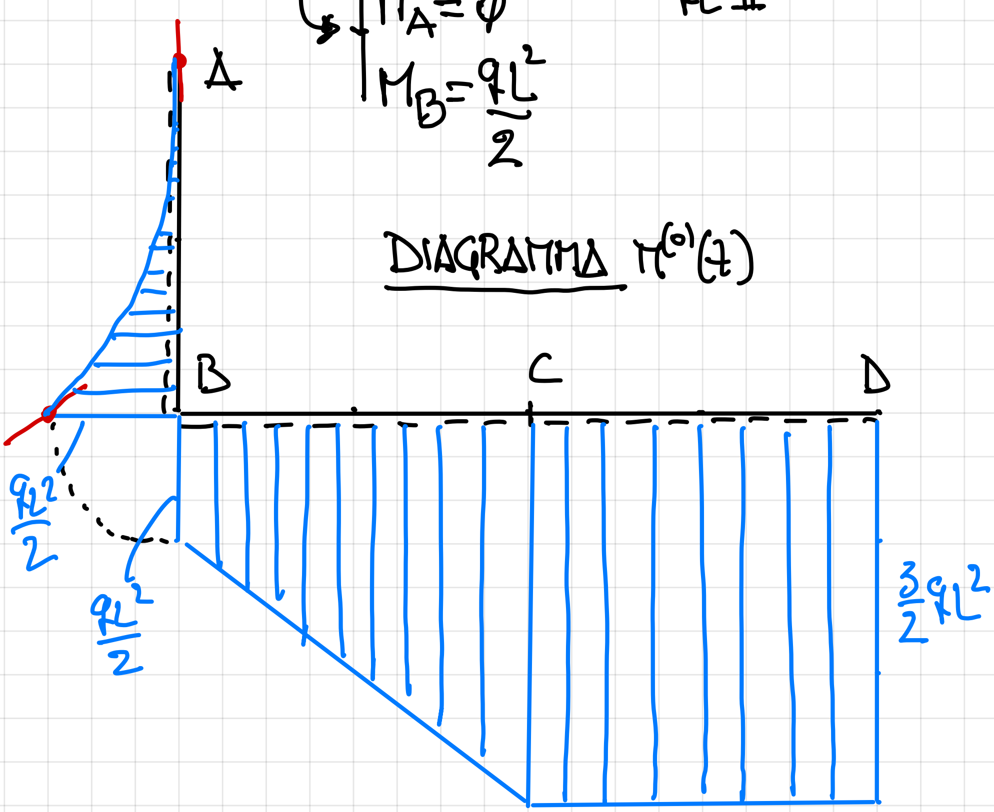
$M_C = \frac{3}{2} qL^2$

TRATTO CD $L \leq z \leq 2L$

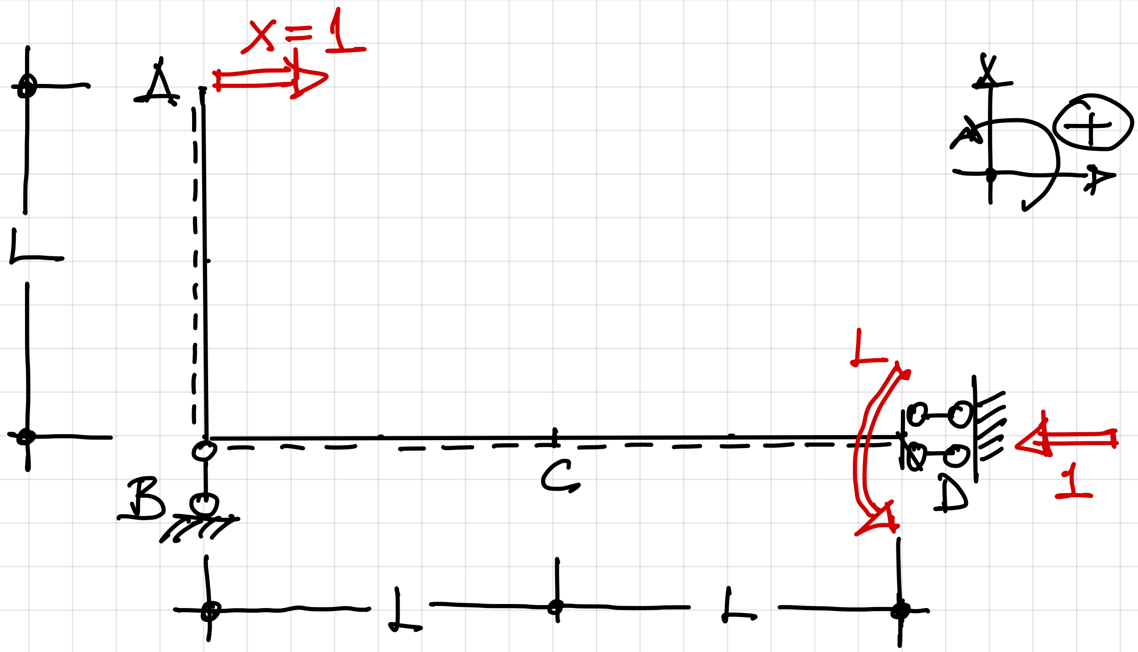


$$\boxed{M^{0}(z) = \frac{3}{2} qL^2}$$ cost.

DIAGRAMMA $M^{(0)}(z)$



SCHEMA [1] – SOLO $X=1$



TRATTO AB $0 \le z \le L$

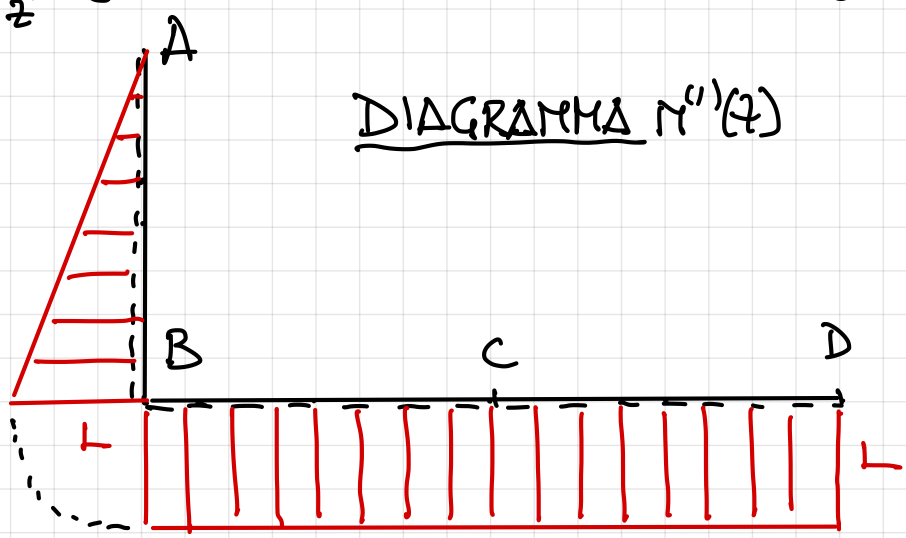
$$\boxed{M^{(1)}(z) = z} \quad \begin{cases} M_A = \phi \\ M_B = L \end{cases}$$

TRATTO BD $0 \le z \le 2L$

$$\boxed{M^{(1)}(z) = L}$$

costante

DIAGRAMMA $M^{(1)}(z)$



$$\underbrace{L_{ve}} = \underset{1}{\overset{\shortparallel}{X_i}}\,\eta_i^{(r)} + \sum_j R_j^{(1)}\eta_j^{(r)} = -\varepsilon_A X + \underbrace{M_D^{(1)}}_{L}(-\bar{\varepsilon}_D)\Big[\underbrace{M_D^{(0)}}_{\frac{3}{2}qL^2} + X\underbrace{M_D^{(1)}}_{L}\Big] =$$

$$= \underbrace{-\varepsilon_A X}\,\underbrace{-\,\bar{\varepsilon}_D L\left[\tfrac{3}{2}qL^2 + L X\right]}$$

$$L_{vi} = \int_{str} M^{(1)} \frac{M^{(r)}}{EI} \, ds_{tr} + \int_{str} M^{(1)} \frac{\alpha \Delta T}{h} \, ds_{tr} =$$

$$= \frac{1}{EI} \int_{str} M^{(1)} M^{(0)} \, ds_{tr} + \frac{X}{EI} \int_{str} \left[M^{(1)} \right]^2 ds_{tr} + \frac{\alpha \Delta T}{h} \int_{str} M^{(1)} \, ds_{tr} =$$

$$= \frac{1}{EI} \left[\int_0^L (z) \, \frac{q z^2}{2} \, dz + \int_0^L L \left[\frac{qL^2}{2} + qL \cdot z \right] dz + \int_L^{2L} L \left[\frac{3}{2} qL^2 \right] dz \right] +$$

$$+ \frac{X}{EI} \left[\int_0^L z^2 \, dz + \int_0^{2L} L^2 \, dz \right] + \frac{\alpha \Delta T}{h} \int_0^L L \, dz =$$

$$= \frac{1}{EI} \left[\frac{q}{2} \left[\frac{z^4}{4} \right]_0^L + \frac{qL^3}{2} \left[z \right]_0^L + qL^2 \left[\frac{z^2}{2} \right]_0^L + \frac{3}{2} qL^3 \left[z \right]_L^{2L} \right] +$$

$$+ \frac{X}{EI} \left[\left[\frac{z^3}{3} \right]_0^L + L^2 \left[z \right]_0^{2L} \right] + \frac{\alpha \Delta T}{h} \cdot L \left[z \right]_0^L =$$

$$= \frac{1}{EI} \left[\frac{q}{8} L^4 + \frac{qL^4}{2} + \frac{qL^2}{2} \cdot L^2 + \frac{3}{2} qL^3 \cdot L \right] +$$

$$+ \frac{X}{EI} \left[\frac{L^3}{3} + L^2 \cdot 2L \right] + \frac{\alpha \Delta T}{h} \cdot L^2 =$$

$$= \frac{qL^4}{EI} \left[\frac{1}{8} + \frac{1}{2} + \frac{1}{2} + \frac{3}{2} \right] + \frac{XL^3}{EI} \left[\frac{1}{3} + 2 \right] + \frac{\alpha \Delta T}{h} \cdot L^2 =$$

$$= \underbrace{\frac{21}{8} \frac{qL^4}{EI} + \frac{7}{3} \frac{XL^3}{EI} + \frac{\alpha \Delta T}{h} \cdot L^2}$$

Lve = Lvi fornisce:

$$-\underline{\varepsilon_A} X - \bar{\varepsilon}_D L\left[\frac{3}{2}qL^2 + \underline{L}X\right] =$$

$$= \frac{21}{8}\frac{qL^4}{EI} + \underbrace{\frac{7}{3}\frac{XL^3}{EI}} + \frac{\alpha\Delta T}{h} L^2$$

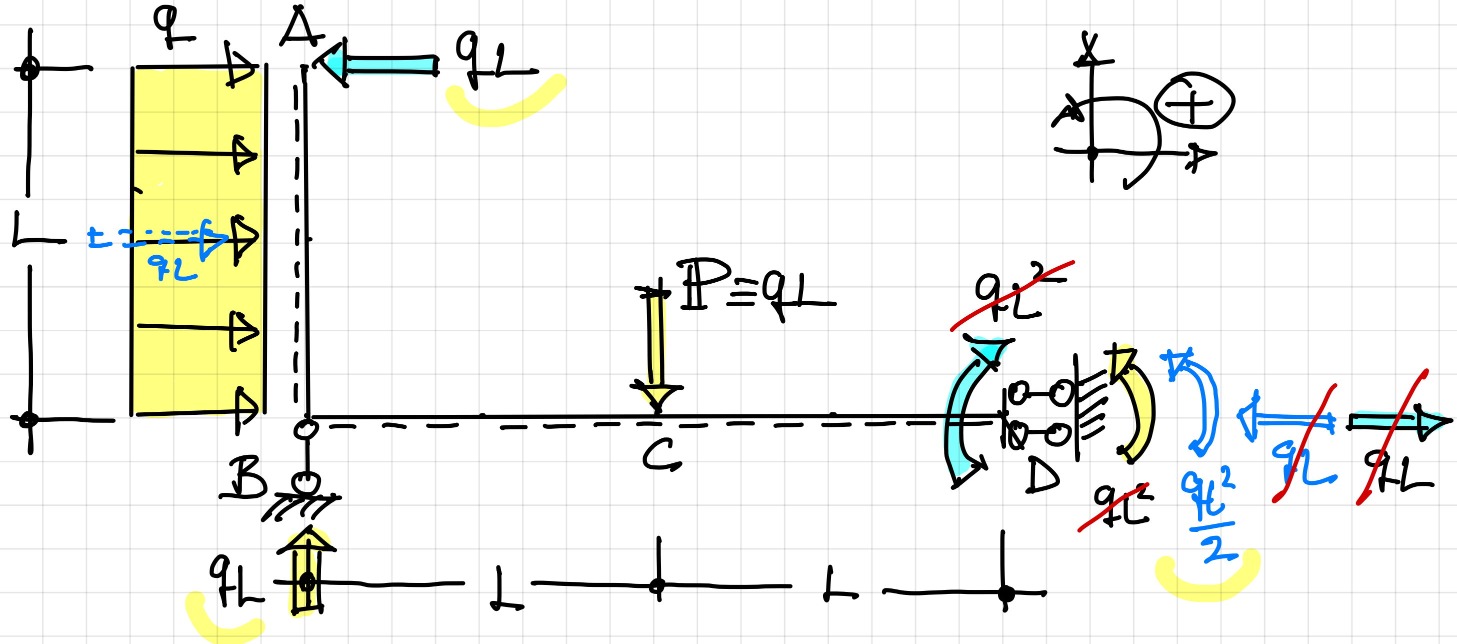
$$X\left[\frac{7}{3}\frac{L^3}{EI} + \underbrace{\varepsilon_A}_{\frac{5}{3}\frac{L^3}{EI}} + \underbrace{\bar{\varepsilon}_D}_{\frac{2L}{EI}} L^2\right] = -\underbrace{\bar{\varepsilon}_D}_{\frac{2L}{EI}}\frac{3}{2}qL^3 - \frac{21}{8}\frac{qL^4}{EI} - \underbrace{\frac{\alpha\Delta T}{h}}_{\frac{3}{8}\frac{qL^2}{EI}} L^2$$

$$X\frac{L^{\cancel{3}}}{\cancel{EI}}\left[\underbrace{\cancel{\frac{7}{3}+\frac{5}{3}+2}}_{6}\right] = -\frac{qL^{\cancel{4}}}{\cancel{EI}}\left[\underbrace{\cancel{3+\frac{21}{8}+\frac{3}{8}}}_{6}\right]$$

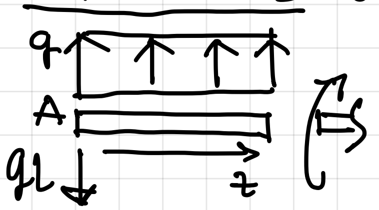
da cui: $\boxed{X = -qL}$ NEGATIVA!

