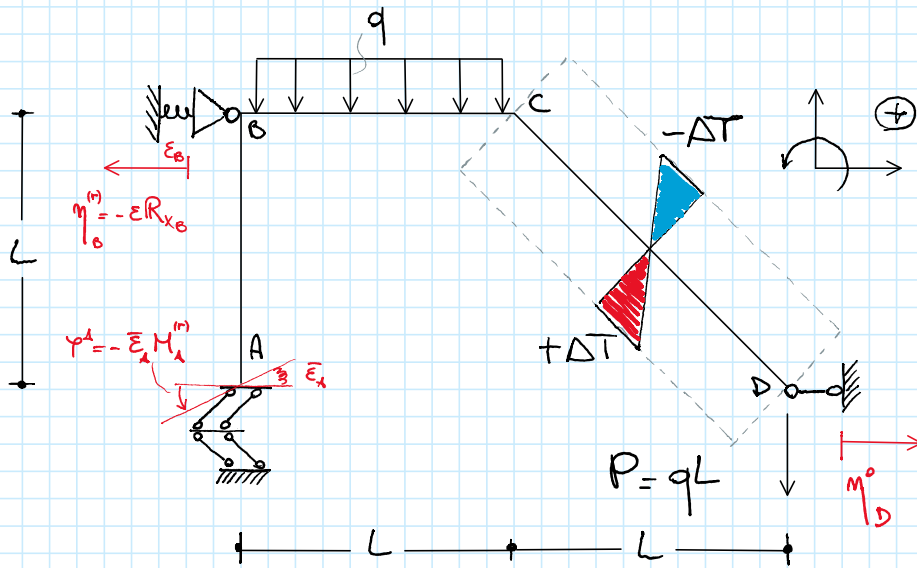


RISOLVERE LA STRUTTURA UNA VOLTA IPERSTATICA  
DETERMINANDO IL DIAGRAMMA DEI MOMENTI



Posizioni

$$|P| = qL$$

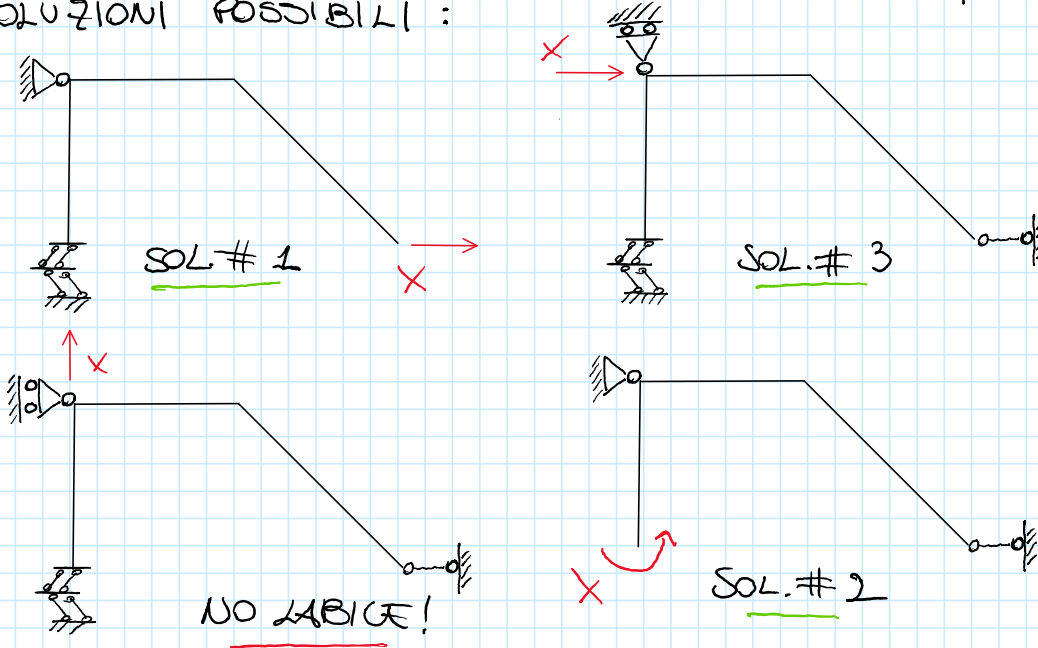
$$|\bar{\varepsilon}| = \frac{2