{ VERSO OPPOSTO A QUELLO IPOTIZZATO

SOLUZIONE SISTEMA PRINCIPALE ISOSTATICO



TRATTO AB $0 \le z \le L$

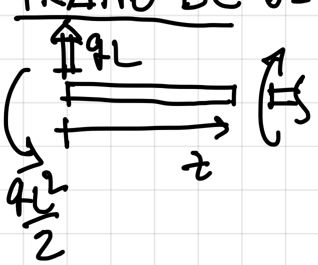


$$\boxed{M^{(r)}(z) = q\frac{z^2}{2} - qL\cdot z}$$

$M_A = \phi$

$M_B = -\dfrac{qL^2}{2}$

TRATTO BC $0 \le z \le L$

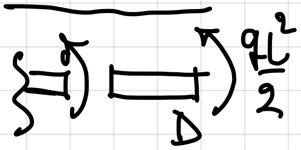


$$\boxed{M^{(r)}(z) = qL\cdot z - \frac{qL^2}{2}}$$

$M_B = -\dfrac{qL^2}{2}$

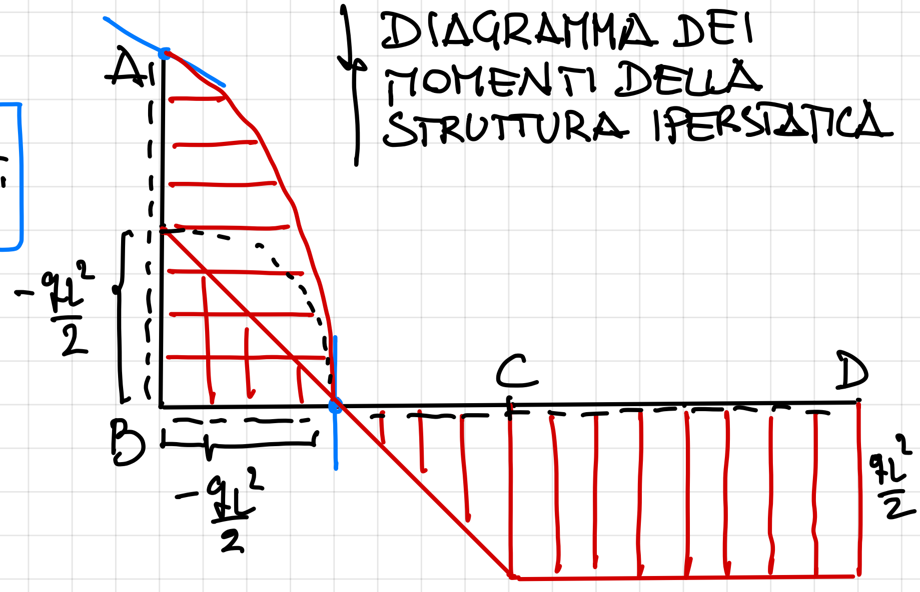
$M_C = \dfrac{qL^2}{2}$

TRATTO CD $L \le z \le 2L$



$$\boxed{M^{(r)}(z) = \frac{qL^2}{2} \text{ cost.}}$$

DIAGRAMMA DEI MOMENTI DELLA STRUTTURA IPERSTATICA

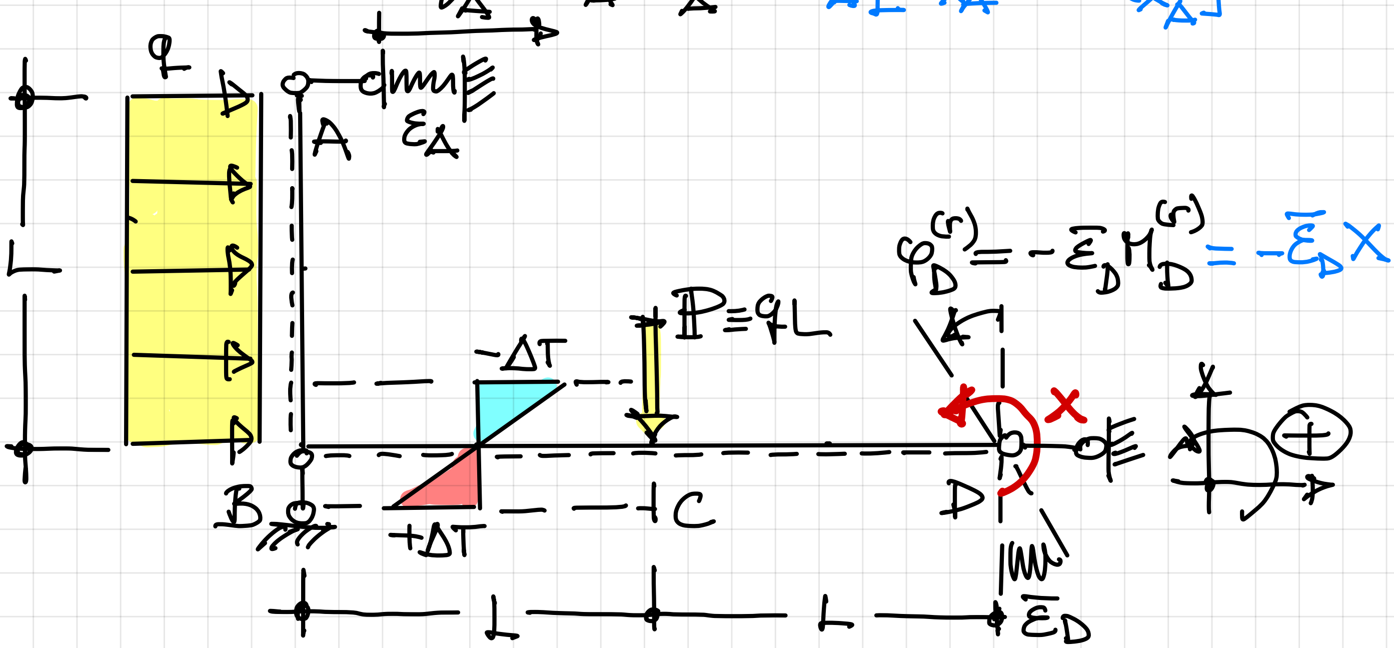


SOLUZIONE #2

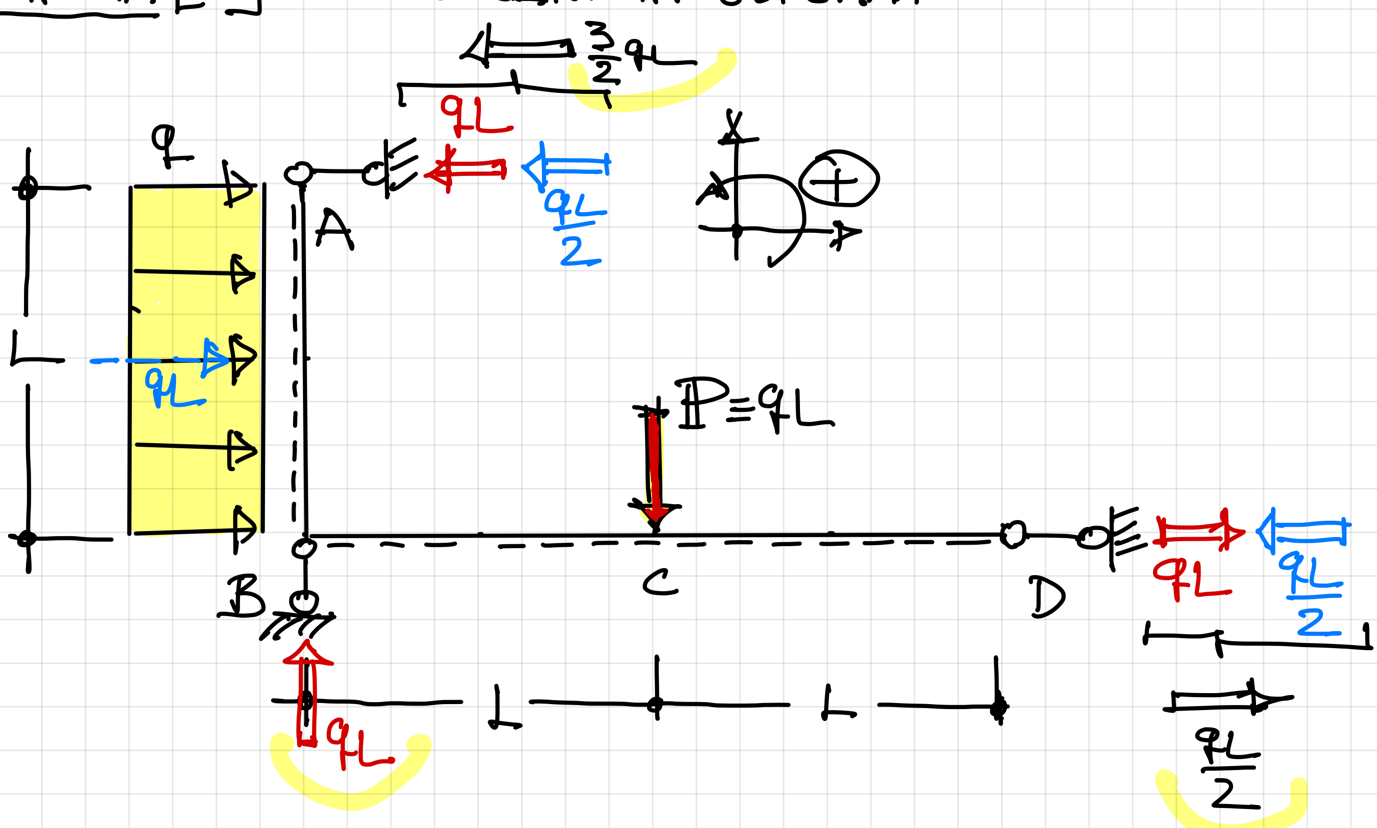
SISTEMA PRINCIPALE ISOSTATICO

$$\eta_A^{(r)} = -\varepsilon_A R_{x_A}^{(r)} = -\varepsilon_A \left[R_{x_A}^{(0)} + X R_{x_A}^{(1)} \right]$$

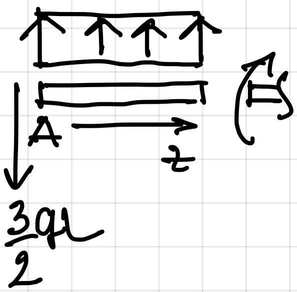
$$\varphi_D^{(r)} = -\bar{\varepsilon}_D M_D^{(r)} = -\bar{\varepsilon}_D X$$



SCHEMA [0] SOLO CARICHI ESTERNI



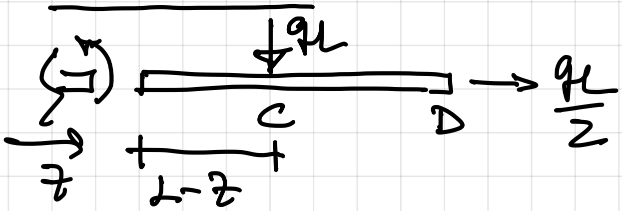
TRATTO AB $0 \le z \le L$



$$\boxed{M^{(0)}(z) = q\frac{z^2}{2} - \frac{3}{2}qL \cdot z}$$

$$\begin{cases} M_A = \phi \\ M_B = -qL^2 \end{cases}$$

TRATTO BC $0 \le z \le L$



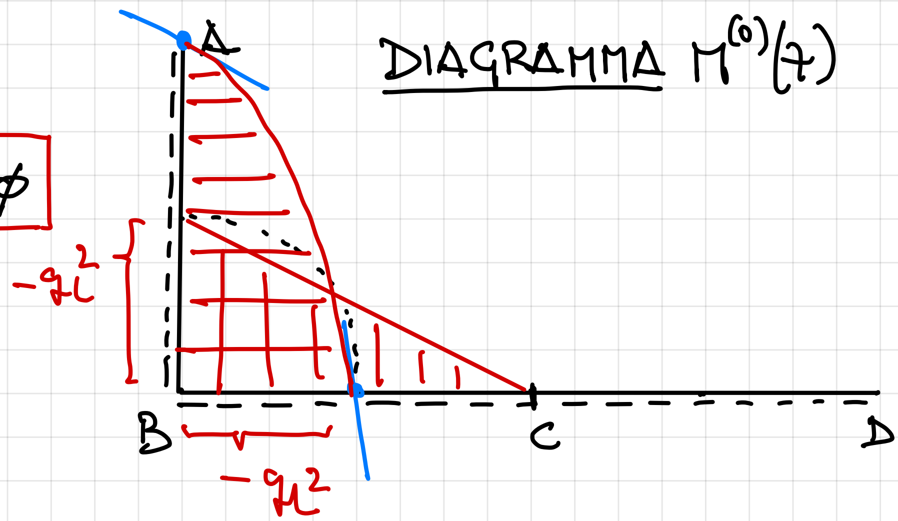
$$\boxed{M^{(0)}(z) = -qL(L-z)}$$

$$\begin{cases} M_B = -qL^2 \\ M_C = \phi \end{cases}$$

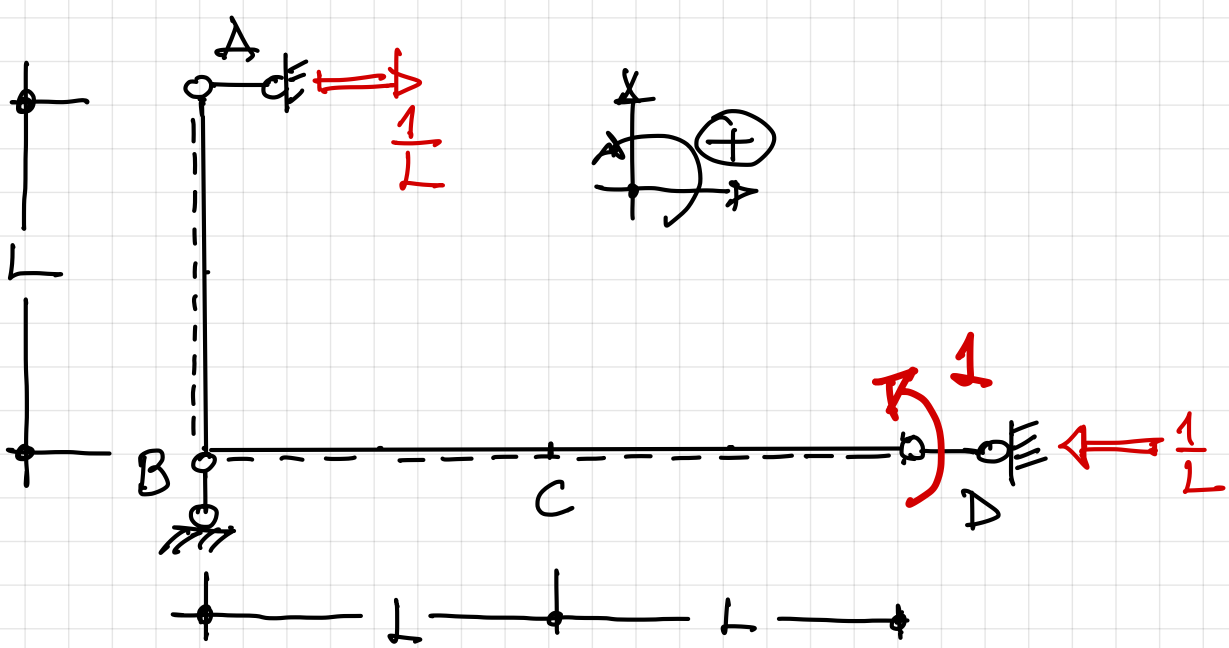
TRATTO CD

SCARICO $\boxed{M^{(0)}(z) = \phi}$

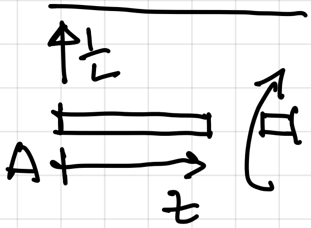
DIAGRAMMA $M^{(0)}(z)$



SCHEMA [1] SOLO $x = 1$

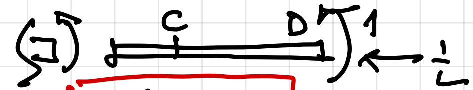


TRATTO AB $0 \le z \le L$



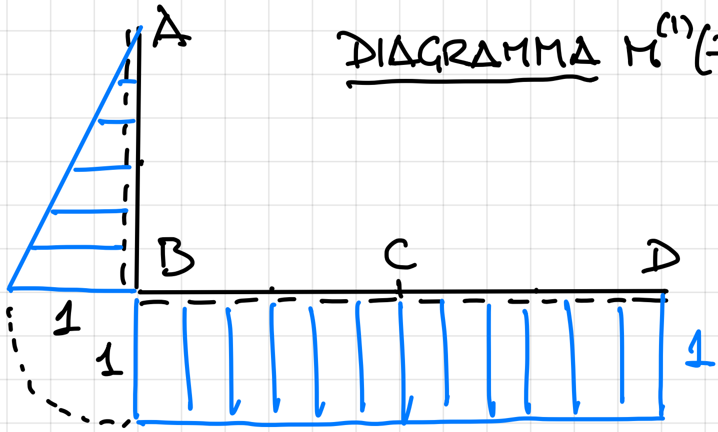
$$\boxed{M^{(1)}(z)=\frac{z}{L}} \qquad \begin{cases} M_A = \phi \\ M_B = 1 \end{cases}$$

TRATTO BD $0 \le z \le 2L$



$$\boxed{M^{(1)}(z)=1} \quad \text{cost.}$$

DIAGRAMMA $M^{(1)}(z)$



$$L_{ve} = \underset{1}{X_i}\,\eta_i^{(r)} + \sum_j R_j^{(1)}\eta_j^{(r)} = -\bar{\varepsilon}_D X + \overbrace{R^{(1)}_{x_A}}^{\frac{1}{L}}(-\varepsilon_A)\Big[\overbrace{R^{(0)}_{x_A}}^{-\frac{3}{2}qL} + X\overbrace{R^{(1)}_{x_A}}^{\frac{1}{L}}\Big] =$$

$$= -\bar{\varepsilon}_D X - \frac{\varepsilon_A}{L}\left[-\frac{3}{2}qL + \frac{X}{L}\right]$$

$$L_{vi} = \int_{str} M^{(1)}\frac{M^{(r)}}{EI}\,ds_{tr} + \int_{str} M^{(1)}\frac{\alpha\Delta T}{h}\,ds_{tr} =$$

$$= \frac{1}{EI}\int_{str} M^{(1)}M^{(0)}\,ds_{tr} + \frac{X}{EI}\int_{str}\left[M^{(1)}\right]^2 ds_{tr} + \frac{\alpha\Delta T}{h}\int_{str} M^{(1)}\,ds_{tr} =$$

$$= \frac{1}{EI}\left\{\int_0^L \frac{z}{L}\left[q\frac{z^2}{2} - \frac{3}{2}qL\cdot z\right]dz + \int_0^L 1\cdot\left[-qL(L-z)\right]dz\right\} +$$

$$+ \frac{X}{EI}\left\{\int_0^L \frac{z^2}{L^2}\,dz + \int_0^{2L} dz\right\} + \frac{\alpha\Delta T}{h}\int_0^L 1\cdot dz =$$

$$= \frac{1}{EI}\left\{\frac{q}{2L}\left[\frac{z^4}{4}\right]_0^L - \frac{3}{2}q\left[\frac{z^3}{3}\right]_0^L - qL^2\left[z\right]_0^L + qL\left[\frac{z^2}{2}\right]_0^L\right\} +$$

$$+ \frac{X}{EI}\left\{\frac{1}{L^2}\left[\frac{z^3}{3}\right]_0^L + \left[z\right]_0^{2L}\right\} + \frac{\alpha\overline{\Delta T}}{h}\cdot\left[z\right]_0^L =$$

$$= \frac{1}{EI}\left\{\frac{q}{2\cancel{L}}\cdot\frac{L^{\cancel{4}\,3}}{4} - \frac{\cancel{3}}{2}q\frac{L^3}{\cancel{3}} - qL^2\cdot L + \frac{qL}{2}\cdot L^2\right\} +$$

$$+ \frac{X}{EI}\left\{\frac{1}{3\cancel{L^2}}\cdot L^{\cancel{3}} + 2L\right\} + \frac{\alpha\overline{\Delta T}}{h}\cdot L =$$

$$= \frac{qL^3}{EI}\left\{\frac{1}{8} - \cancel{\frac{1}{2}} - 1 + \cancel{\frac{1}{2}}\right\} + \frac{XL}{EI}\left[\frac{1}{3} + 2\right] + \frac{\alpha\overline{\Delta T}}{h}\cdot L =$$

$$= \underbrace{-\frac{7}{8}\frac{qL^3}{EI} + \frac{7}{3}\frac{XL}{EI} + \frac{\alpha\overline{\Delta T}}{h}L}$$

$L_{ve} = L_{vi}$ fornisce:

$$-\bar{\varepsilon}_D X - \frac{\varepsilon_A}{L}\left[-\frac{3}{2}qL + \frac{X}{L}\right] = -\frac{7}{8}\frac{qL^3}{EI} + \frac{7}{3}\frac{XL}{EI} + \frac{\alpha\overline{\Delta T}}{h}L$$

$$X\left[\frac{7}{3}\frac{L}{EI} + \underbrace{\bar{\varepsilon}_D}_{\frac{2L}{EI}} + \frac{\overbrace{\varepsilon_A}^{\frac{5}{3}\frac{L^3}{EI}}}{L^2}\right] = +\frac{7}{8}\frac{qL^3}{EI} - \underbrace{\frac{\alpha\overline{\Delta T}}{h}}_{\frac{3}{8}\frac{qL^2}{EI}}L + \underbrace{\varepsilon_A}_{\frac{5}{3}\frac{L^3}{EI}}\frac{3}{2}q$$

$$X\frac{\cancel{L}}{\cancel{EI}}\underbrace{\left[\frac{7}{3} + 2 + \frac{5}{3}\right]}_{6} = \frac{qL^{\cancel{3}\,2}}{\cancel{EI}}\underbrace{\left\{\frac{7}{8} - \frac{3}{8} + \frac{5}{2}\right\}}_{3}$$

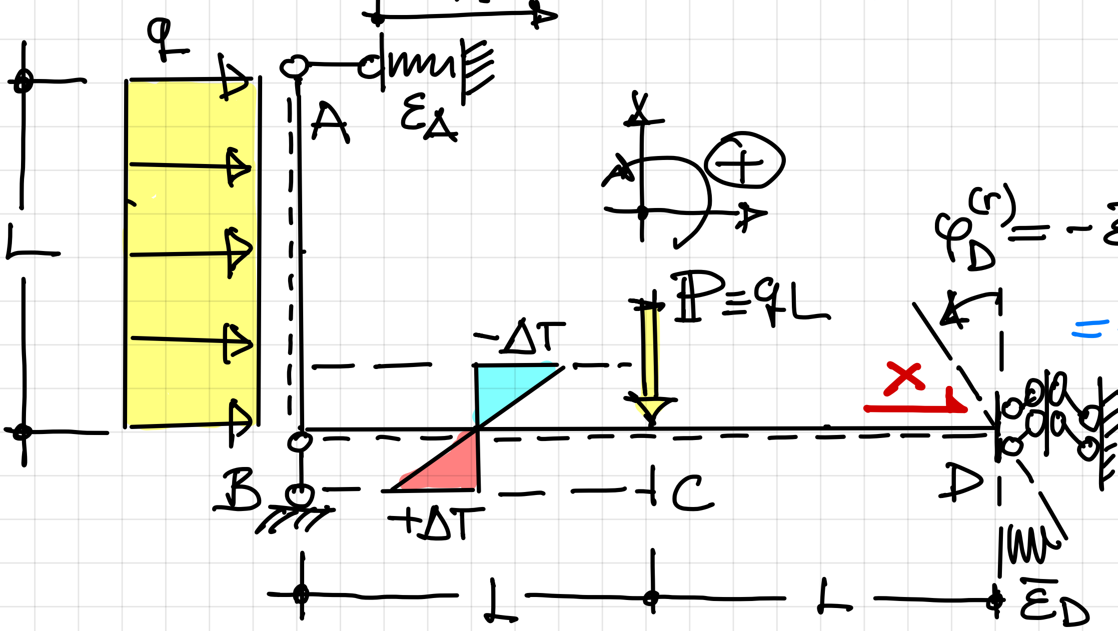
$6X = 3qL^2$ da cui $\boxed{X = \frac{qL^2}{2}}$

Ok!
cfr. RV
di pag. VI

SOLUZIONE #3

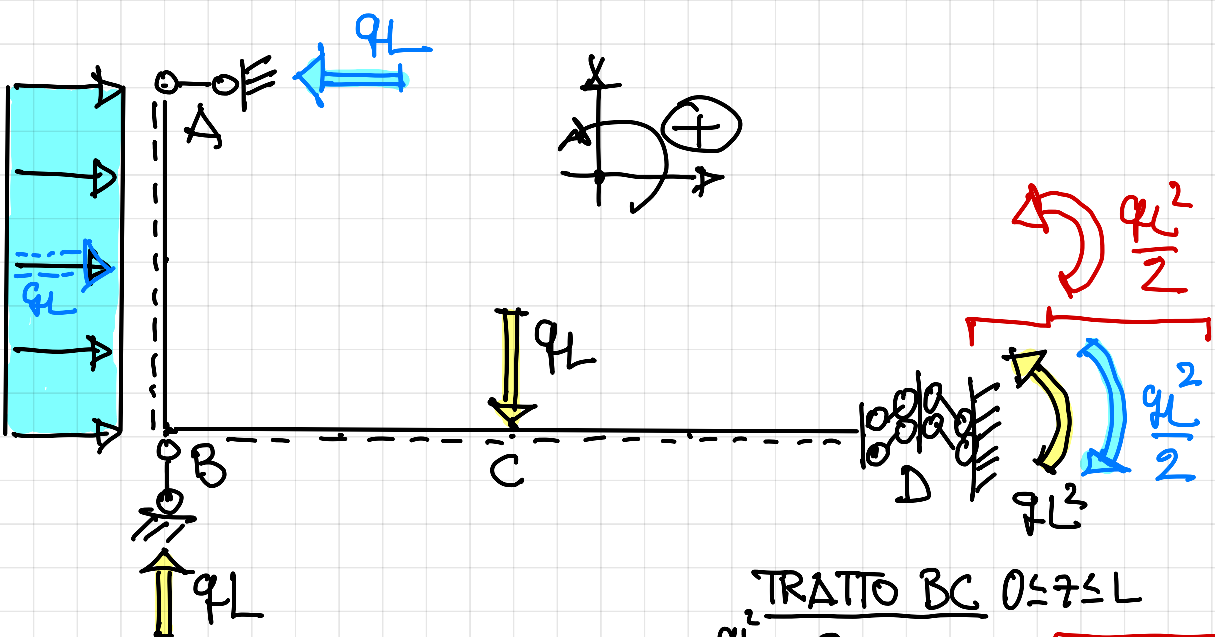
SISTEMA PRINCIPALE ISOSTATICO

$$\eta_A^{(r)} = -\varepsilon_A R_{x_A}^{(r)} = -\varepsilon_A \left[R_{x_A}^{(0)} + X R_{x_A}^{(1)} \right]$$

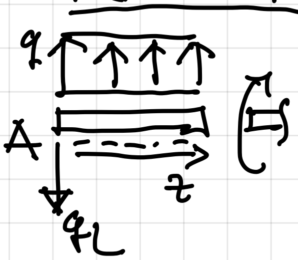


$$\varphi_D^{(r)} = -\bar{\varepsilon}_D M_D^{(r)} = -\bar{\varepsilon}_D \left[M_D^{(0)} + X M_D^{(1)} \right]$$

SCHEMA [0] SOLO CARICHI ESTERNI



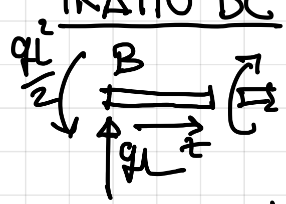
TRATTO AB $0 \le z \le L$



$$\boxed{M^{(0)}(z) = q\frac{z^2}{2} - qL\cdot z}$$

$$\begin{cases} M_A = 0 \\ M_B = -\frac{qL^2}{2} \end{cases}$$

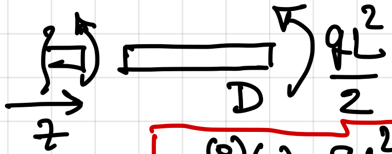
TRATTO BC $0 \le z \le L$



$$\boxed{M^{(0)}(z) = qL\cdot z - \frac{qL^2}{2}}$$

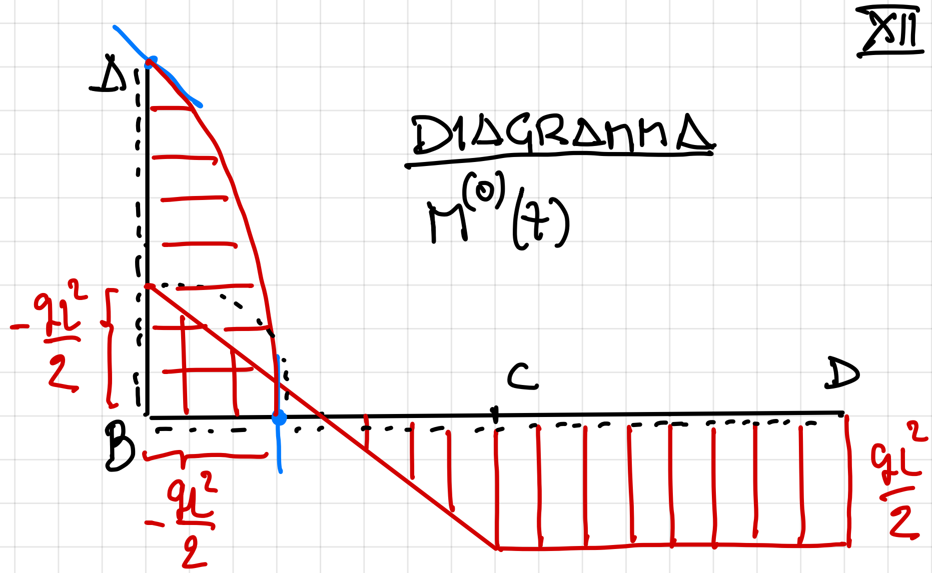
$$M_B = -\frac{qL^2}{2}; \quad M_C = \frac{qL^2}{2}$$

TRATTO CD $L \le z \le 2L$

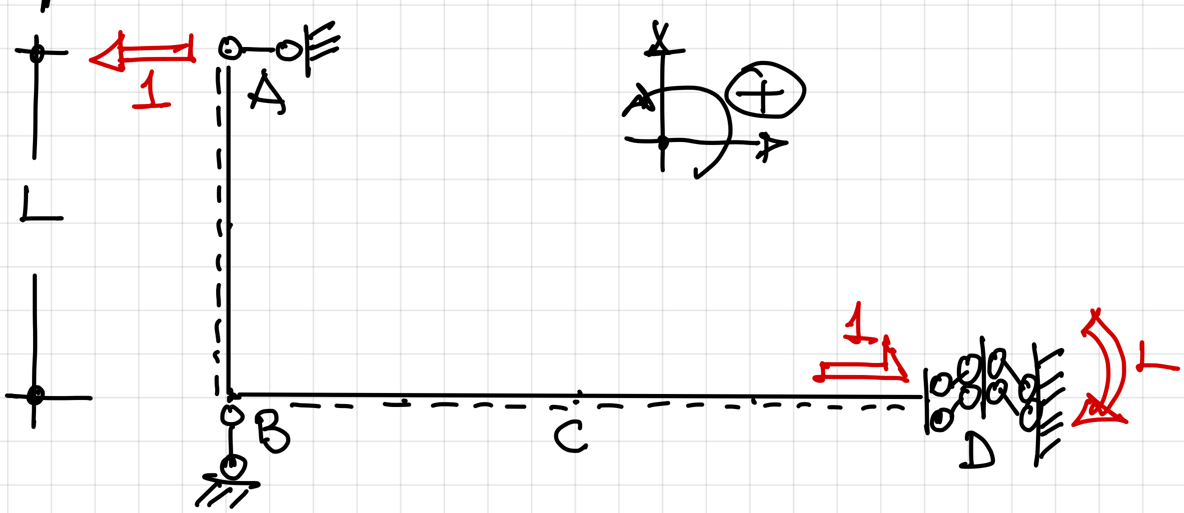


$$\boxed{M^{(0)}(z) = \frac{qL^2}{2}}\ \text{cost.}$$

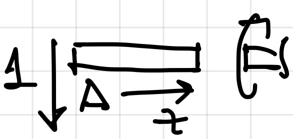
DIAGRAMMA $M^{(0)}(z)$



SCHEMA [1] Solo $X = 1$



TRATTO AB $0 \le z \le L$

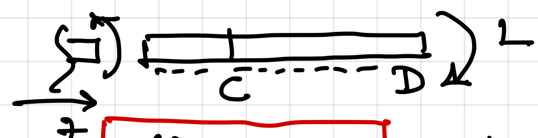


$$\boxed{M^{(1)}(z) = -z}$$

$M_A = \emptyset$

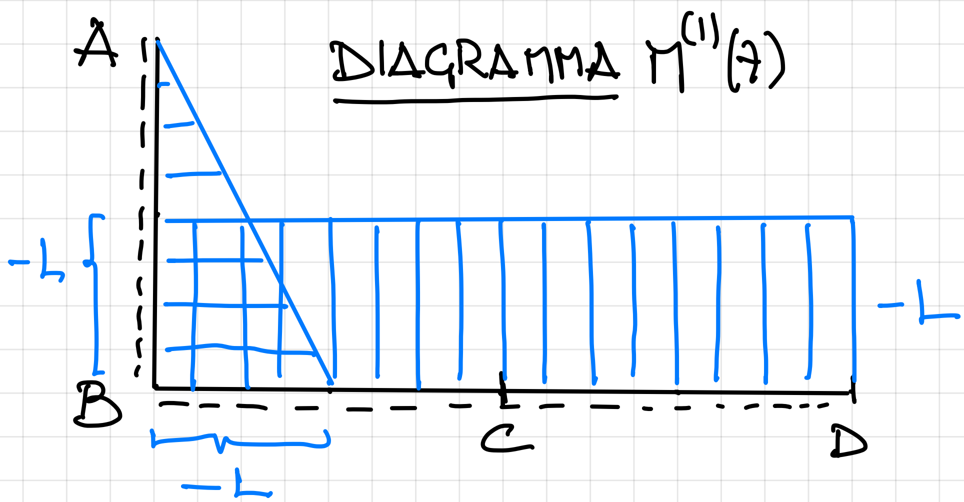
$M_B = -L$

TRATTO BD $0 \le z \le 2L$



$$\boxed{M^{(1)}(z) = -L}\ \text{cost.}$$

DIAGRAMMA $M^{(1)}(z)$



$$L_{ve} = \underbrace{X_i}_{=1} \underbrace{\eta_i^{(r)}}_{=\phi} + \sum_j R_j^{(1)} \eta_j^{(r)} =$$

$$= \underbrace{R_{x_A}^{(1)}}_{-1} (-\varepsilon_A)\left[\underbrace{R_{x_A}^{(0)}}_{-qL} + X \underbrace{R_{x_A}^{(1)}}_{-1}\right] + \underbrace{M_D^{(1)}}_{-L} (-\bar{\varepsilon}_D)\left[\underbrace{M_D^{(0)}}_{\frac{qL^2}{2}} + X \underbrace{M_D^{(1)}}_{-L}\right] =$$

$$= -\varepsilon_A \left[qL + X\right] + \bar{\varepsilon}_D L \left[\frac{qL^2}{2} - XL\right]$$

$$L_{vi} = \int_{str} M^{(1)} \frac{M^{(r)}}{EI} \, ds_{tr} + \int_{str} M^{(1)} \frac{\alpha \Delta T}{h} \, ds_{tr} =$$

$$= \frac{1}{EI} \int_{str} M^{(1)} M^{(0)} \, ds_{tr} + \frac{X}{EI} \int_{str} \left[M^{(1)}\right]^2 ds_{tr} + \frac{\alpha \Delta T}{h} \int_{str} M^{(1)} \, ds_{tr} =$$

$$= \frac{1}{EI} \left\{ \int_0^L (-z)\left[\frac{qz^2}{2} - qL \cdot z\right] dz + \int_0^L -L\left[qL \cdot z - \frac{qL^2}{2}\right] dz + \int_L^{2L} -L\left[\frac{qL^2}{2}\right] dz \right\} +$$

$$+ \frac{X}{EI} \left\{ \int_0^L z^2 \, dz + \int_0^{2L} L^2 \, dz \right\} + \frac{\alpha \Delta T}{h} \int_0^L -L \, dz =$$

$$= \frac{1}{EI} \left[-\frac{q}{2} \left[\frac{z^4}{4}\right]_0^L + qL \left[\frac{z^3}{3}\right]_0^L - qL^2 \left[\frac{z^2}{2}\right]_0^L + \frac{qL^3}{2} \left[z\right]_0^L - \frac{qL^3}{2} \left[z\right]_L^{2L} \right] +$$

$$+ \frac{X}{EI} \left\{ \left[\frac{z^3}{3}\right]_0^L + L^2 \left[z\right]_0^{2L} \right\} - \frac{\alpha \Delta T}{h} L \cdot \left[z\right]_0^L =$$

$$= \frac{1}{EI} \left[-\frac{q}{8} \cdot L^4 + \frac{qL}{3} \cdot L^3 - \frac{qL^2}{2} \cdot L^2 + \frac{qL^3}{2} \cdot L - \frac{qL^3}{2} \cdot L \right] +$$

$$+ \frac{X}{EI} \left\{ \frac{L^3}{3} + L^2 \cdot 2L \right\} - \frac{\alpha \Delta T}{h} \cdot L \cdot L =$$

$$= \frac{qL^4}{EI}\left[-\frac{1}{8}+\frac{1}{3}-\frac{1}{2}+\cancel{\frac{1}{2}}-\cancel{\frac{1}{2}}\right]+\frac{XL^3}{EI}\left[\frac{1}{3}+2\right]-\alpha\frac{\Delta T}{h}\cdot L^2 =$$

$$\underbrace{= -\frac{qL^4}{EI}\frac{7}{24}+\frac{7}{3}X\frac{L^3}{EI}-\alpha\frac{\Delta T}{h}L^2}$$

$\Rightarrow$ $L_{ve}=L_{vi}$ fornisce

$$-\varepsilon_A\left[qL+X\right]+\bar{\varepsilon}_D L\left[\frac{qL^2}{2}-XL\right]=$$

$$= -\frac{qL^4}{EI}\frac{7}{24}+\frac{7}{3}X\frac{L^3}{EI}-\alpha\frac{\Delta T}{h}L^2$$

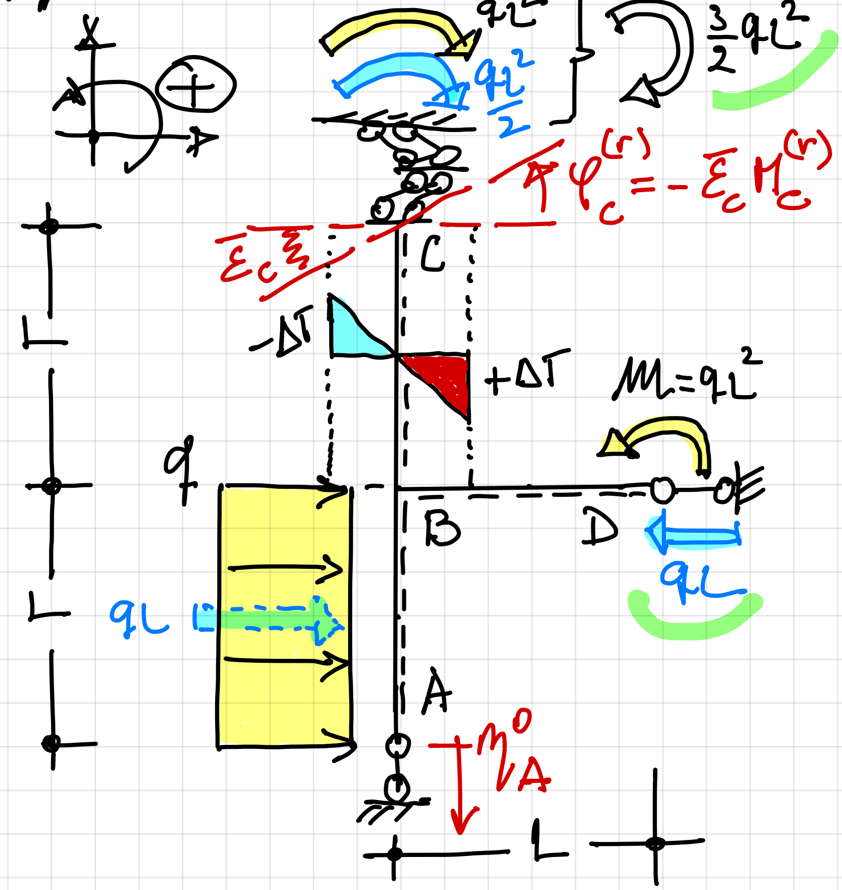
$$X\left[\frac{7}{3}\frac{L^3}{EI}+\underbrace{\varepsilon_A}_{\frac{5}{3}\frac{L^3}{EI}}+\underbrace{\bar{\varepsilon}_D}_{\frac{2L}{EI}}L^2\right]=\frac{qL^4}{EI}\cdot\frac{7}{24}+\underbrace{\alpha\frac{\Delta T}{h}}_{\frac{3}{8}\frac{qL^2}{EI}}\cdot L^2-\underbrace{\varepsilon_A}_{\frac{5}{3}\frac{L^3}{EI}}qL+\underbrace{\bar{\varepsilon}_D}_{\frac{2L}{EI}}\frac{qL^3}{2}$$

$$X\frac{L^3}{EI}\left[\frac{7}{3}+\frac{5}{3}+2\right]=\frac{qL^4}{EI}\underbrace{\left[\frac{7}{24}+\frac{3}{8}-\frac{5}{3}+1\right]}_{=\phi}$$

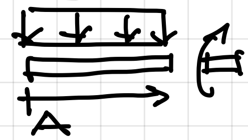
da cui $\boxed{X=\phi}$ ok! cfr. RV di p. VI

ES.#2 – TIPO 1 — SOLUZIONE

STRUTTURA REALE



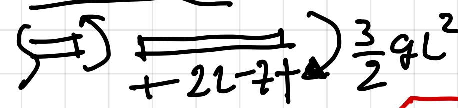
TRATTO AB $0 \le z \le L$



$$\boxed{M^{(r)}(z) = -\frac{qz^2}{2}}$$

$$\begin{cases} M_A = \phi \\ M_B = -\frac{qL^2}{2} \end{cases}$$

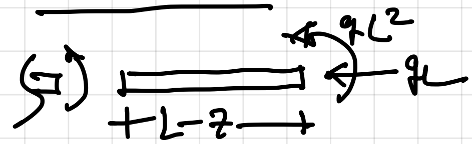
TRATTO BC $L \le z \le 2L$



$$\boxed{M^{(r)}(z) = -\frac{3}{2}qL^2}$$

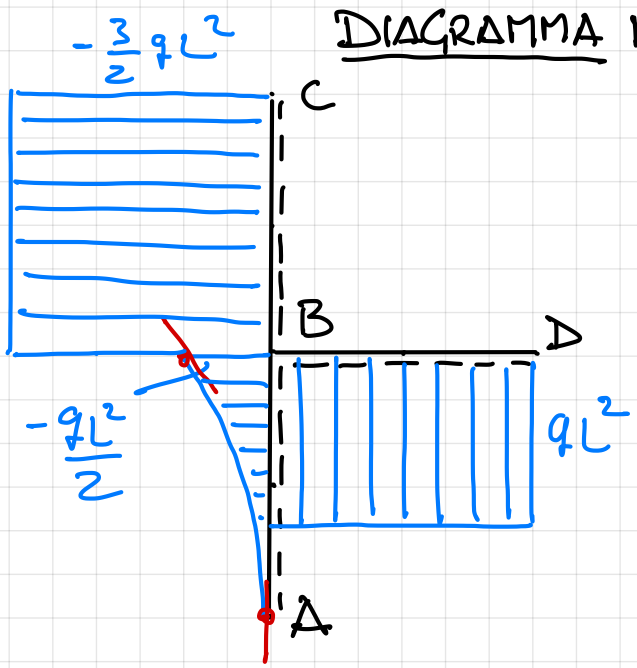
cost.

TRATTO BD $0 \le z \le L$

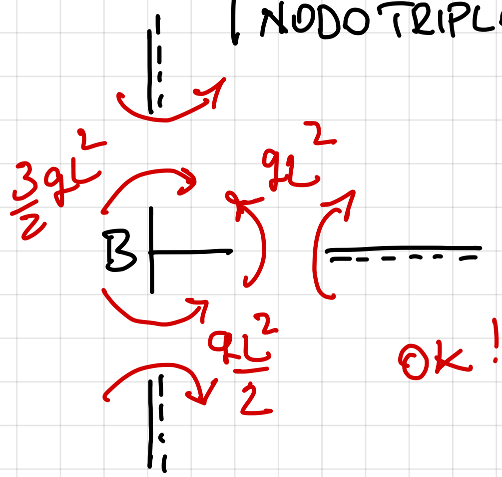


$$\boxed{M^{(r)}(z) = qL^2}$$ cost.

DIAGRAMMA $M^{(r)}(z)$

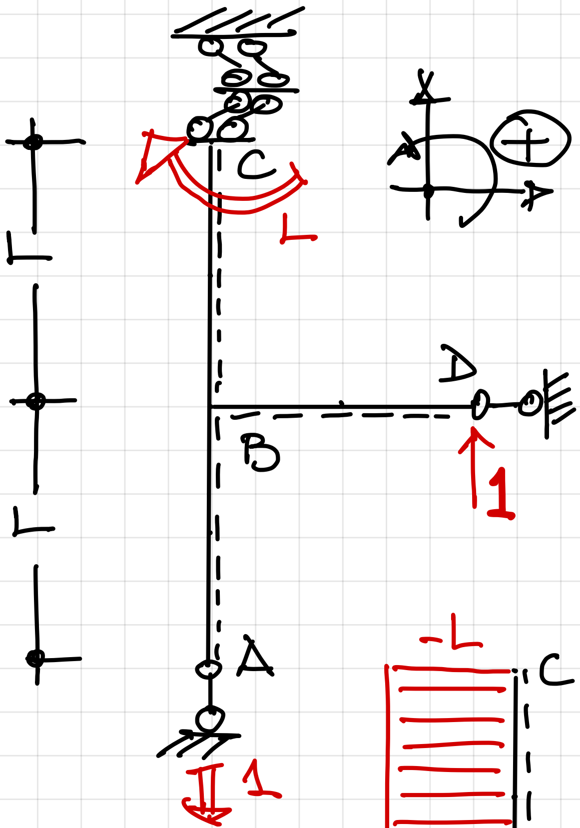


VERIFICA AL NODO TRIPLO B



OK!

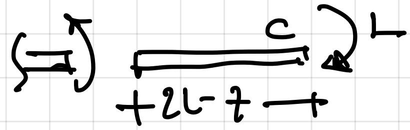
STRUTTURA FITTIZIA PER IL CALCOLO DELLO SPOSTAMENTO VERTICALE DELLA SEZIONE D



TRATTO AB $0 \le z \le L$

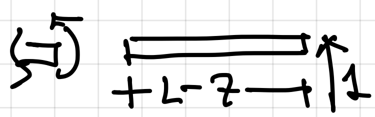
SCARICO $\boxed{M^{(f)}(z) = \phi}$

TRATTO BC $L \le z \le 2L$



$\boxed{M^{(f)}(z) = -L}$ cost

TRATTO BD $0 \le z \le L$



$\boxed{M^{(f)}(z) = L - z}$ $\begin{cases} M_B = L \\ M_D = \phi \end{cases}$

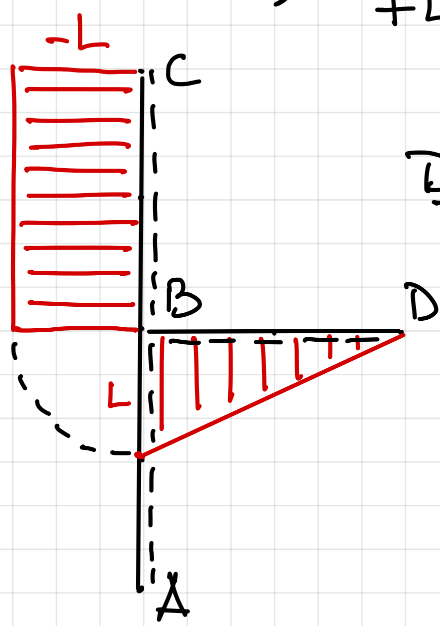


DIAGRAMMA $M^{(f)}(z)$

$$\underbrace{L_{ve}} = 1 \cdot \eta_D + \sum_j R_j^{(f)} \eta_j^{(r)} =$$

$$= 1 \cdot \eta_D + \underbrace{R_{y_A}^{(f)}}_{-1}\left(-\eta_A^0\right) + \underbrace{M_c^{(f)}}_{-L} \overbrace{\varphi_c^{(r)}}^{-\bar{E}_c \overbrace{M_c^{(r)}}^{-\frac{3}{2}qL^2}} =$$

$$= \underbrace{\eta_D + \eta_A^0 - \bar{E}_c \frac{3}{2} qL^3}$$

$$L_{vi} = \int_{str} M^{(f)} \frac{M^{(r)}}{EI}\, ds_{tr} + \int_{str} M^{(f)} \frac{\alpha \overline{\Delta T}}{h}\, ds_{tr} =$$

$$= \frac{1}{EI}\left\{\int_L^{2L}(-L)\left(-\frac{3}{2}qL^2\right)dz + \int_0^L (L-z)\, qL^2\, dz\right\} + \frac{\alpha\overline{\Delta T}}{h}\int_L^{2L}(-L)\,dz =$$

$$= \frac{1}{EI}\left\{\frac{3}{2}qL^3\left[z\right]_L^{2L} + qL^3\left[z\right]_0^L - qL^2\left[\frac{z^2}{2}\right]_0^L\right\} - \frac{\alpha\overline{\Delta T}}{h}L\left[z\right]_L^{2L} =$$

$$= \frac{1}{EI}\left\{\frac{3}{2}qL^4 + qL^4 - \frac{qL^4}{2}\right\} - \frac{\alpha\overline{\Delta T}}{h}L^2 =$$

$$= 2\frac{qL^4}{EI} - \frac{\alpha\overline{\Delta T}}{h}L^2$$

$L_{ve} = L_{vi}$ fornisce

$$\eta_D + \underbrace{\eta_A^0}_{\frac{qL^4}{2EI}} - \underbrace{\bar{\varepsilon}_c}_{\frac{L}{EI}}\,\frac{3}{2}qL^3 = \cancel{2\frac{qL^4}{EI}} - \underbrace{\cancel{\frac{\alpha\overline{\Delta T}}{h}}}_{\frac{2qL^2}{EI}}L^2$$

da cui

$$\eta_D = \frac{3}{2}\frac{qL^4}{EI} - \frac{qL^4}{2EI} = \frac{qL^4}{EI} \uparrow$$

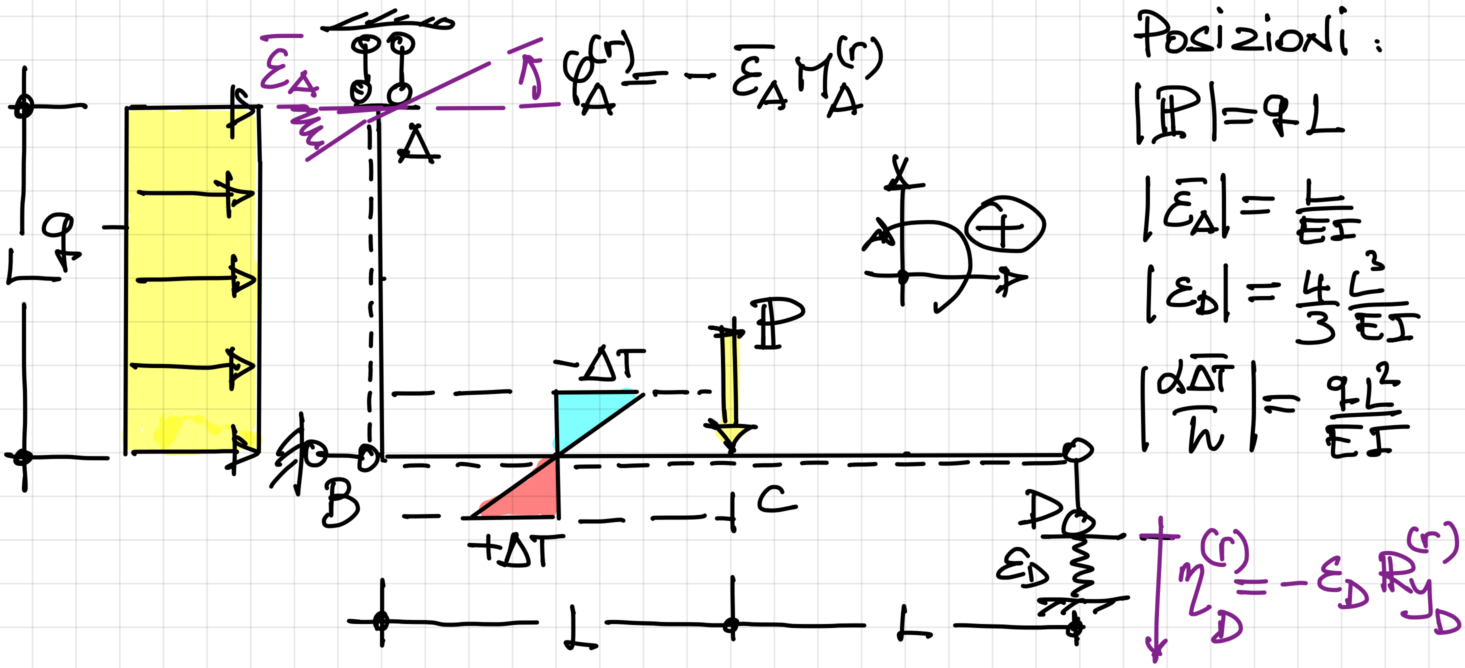
POSITIVO VERSO L'ALTO

MECCANICA delle STRUTTURE – P.FUSCHI
PROVA SCRITTA del 29 GENNAIO 2025

TIPO 2

ES. #1

RISOLVERE LA STRUTTURA UNA VOLTA IPERSTATICA SEGUENTE DETERMINANDO IL DIAGRAMMA DEI MOMENTI



Posizioni:

$|\mathbb{P}| = qL$

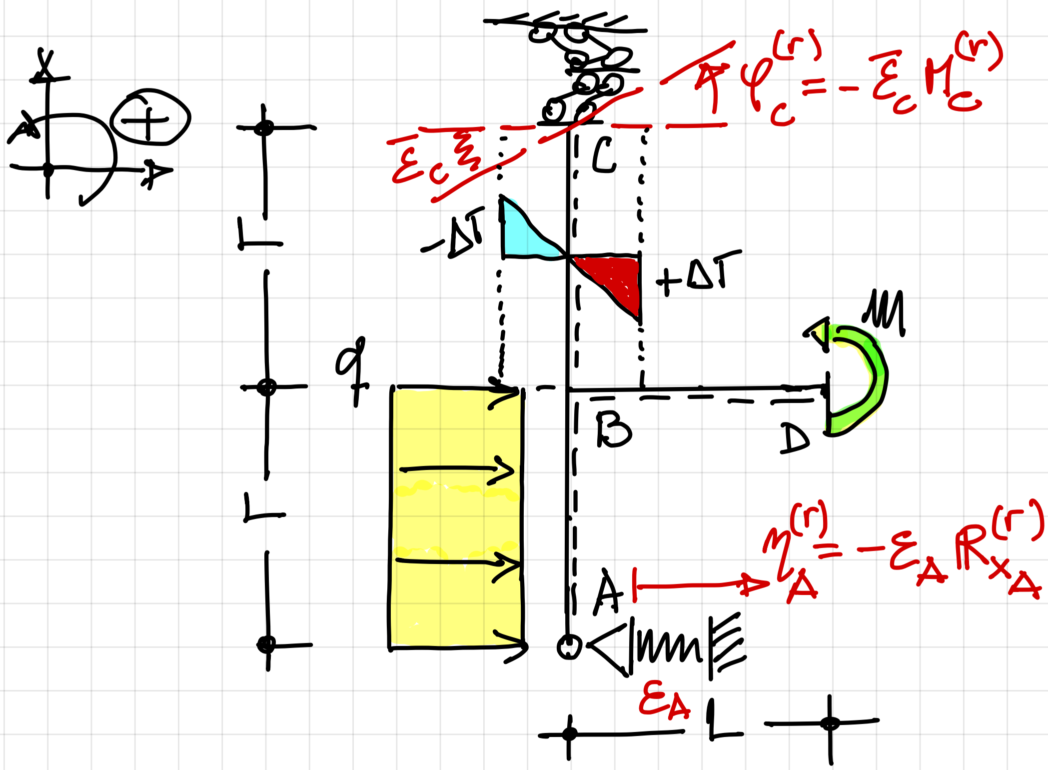
$|\bar{\varepsilon}_A| = \frac{L}{EI}$

$|\varepsilon_D| = \frac{4}{3}\frac{L^3}{EI}$

$\left|\frac{\alpha \Delta T}{h}\right| = \frac{qL^2}{EI}$

ES. #2

CALCOLARE LO SPOST. ORIZZONTALE DELLA SEZ. D DELLA STRUTTURA SEGUENTE CON IL METODO DELLA FORZA UNITARIA



Posizioni:

$|\mathcal{M}| = qL^2$

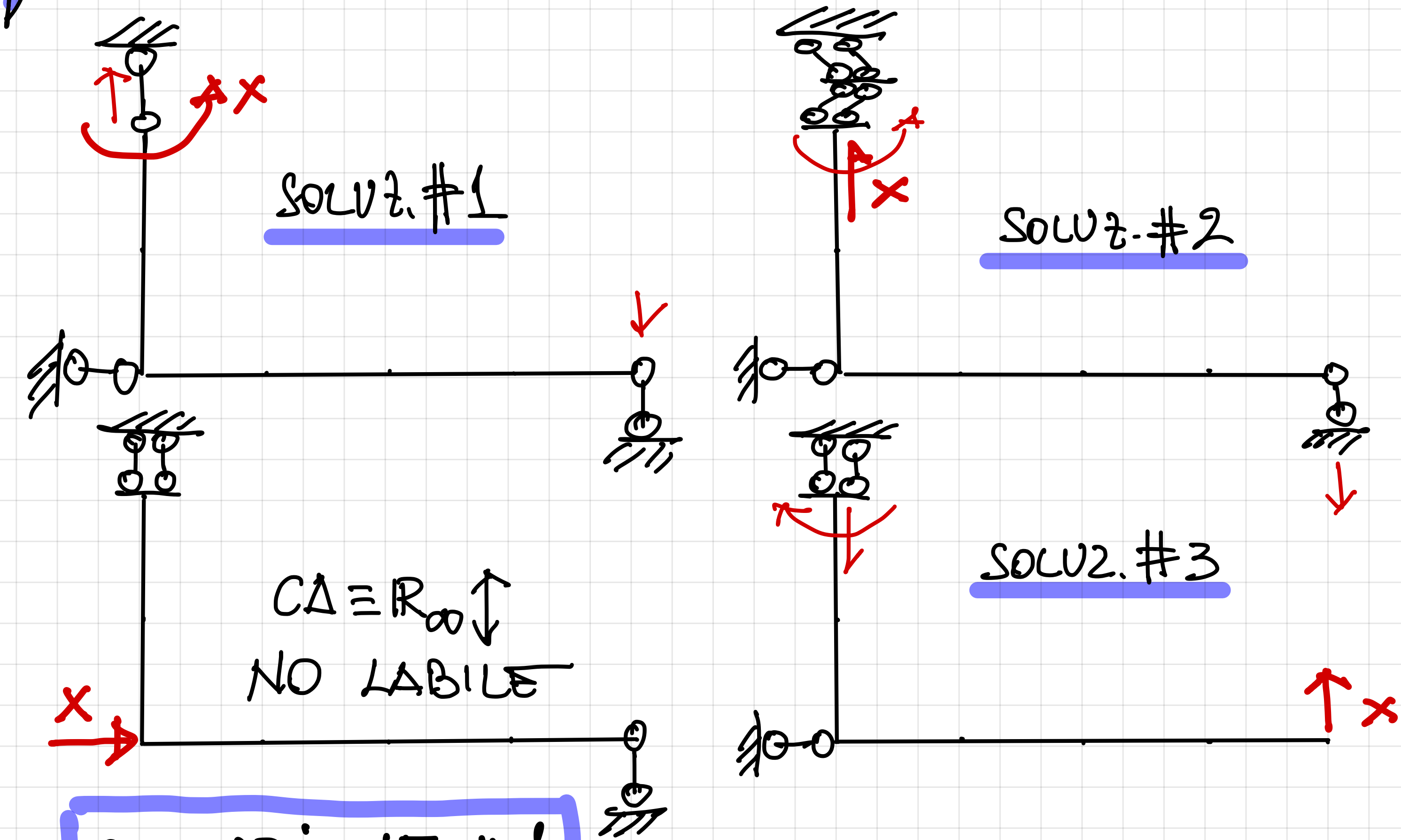
$|\varepsilon_A| = \frac{2}{3}\frac{L^3}{EI}$

$|\bar{\varepsilon}_C| = \frac{L}{EI}$

$\left|\frac{\alpha \Delta T}{h}\right| = \frac{13}{8}\frac{qL^2}{EI}$

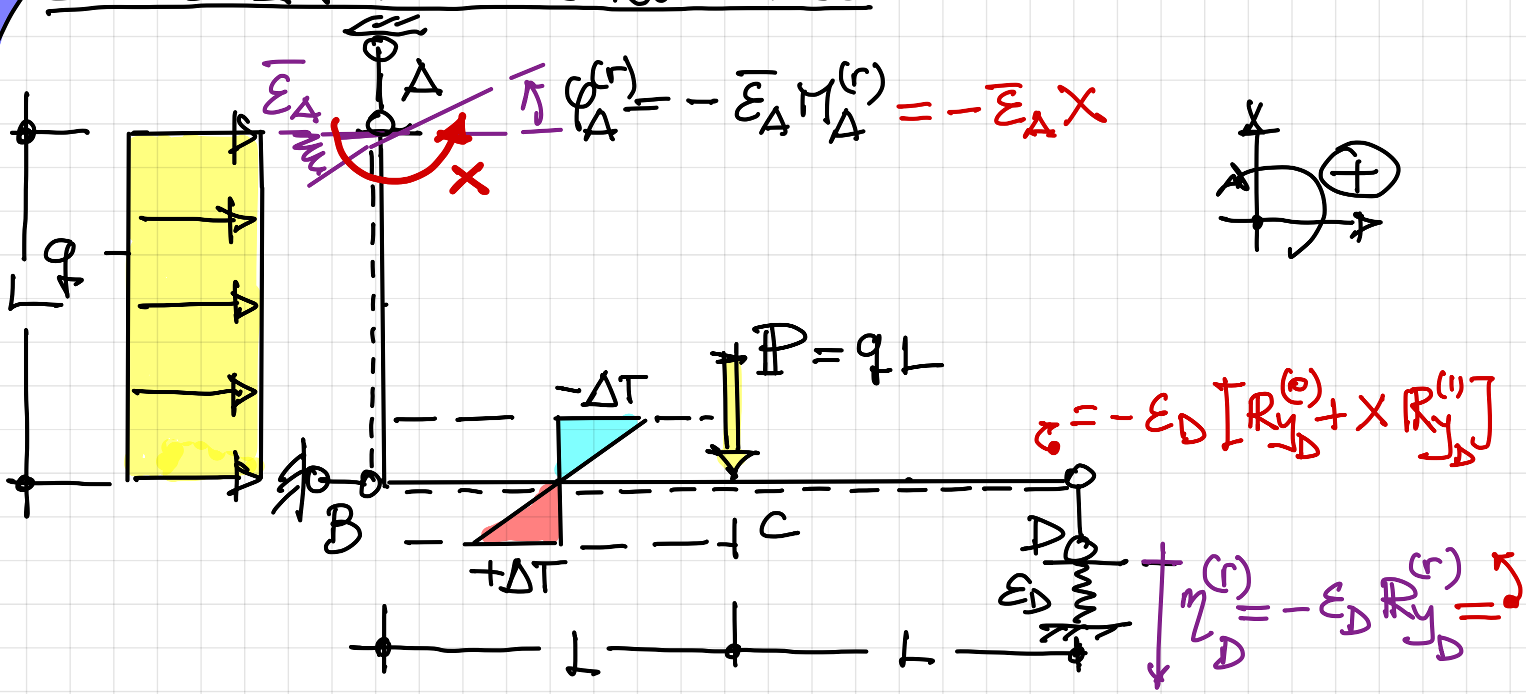
ES. #1 – TIPO 2 SOLUZIONI

SOLUZIONI POSSIBILI

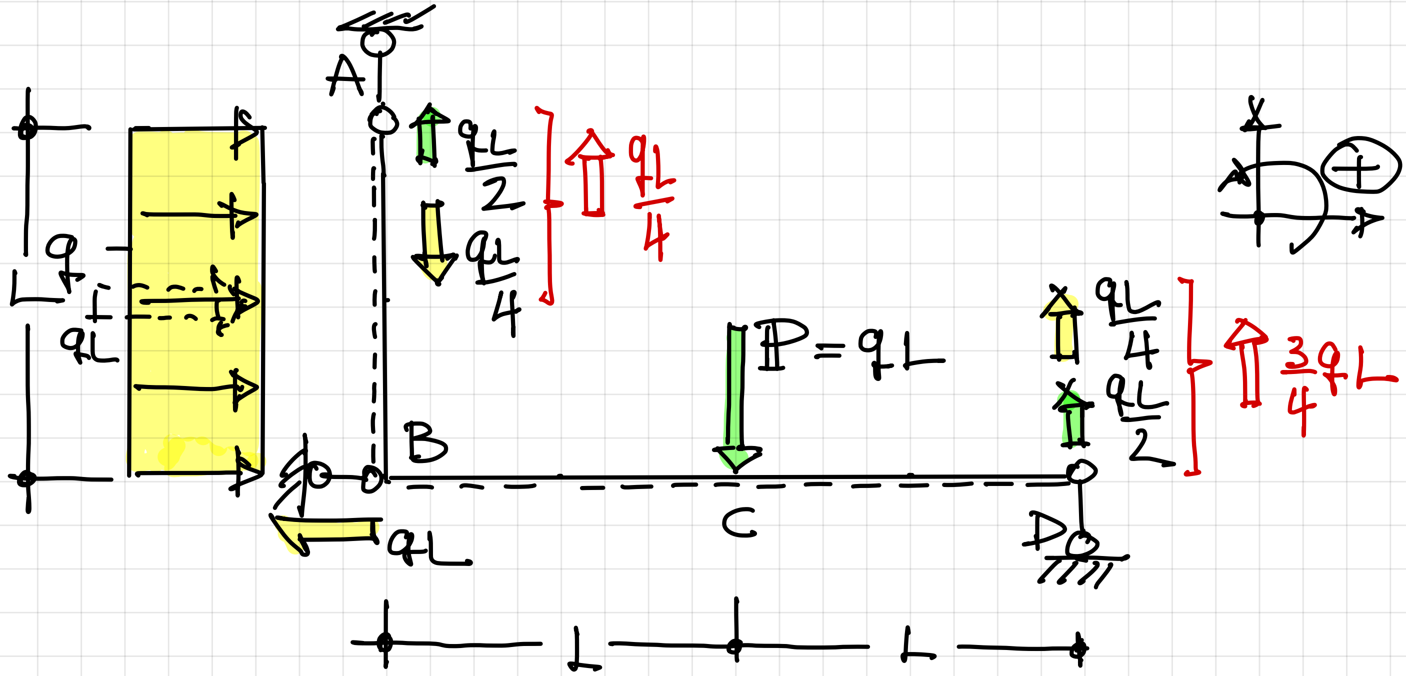


SOLUZIONE #1

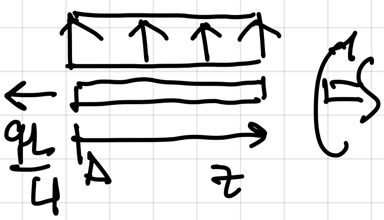
SISTEMA PRINCIPALE ISOSTATICO



SCHEMA [0] - SOLO CARICHI ESTERNI

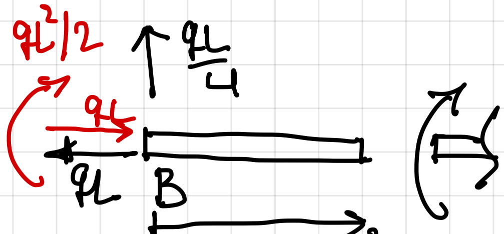


TRATTO AB $0 \le z \le L$



$$\boxed{M^{(0)}(z) = \frac{q z^2}{2}} \qquad \begin{cases} M_A = \emptyset \\ M_B = \frac{qL^2}{2} \end{cases}$$

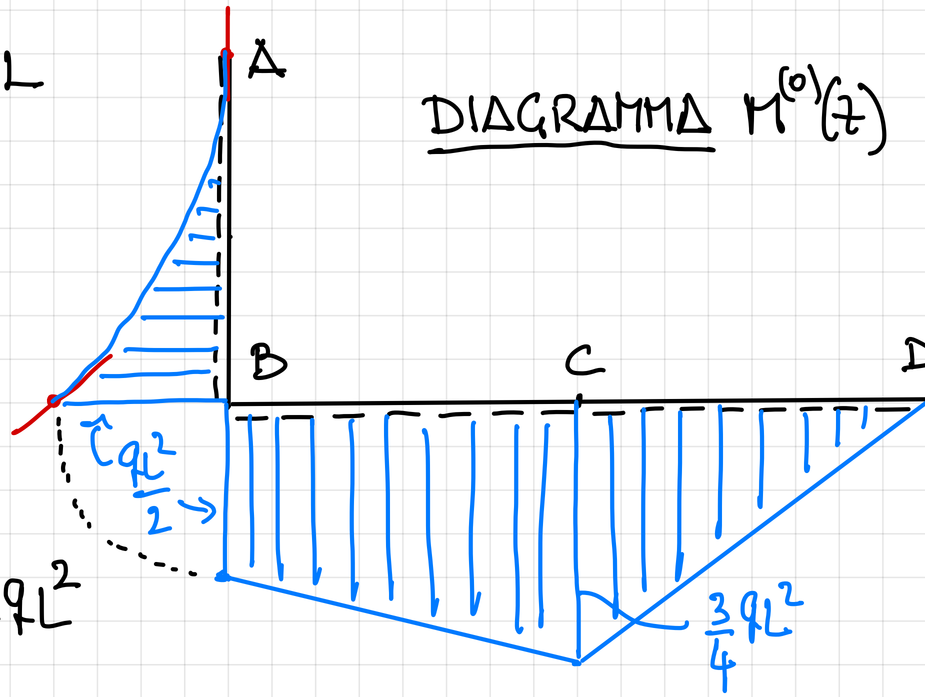
TRATTO BC $0 \le z \le L$



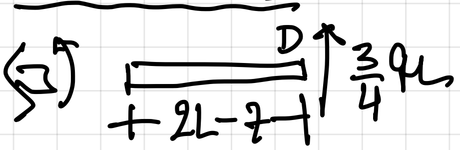
$$\boxed{M^{(0)}(z) = \frac{qL^2}{2} + \frac{qL}{4} \cdot z}$$

$$M_B = \frac{qL^2}{2} \; ; \; M_C = \frac{3}{4} qL^2$$

DIAGRAMMA $M^{(0)}(z)$

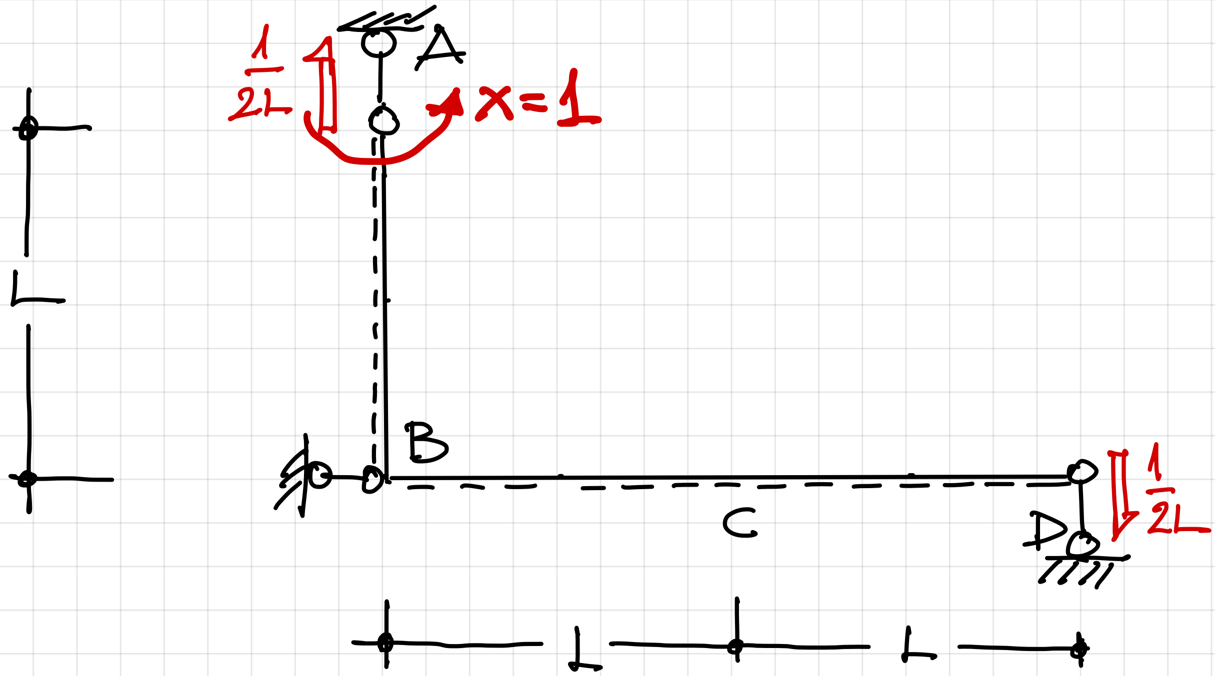


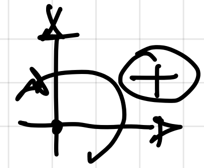
TRATTO CD $L \le z \le 2L$



$$\boxed{M^{0}(z) = \frac{3}{4} qL (2L - z)} \qquad \begin{cases} M_C = \frac{3}{4} qL^2 \\ M_D = \emptyset \end{cases}$$

SCHEMA [1] SOLO $X=1$

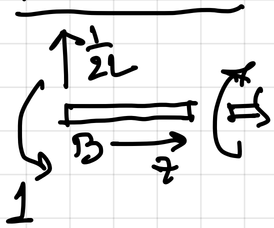




TRATTO AB $0 \le z \le L$

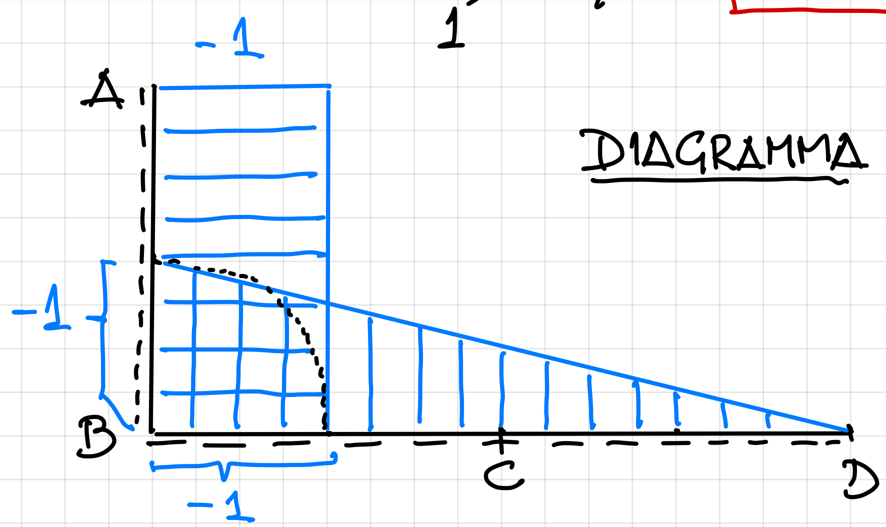
$$\boxed{M^{(1)}(z) = -1} \text{ cost.}$$

TRATTO BD $0 \le z \le 2L$



$$\boxed{M^{(1)}(z) = \frac{z}{2L} - 1} \quad \begin{cases} M_B = -1 \\ M_D = 0 \end{cases}$$

DIAGRAMMA $M^{(1)}(z)$



$$\underbrace{L_{ve}} = \underset{=1}{X_i}\, \eta_i^{(r)} + \sum_j R_j^{(1)} \eta_j^{(r)} =$$

$$= -\varepsilon_A X + \underbrace{R_{y_D}^{(1)}}_{-\frac{1}{2L}} (-\varepsilon_D)\Big[\underbrace{R_{y_D}^{(0)}}_{\frac{3}{4}qL} + X \underbrace{R_{y_D}^{(1)}}_{-\frac{1}{2L}}\Big] =$$

$$= \underbrace{-\varepsilon_A X + \frac{\varepsilon_D}{2L}\left[\frac{3}{4}qL - \frac{X}{2L}\right]}$$

$$\underbrace{L_{vi}} = \int_{str} M^{(1)} \frac{M^{(r)}}{EI}\, dstr + \int_{str} M^{(1)} \frac{\alpha \Delta T}{h}\, dstr =$$

$$= \frac{1}{EI}\int_{str} M^{(1)} M^{(0)}\, dstr + \frac{X}{EI}\int_{str} \left[M^{(1)}\right]^2 dstr + \frac{\alpha \Delta T}{h}\int_{str} M^{(1)}\, dstr =$$

$$= \frac{1}{EI}\left\{\left[\int_0^L (-1)\frac{qz^2}{2}\, dstr + \int_0^L \underbrace{\left(\frac{z}{2L}-1\right)\left[\frac{qL^2}{2} + \frac{qL}{4}\cdot z\right]}_{\frac{qL}{4}\cdot z - \frac{qL^2}{2} + \frac{q}{8}z^2 - \frac{qL}{4}z} dstr \, + \right.\right.$$

$$\underbrace{\left(\frac{z}{2L}-1\right)\left(\frac{3}{2}qL^2 - \frac{3}{4}qLz\right)}_{\frac{3}{4}qL\cdot z - \frac{3}{2}qL^2 - \frac{3}{8}qz^2 + \frac{3}{4}qL\cdot z}$$

$$\left.\left. + \int_L^{2L} \left(\frac{z}{2L}-1\right)\left[\frac{3}{4}qL(2L-z)\right] dstr \right]\right\} +$$

$$+ \frac{X}{EI}\left[\int_0^L 1\, dz + \int_0^{2L} \underbrace{\left(\frac{z}{2L}-1\right)^2}_{\frac{z^2}{4L^2} + 1 - \frac{z}{L}} dz\right] + \frac{\alpha \Delta T}{h}\int_0^L \left(\frac{z}{2L}-1\right) dstr =$$

$$= \frac{1}{EI}\left[-\frac{q}{2}\left[\frac{z^3}{3}\right]_0^L + \frac{qL}{4}\left[\frac{z^2}{2}\right]_0^L - \frac{qL^2}{2}\left[z\right]_0^L + \frac{q}{8}\left[\frac{z^3}{3}\right]_0^L - \frac{qL}{4}\left[\frac{z^2}{2}\right]_0^L + \right.$$

$$\left. + \frac{3}{4}qL\left[\frac{z^2}{2}\right]_L^{2L} - \frac{3}{2}qL^2\left[z\right]_L^{2L} - \frac{3}{8}q\left[\frac{z^3}{3}\right]_L^{2L} + \frac{3}{4}qL\left[\frac{z^2}{2}\right]_L^{2L}\right] +$$

$$+ \frac{X}{EI}\left[\left[z\right]_0^L + \frac{1}{4L^2}\left[\frac{z^3}{3}\right]_0^{2L} + \left[z\right]_0^{2L} - \frac{1}{L}\left[\frac{z^2}{2}\right]_0^{2L}\right] +$$

$$+ \frac{\alpha \Delta T}{h}\left[\frac{1}{2L}\left[\frac{z^2}{2}\right]_0^L - \left[z\right]_0^L\right] =$$

$$= \frac{1}{EI}\left[-\frac{q}{6}L^3 + \cancel{\frac{qL}{8}\cdot L^2} - \frac{qL^2}{2}\cdot L + \frac{q}{24}L^3 - \cancel{\frac{qL}{8}L^2} + \right.$$

$$\left. + \frac{3}{8}qL\cdot 3L^2 - \frac{3}{2}qL^2\cdot L - \frac{1}{8}q\cdot 7L^3 + \frac{3}{8}qL\cdot 3L^2 \right] +$$

$$+ \frac{X}{EI}\left[L + \frac{2}{3}L + \cancel{2L} - \cancel{2L} \right] + \frac{\alpha\overline{\Delta T}}{h}\left[\frac{L}{4} - L \right] =$$

$$= \frac{qL^3}{EI}\left[\underbrace{-\frac{1}{6} - \frac{1}{2} + \frac{1}{24} + \frac{9}{8} - \frac{3}{2} - \frac{7}{8} + \frac{9}{8}}_{\to\, = -\frac{3}{4}} \right] + \frac{X}{EI}\cdot\frac{5}{3}L - \frac{\alpha\overline{\Delta T}}{h}\cdot\frac{3}{4} =$$

$$= \underbrace{-\frac{3}{4}\frac{qL^3}{EI} + \frac{5}{3}\frac{XL}{EI} - \frac{3}{4}\frac{\alpha\overline{\Delta T}}{h}}$$

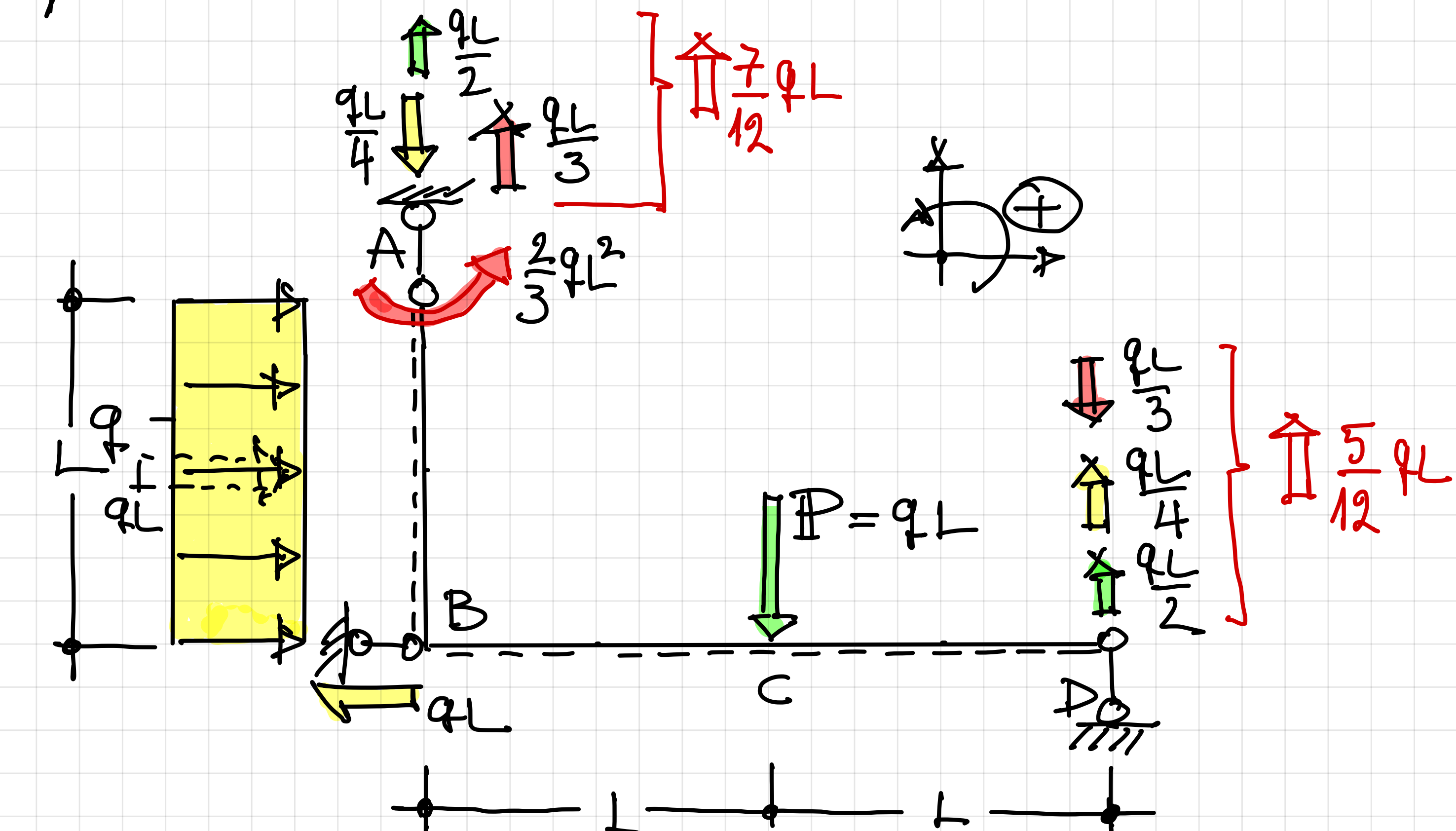
$L_{ve} = L_{vi}$ fornisce

$$-\overline{\varepsilon}_A X + \frac{\varepsilon_D}{2L}\left[\frac{3}{4}qL - \frac{X}{2L} \right] = -\frac{3}{4}\frac{qL^3}{EI} + \frac{5}{3}\frac{XL}{EI} - \frac{3}{4}\frac{\alpha\overline{\Delta T}}{h}$$

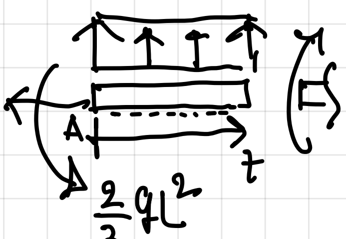
$$X\left[\frac{5}{3}\frac{L}{EI} + \underbrace{\overline{\varepsilon}_A}_{\frac{L}{EI}} + \frac{\overbrace{\varepsilon_D}^{\to \frac{4}{3}\frac{L^3}{EI}}}{4L^2} \right] = \frac{3}{8}q\underbrace{\varepsilon_D}_{\frac{4}{3}\frac{L^3}{EI}} + \frac{3}{4}\frac{qL^3}{EI} + \frac{3}{4}\underbrace{\frac{\alpha\overline{\Delta T}}{h}}_{\frac{qL^3}{EI}}$$

$$\frac{X\cancel{L}}{\cancel{EI}}\left[\underbrace{\frac{5}{3} + 1 + \frac{1}{3}}_{3} \right] = \frac{qL^{\cancel{3}\,2}}{\cancel{EI}}\left[\underbrace{\frac{1}{2} + \frac{3}{4} + \frac{3}{4}}_{2} \right]$$

da cui $\boxed{X = \frac{2}{3}qL^2}$ POSITIVO! VERSO IPOTIZZATO CORRETTO!



TRATTO AB $0 \le z \le L$

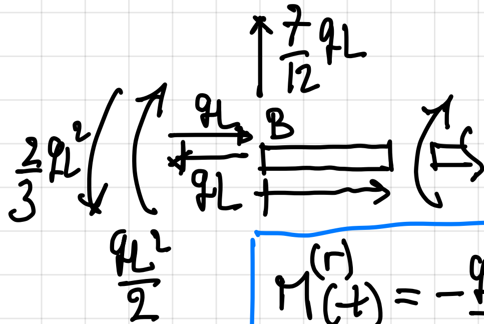


$$\boxed{M^{(r)}(z) = q\frac{z^2}{2} - \frac{2}{3}qL^2}$$

$M_A = -\frac{2}{3}qL^2$

$M_B = -\frac{qL^2}{6}$

TRATTO BC $0 \le z \le L$



$$\boxed{M^{(r)}(z) = -\frac{qL^2}{6} + \frac{7}{12}qL\cdot z}$$

$M_B = -\frac{qL^2}{6}$

$M_C = \frac{5}{12}qL^2$

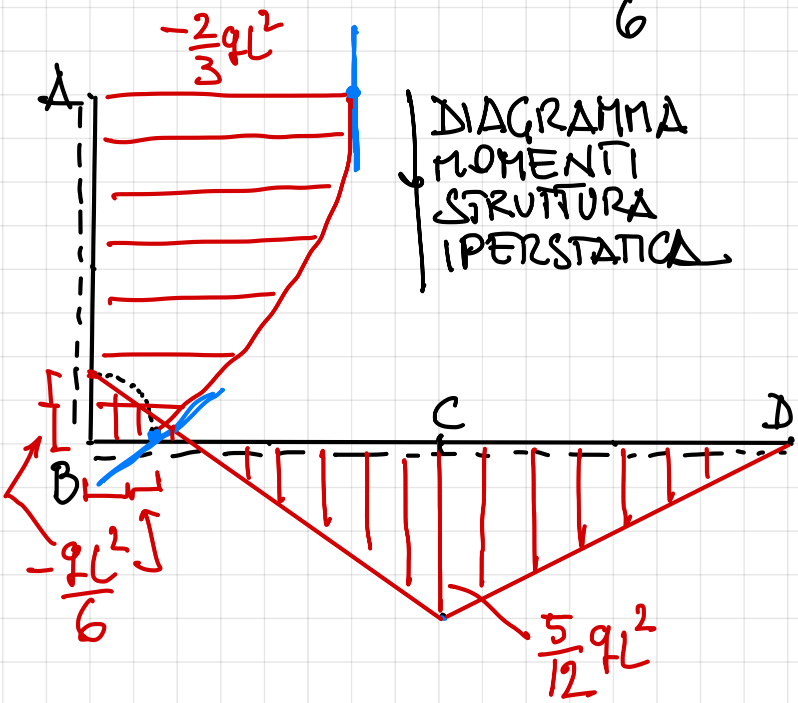
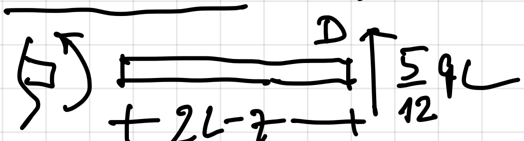


DIAGRAMMA MOMENTI STRUTTURA IPERSTATICA

TRATTO CD $L \le z \le 2L$



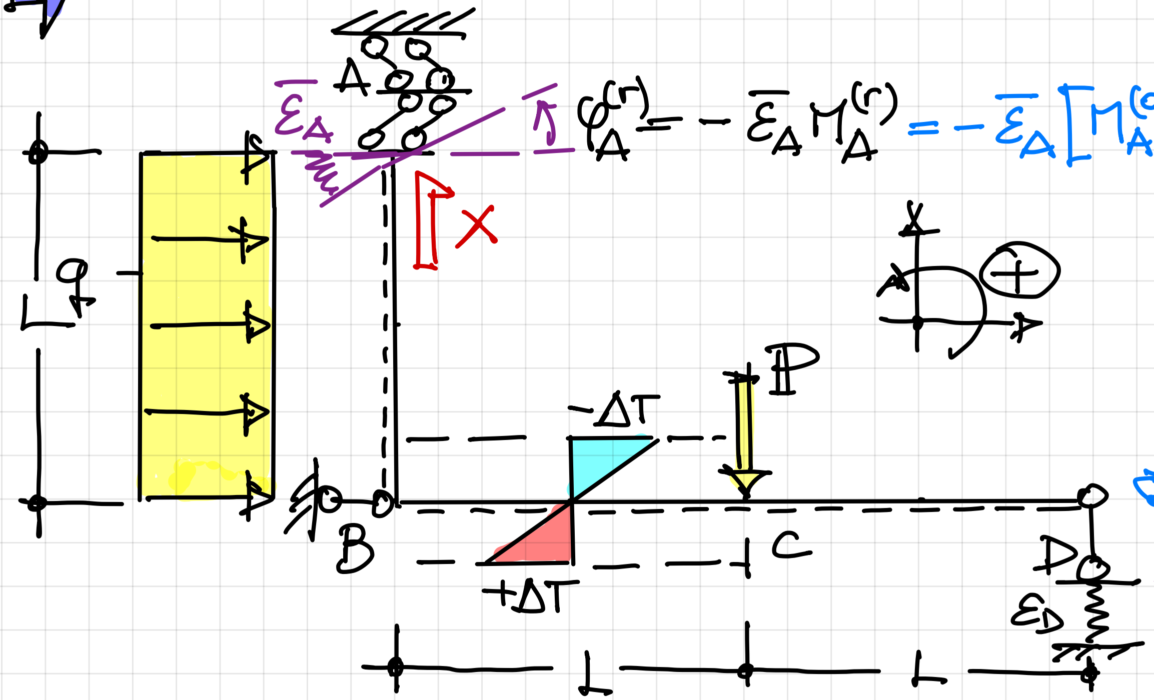
$$\boxed{M^{(r)}(z) = \frac{5}{12}qL(2L-z)}$$

$M_C = \frac{5}{12}qL^2$

$M_D = \emptyset$

SOLUZIONE #2

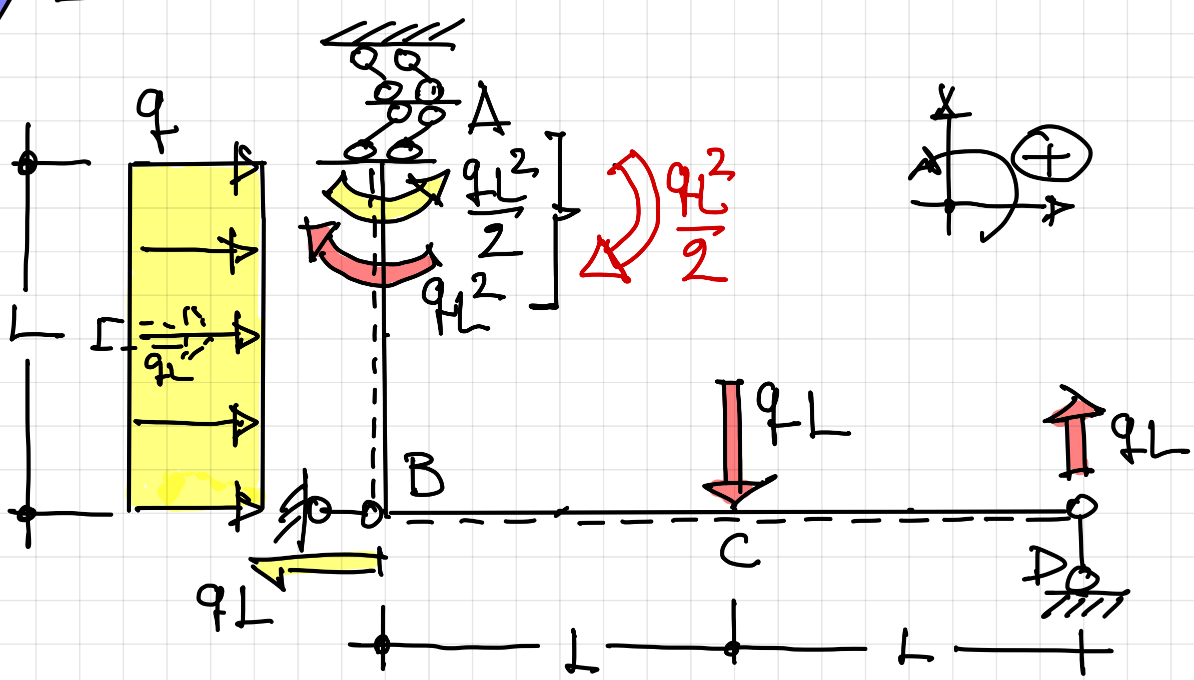
SISTEMA PRINCIPALE ISOSTATICO



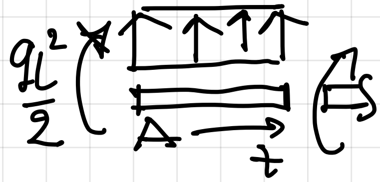
$$\varphi_A^{(r)} = -\bar{\varepsilon}_A M_A^{(r)} = -\bar{\varepsilon}_A\left[M_A^{(0)} + X\, M_A^{(1)}\right]$$

$$\eta_D^{(r)} = -\varepsilon_D R_{y_D}^{(r)} = -\varepsilon_D\left[R_{y_D}^{(0)} + X\, R_{y_D}^{(1)}\right]$$

SCHEMA [0] SOLO CARICHI ESTERNI



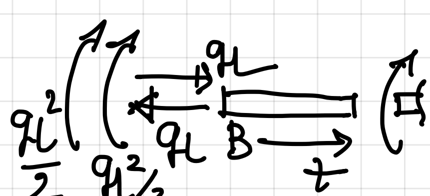
TRATTO AB $0 \le z \le L$



$$\boxed{M^{(0)}(z) = \frac{qL^2}{2} + \frac{qz^2}{2}}$$

$M_A = \frac{qL^2}{2}$ $\quad M_B = qL^2$

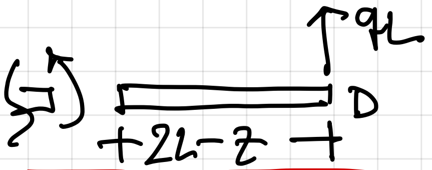
TRATTO BC $0 \le z \le L$



$$\boxed{M^{(0)}(z) = qL^2}$$

cost.

TRATTO CD $L \leq z \leq 2L$



$$\boxed{M^{(0)}(z) = qL(2L-z)}$$

$M_C = qL^2$; $M_D = \emptyset$

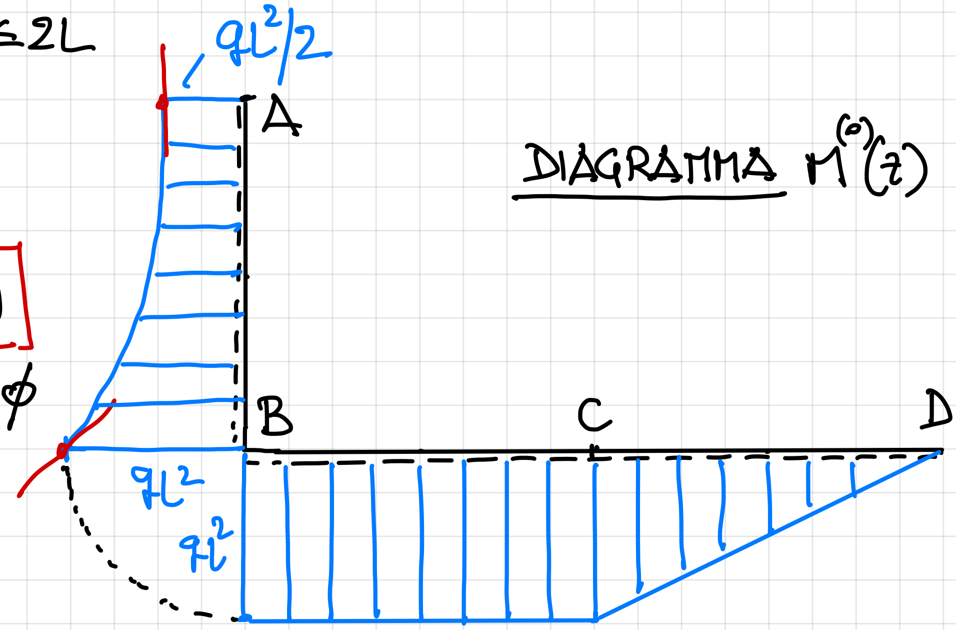
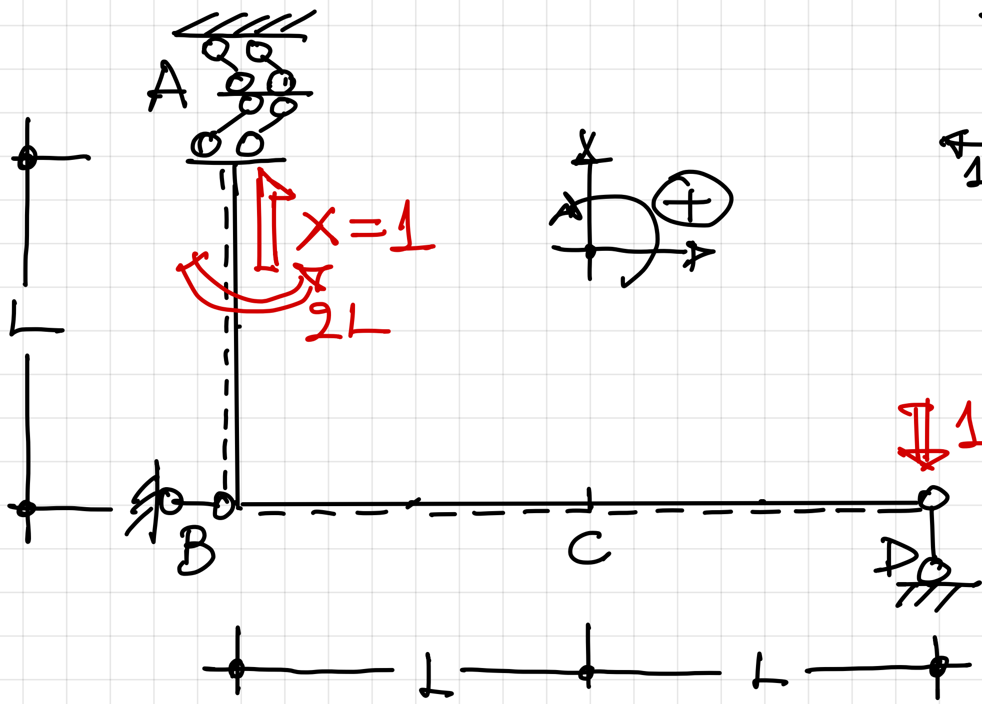
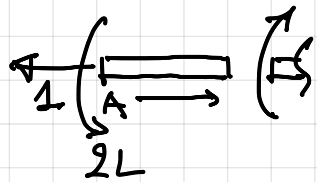


DIAGRAMMA $M^{(0)}(z)$

SCHEMA [1] SOLO $X = 1$



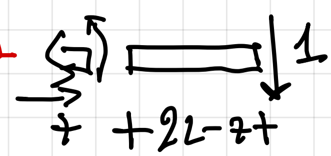
TRATTO AB $0 \leq z \leq L$



$$\boxed{M^{(1)}(z) = -2L}$$

cost

TRATTO BD $0 \leq z \leq 2L$



$$\boxed{M^{(1)}(z) = -(2L-z)}$$

$M_B = -2L$; $M_D = \emptyset$

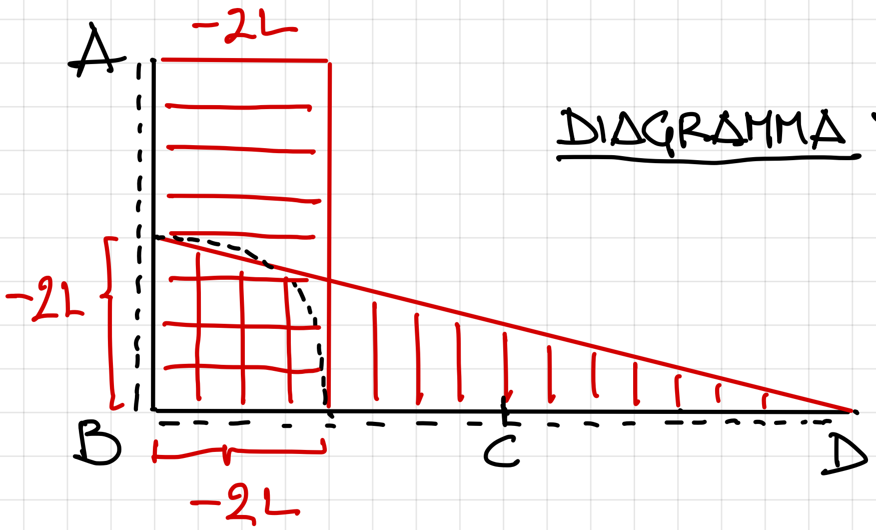


DIAGRAMMA $M^{(1)}(z)$

$$L_{ve} = \sum_{1}^{n} X_i\, \eta_j^{(r)} + \sum_j R_j^{(1)} \eta_j^{(r)} =$$

$$= 1 \cdot \phi + M_A^{(1)} \varphi_A^{(r)} + R_{y_D}^{(1)} \eta_D^{(r)} =$$

$$= \underbrace{M_A^{(1)}}_{2L} (-\mathcal{E}_A) \left[\underbrace{M_A^{(0)}}_{-\frac{qL^2}{2}} + X \underbrace{M_A^{(1)}}_{2L} \right] + \underbrace{R_{y_D}^{(1)}}_{-1} (-\mathcal{E}_D) \left[\underbrace{R_{y_D}^{(0)}}_{qL} + X \underbrace{R_{y_D}^{(1)}}_{-1} \right] =$$

$$= -\mathcal{E}_A\, 2L \left[-\frac{qL^2}{2} + 2LX \right] + \mathcal{E}_D \left[qL - X \right]$$

$$L_{vi} = \int_{str} M^{(1)} \frac{M^{(r)}}{EI} ds_{tr} + \int_{str} M^{(1)} \frac{\alpha \Delta T}{h} ds_{tr} =$$

$$= \frac{1}{EI} \int_{str} M^{(1)} M^{(0)} ds_{tr} + \frac{X}{EI} \int_{str} \left[M^{(1)} \right]^2 ds_{tr} + \frac{\alpha \Delta T}{h} \int_{str} M^{(1)} ds_{tr} =$$

$$= \frac{1}{EI} \left[\int_0^L (-\cancel{2}L) \left[\frac{qL^2}{\cancel{2}} + \frac{qz^2}{\cancel{2}} \right] dz + \int_0^L -(2L-z)\, qL^2\, dz + \right.$$

$$\left. + \int_L^{2L} -\underbrace{(2L-z)^2}_{\to 4L^2 + z^2 - 4Lz} qL\, dz \right] +$$

$$+ \frac{X}{EI} \left\{ \int_0^L 4L^2 dz + \int_0^{2L} (2L-z)^2 dz \right\} - \frac{\alpha \Delta T}{h} \int_0^L (2L-z)\, dz =$$

$$= \frac{1}{EI} \left[-qL^3 [z]_0^L - qL \left[\frac{z^3}{3} \right]_0^L - 2qL^3 [z]_0^L + qL^2 \left[\frac{z^2}{2} \right]_0^L \right.$$

$$\left. - 4qL^3 [z]_L^{2L} - qL \left[\frac{z^3}{3} \right]_L^{2L} + 4qL^2 \left[\frac{z^2}{2} \right]_L^{2L} \right] +$$

$$+ \frac{X}{EI} \left[4L^2 [z]_0^L + 4L^2 [z]_0^{2L} + \left[\frac{z^3}{3} \right]_0^{2L} - 4L \left[\frac{z^2}{2} \right]_0^{2L} \right] +$$

$$- \frac{\alpha \Delta T}{h} \left\{ 2L [z]_0^L - \left[\frac{z^2}{2} \right]_0^L \right\} =$$

$$= \frac{qL^4}{EI}\left[-1-\frac{1}{3}-2+\frac{1}{2}-4-\frac{7}{3}+6\right]+$$

$$+X\frac{L^3}{EI}\left[4+\cancel{8}+\frac{8}{3}-\cancel{8}\right]-\frac{\alpha\Delta T}{h}\left[2L^2-\frac{L^2}{2}\right]=$$

$$=\underbrace{-\frac{qL^4}{EI}\frac{19}{6}+X\frac{L^3}{EI}\frac{20}{3}-\frac{\alpha\Delta T}{h}\frac{3}{2}L^2}$$

$$\frac{-6-2-12+3-24}{6}$$

$-14, +36$

$+39 - 58$

$L_{ve} = L_{vi}$ fornisce:

$$-\bar{\varepsilon}_A 2L\left[-\frac{qL^2}{2}+2LX\right]+\varepsilon_D\left[qL-X\right]=$$

$$=-\frac{qL^4}{EI}\frac{19}{6}+X\frac{L^3}{EI}\frac{20}{3}-\frac{\alpha\Delta T}{h}\frac{3}{2}L^2$$

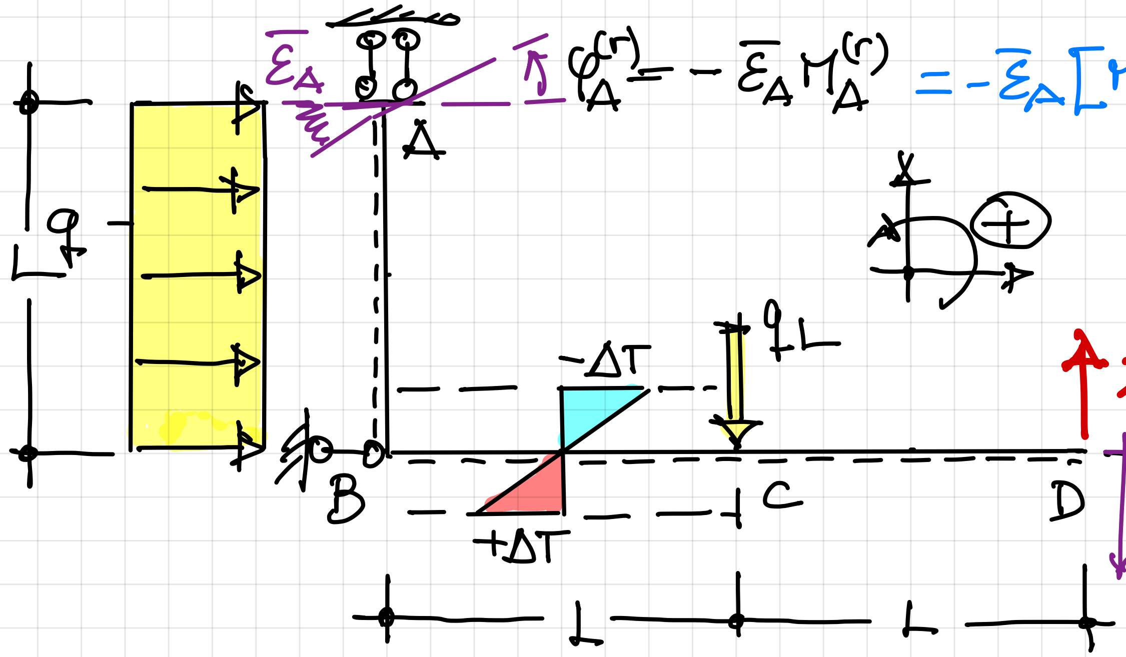
$$X\left[\frac{L^3}{EI}\frac{20}{3}+\underbrace{\bar{\varepsilon}_A}_{\frac{L}{EI}}4L^2+\underbrace{\varepsilon_D}_{\frac{4}{3}\frac{L^3}{EI}}\right]=+\frac{qL^4}{EI}\frac{19}{6}+\underbrace{\frac{\alpha\Delta T}{h}}_{\frac{qL^2}{EI}}\frac{3}{2}L^2+\underbrace{\varepsilon_D}_{\frac{4}{3}\frac{L^3}{EI}}qL+\underbrace{\bar{\varepsilon}_A}_{\frac{L}{EI}}qL^3$$

$$X\cancel{\frac{L^3}{EI}}\underbrace{\left[\frac{20}{3}+4+\frac{4}{3}\right]}_{12}=\cancel{\frac{qL^4}{EI}}\underbrace{\left[+\frac{19}{6}+\frac{3}{2}+\frac{4}{3}+1\right]}_{7}$$

da cui $\boxed{X=\frac{7}{12}qL}$ ok! cfr. RV di pag. 6

SOLUZIONE #3

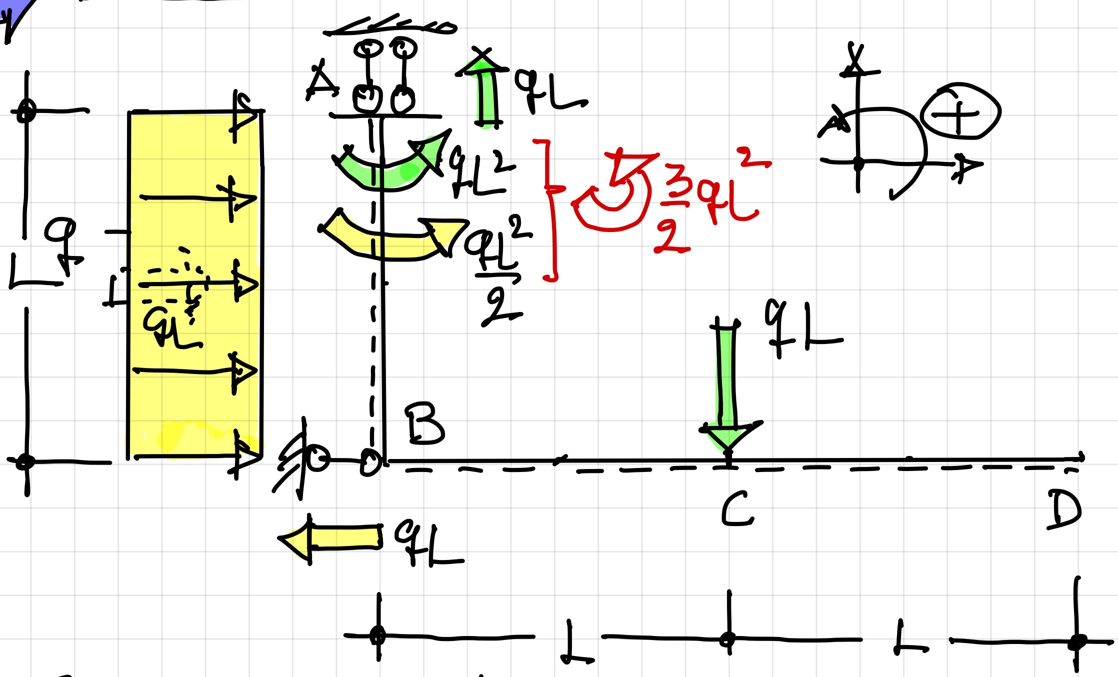
SISTEMA PRINCIPALE ISOSTATICO



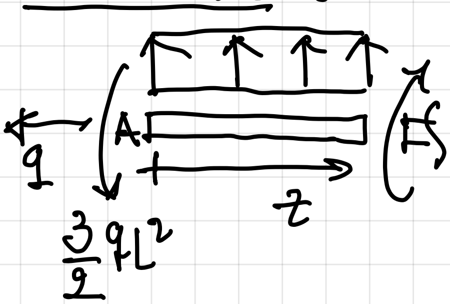
$$\varphi_A^{(r)} = -\varepsilon_A M_A^{(r)} = -\varepsilon_A\left[M_A^{(0)} + X\, M_A^{(1)}\right]$$

$$\eta_D^{(r)} = -\varepsilon_D R_{yD}^{(r)} = -\varepsilon_D X$$

SCHEMA [0] SOLO CARICHI ESTERNI



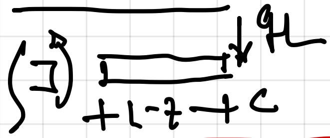
TRATTO AB $0 \le z \le L$



$$\boxed{M^{(0)}(z) = \frac{qz^2}{2} - \frac{3}{2}qL^2}$$

$$\begin{cases} M_A = -\frac{3}{2}qL^2 \\ M_B = -qL^2 \end{cases}$$

TRATTO BC $0 \le z \le L$



$$\boxed{M^{(0)}(z) = -qL(L-z)}$$

$$\begin{cases} M_B = -qL^2 \\ M_C = \emptyset \end{cases}$$

TRATTO CD $L \leq z \leq 2L$

SCARICO $\boxed{M^{(0)}(z) = \phi}$

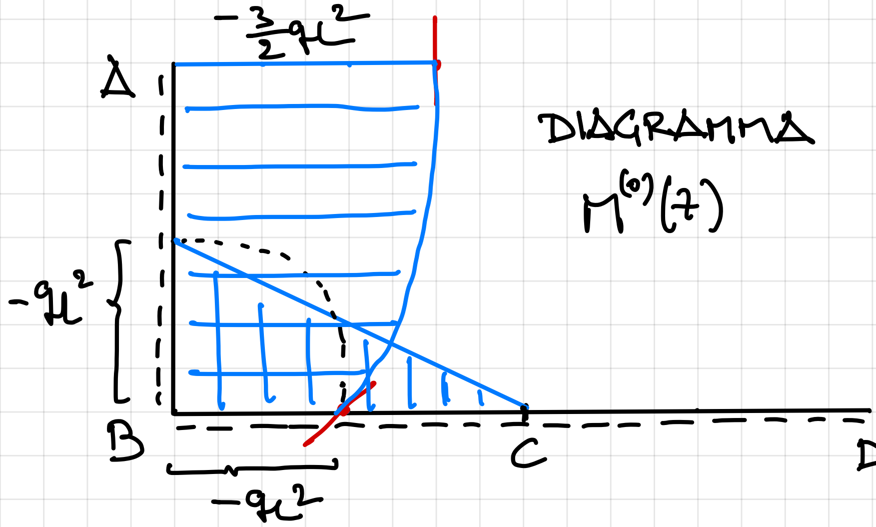
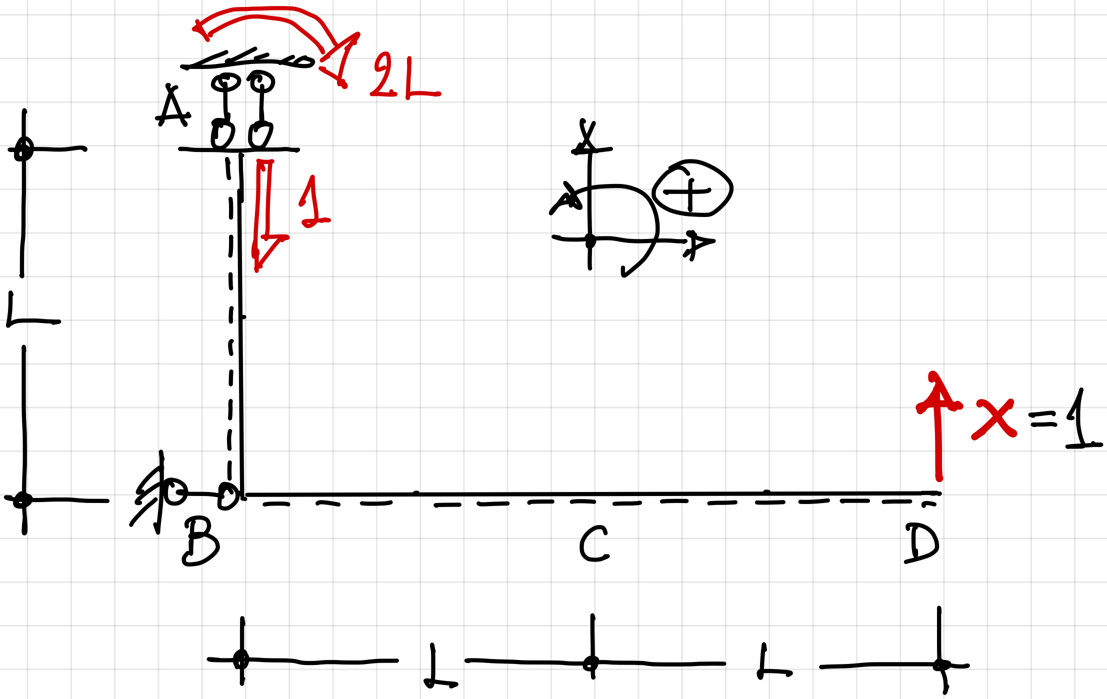
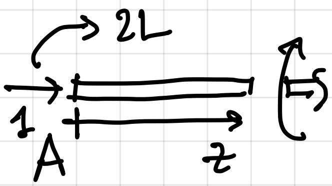


DIAGRAMMA $M^{(0)}(z)$

SCHEMA [1] SOLO $X=1$



TRATTO AB $0 \leq x \leq L$



$\boxed{M^{(1)}(z) = 2L}$ cost.

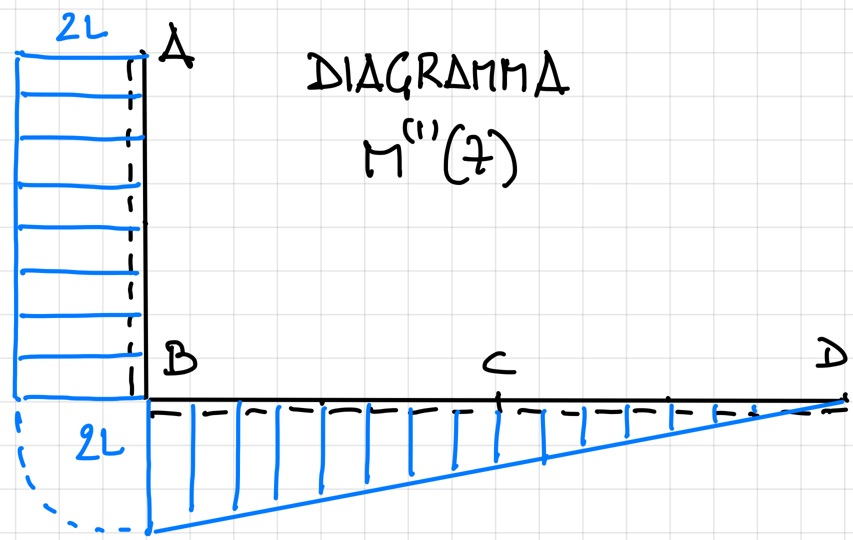
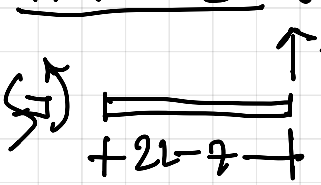


DIAGRAMMA $M^{(1)}(z)$

TRATTO BD $0 \leq z \leq 2L$



$\boxed{M^{(1)}(z) = 2L - z}$

$\begin{cases} M_B = 2L \\ M_D = \phi \end{cases}$

$$L_{ve} = \underset{1}{\overset{\|}{X_i}}\, \eta_i^{(r)} + \sum_j R_j^{(1)} \eta_j^{(r)} = 1 \cdot \underbrace{\eta_D^{(r)}}_{-\varepsilon_D X} + \underbrace{M_A^{(1)}}_{-2L} (-\overline{\varepsilon}_A) \Big[\underbrace{M_A^{(0)}}_{\frac{3}{2}qL^2} + X \underbrace{M_A^{(1)}}_{-2L} \Big] =$$

$$= -\varepsilon_D X + 2L\, \overline{\varepsilon}_A \left[\frac{3}{2} qL^2 - 2L X \right]$$

$$L_{vi} = \int_{str} M^{(1)} \frac{M^{(r)}}{EI}\, ds_{tr} + \int_{str} M^{(1)} \frac{\alpha \overline{\Delta T}}{h}\, ds_{tr} =$$

$$= \frac{1}{EI} \int_{str} M^{(1)} M^{(0)} ds_{tr} + \frac{X}{EI} \int_{str} \left[M^{(1)}\right]^2 ds_{tr} + \frac{\alpha \overline{\Delta T}}{h} \int_{str} M^{(1)} ds_{tr} =$$

$$= \frac{1}{EI} \left\{ \int_0^L \cancel{2} L \left(\frac{q z^2}{\cancel{2}} - \frac{3}{\cancel{2}} qL^2 \right) dz + \int_0^L \underbrace{(2L - z)\left[-qL(L - z)\right]}_{-qL\left(2L^2 - 2Lz - zL + z^2\right)} dz \right\} +$$

$$+ \frac{X}{EI} \left\{ \int_0^L 4L^2 dz + \int_0^{2L} \underbrace{(2L - z)^2}_{4L^2 + z^2 - 4Lz} dz \right\} + \frac{\alpha \overline{\Delta T}}{h} \int_0^{2L} (2L - z)\, dz =$$

$$= \frac{1}{EI} \left\{ qL \left[\frac{z^3}{3} \right]_0^L - 3qL^3 [z]_0^L - 2qL^3 [z]_0^L + 2qL^2 \left[\frac{z^2}{2} \right]_0^L + \right.$$

$$\left. + qL^2 \left[\frac{z^2}{2} \right]_0^L - qL \left[\frac{z^3}{3} \right]_0^L \right\} +$$

$$+ \frac{X}{EI} \left\{ 4L^2 [z]_0^L + 4L^2 [z]_0^{2L} + \left[\frac{z^3}{3} \right]_0^{2L} - 4L \left[\frac{z^2}{2} \right]_0^{2L} \right\} +$$

$$+ \frac{\alpha \overline{\Delta T}}{h} \left\{ 2L [z]_0^L - \left[\frac{z^2}{2} \right]_0^L \right\} =$$

$$= \frac{qL^4}{EI}\left[\frac{1}{3} - 3 - 2 + 1 + \frac{1}{2} - \frac{1}{3}\right] +$$

$$+ \frac{XL^3}{EI}\left[4 + \cancel{8} + \frac{8}{3} - \cancel{8}\right] + \alpha\frac{\Delta T}{h}\left[2L^2 - \frac{L^2}{2}\right] =$$

$$= -\frac{21}{6}\frac{qL^4}{EI} + \frac{20}{3}\frac{XL^3}{EI} + \alpha\frac{\Delta T}{h}\frac{3}{2}L^2$$

$L_{ve} = L_{vi}$ fornisce!

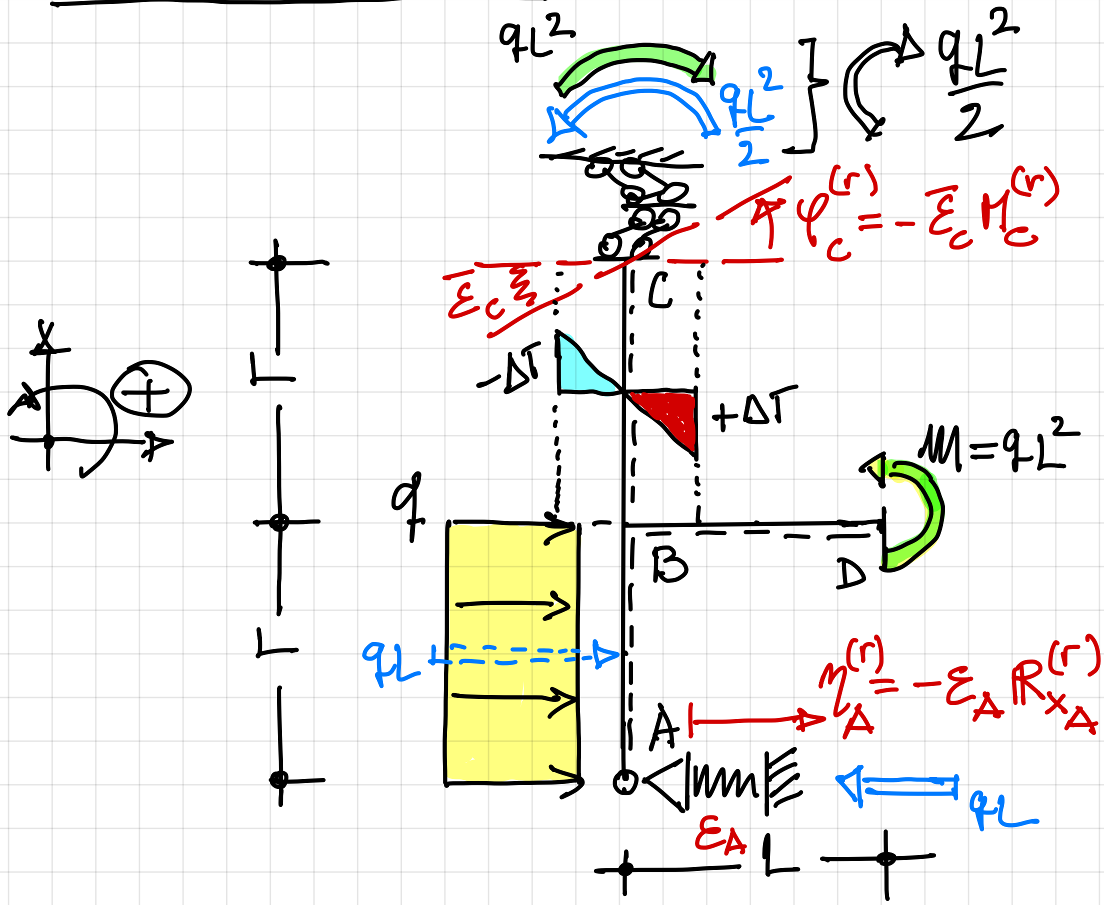
$$-\underbrace{\mathcal{E}_D}_{\frac{4}{3}\frac{L^3}{EI}} X + 2L\underbrace{\mathcal{E}_A}_{\frac{L}{EI}}\left[\frac{3}{2}qL^2 - 2LX\right] = -\frac{21}{6}\frac{qL^4}{EI} + \frac{20}{3}\frac{XL^3}{EI} + \underbrace{\alpha\frac{\Delta T}{h}}_{\frac{qL^2}{EI}}\frac{3}{2}L^2$$

$$X\frac{\cancel{L^3}}{\cancel{EI}}\underbrace{\left[\frac{20}{3} + \frac{4}{3} + 4\right]}_{12} = \frac{qL^{\cancel{4}}}{\cancel{EI}}\underbrace{\left[3 + \frac{21}{6} - \frac{3}{2}\right]}_{5}$$

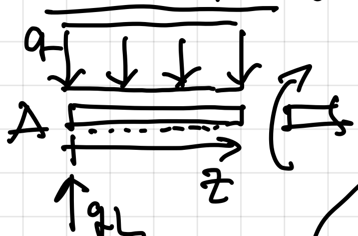
da cui $\boxed{X = \frac{5}{12}qL}$ ok! cfr. RV di pag. 6

ES.#2 – TIPO 2 SOLUZIONE

STRUTTURA REALE



TRATTO AB $0 \le z \le L$



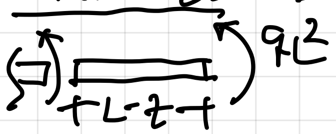
$$\boxed{M^{(r)}(z) = qL \cdot z - \frac{qz^2}{2}}$$

$$\begin{cases} M_A = 0 \\ M_B = \frac{qL^2}{2} \end{cases}$$

TRATTO BC $L \le z \le 2L$

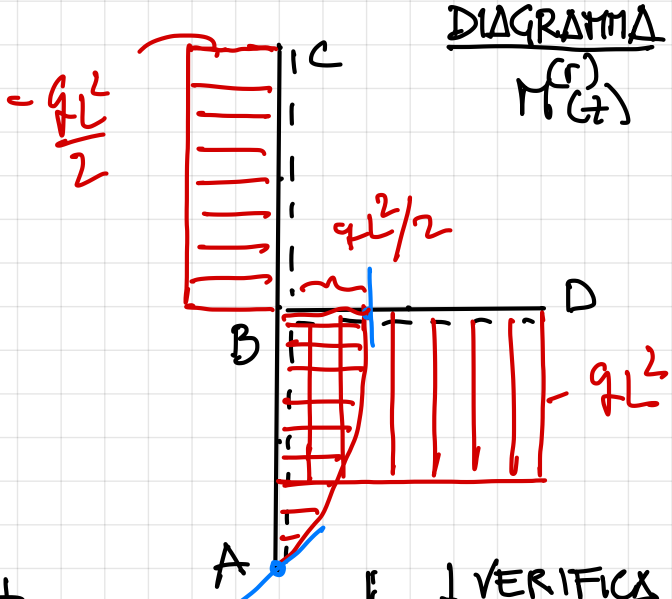
$$\boxed{M^{(r)}(z) = -\frac{qL^2}{2}}$$ cost.

TRATTO BD $0 \le z \le L$

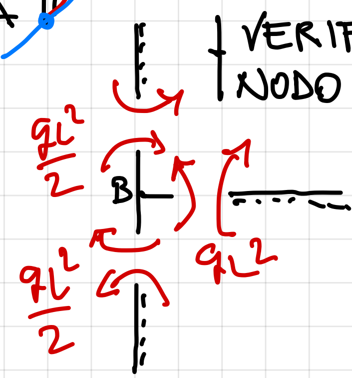


$$\boxed{M^{(r)}(z) = qL^2}$$ cost

DIAGRAMMA $M^{(r)}(z)$

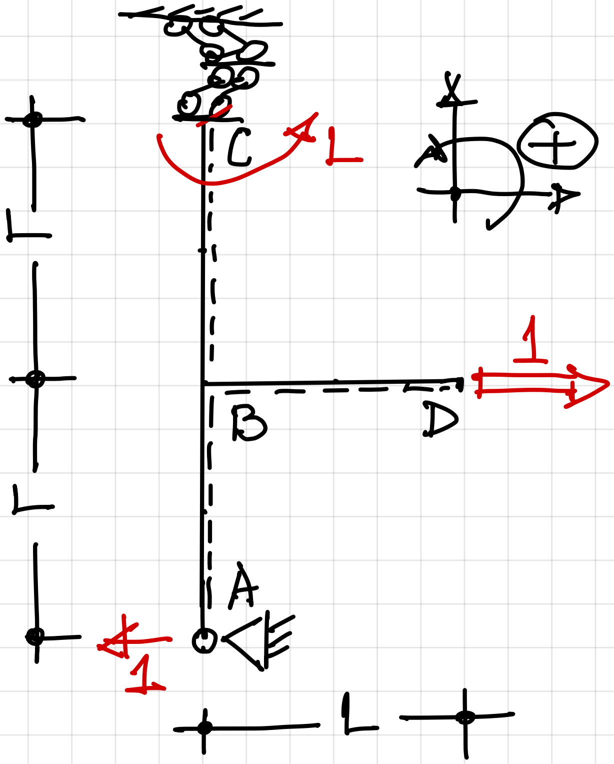


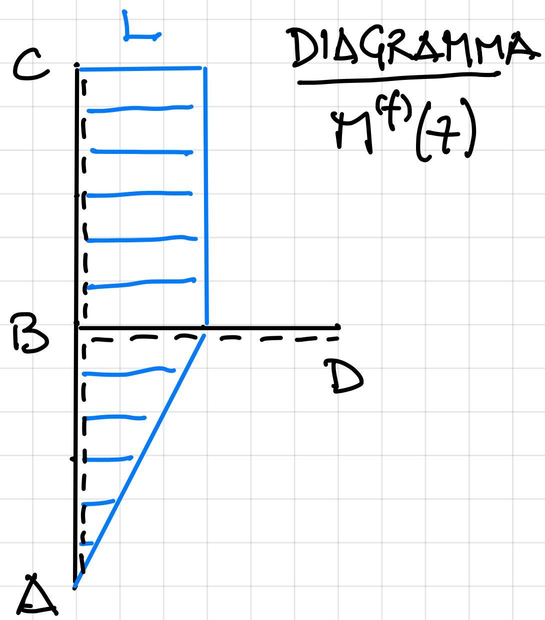
VERIFICA NODO B



OK!

STRUTTURA FITTIZIA PER IL CALCOLO DELLO SPOSTAMENTO ORIZZONTALE DELLA SEZIONE D





TRATTO AB $0 \le z \le L$

$$\boxed{M^{(f)}(z) = z}$$

$$\begin{cases} M_A = \emptyset \\ M_B = L \end{cases}$$

TRATTO BC $L \le z \le 2L$

$$\boxed{M^{(f)}(z) = L} \quad \text{cost.}$$

TRATTO BD

SCARICO

$$\boxed{M^{(f)}(z) = \emptyset}$$

$$\underbrace{L_{ve}} = 1 \cdot \eta_D + \sum_j R_j^{(f)} \eta_j^{(r)} =$$

$$= \eta_D + \underbrace{R_{x_A}^{(f)}}_{-1} \underbrace{\eta_A^{(r)}}_{-\varepsilon_A \underbrace{R_{x_A}^{(r)}}_{-qL}} + \overbrace{M_C^{(f)}}^{L} \overbrace{\varphi_C^{(r)}}^{-\bar{\varepsilon}_C \overbrace{M_C^{(r)}}^{-\frac{qL^2}{2}}} =$$

$$= \underbrace{\eta_D - \varepsilon_A qL + \bar{\varepsilon}_C \frac{qL^3}{2}}$$

$$\underbrace{L_{vi}} = \int_{str} M^{(f)} \frac{M^{(r)}}{EI} ds_{tr} + \int_{str} M^{(f)} \frac{\alpha \overline{\Delta T}}{h} ds_{tr} =$$

$$= \frac{1}{EI}\left\{ \int_0^L z\left[qL\cdot z - \frac{qz^2}{2}\right] dz + \int_L^{2L} L\left[-\frac{qL^2}{2}\right] dz \right\} + \frac{\alpha \overline{\Delta T}}{h}\int_L^{2L} L\, dz =$$

$$= \frac{1}{EI}\left\{ qL\left[\frac{z^3}{3}\right]_0^L - \frac{q}{2}\left[\frac{z^4}{4}\right]_0^L - \frac{qL^3}{2}\left[z\right]_L^{2L} \right\} + \frac{\alpha \overline{\Delta T}}{h} L \left[z\right]_L^{2L} =$$

$$= \frac{1}{EI}\left\{ \frac{qL^4}{3} - \frac{qL^4}{8} - \frac{qL^4}{2} \right\} + \alpha \frac{\overline{\Delta T}}{h} L^2 =$$

$$= \underbrace{-\frac{qL^4}{EI}\frac{7}{24} + \frac{\alpha \overline{\Delta T}}{h}\cdot L^2}$$

$L_{ve} = L_{vi}$ fornisce

$$\eta_D - \underbrace{\varepsilon_A}_{\frac{2}{3}\frac{L^3}{EI}} qL + \underbrace{\overline{\varepsilon}_C}_{\frac{L}{EI}} \frac{qL^3}{2} = -\frac{qL^4}{EI}\frac{7}{24} + \underbrace{\frac{\alpha \overline{\Delta T}}{h}}_{\frac{13}{8}\frac{qL^2}{EI}} \cdot L^2$$

da cui:

$$\eta_D = \frac{qL^4}{EI}\underbrace{\left[\frac{2}{3} - \frac{1}{2} - \frac{7}{24} + \frac{13}{8}\right]}_{\cancel{\frac{36}{24}}\ \frac{3}{2}} \Rightarrow \underline{X = \frac{3}{2}\frac{qL^4}{EI}}$$

POSITIVO! VERSO DESTRA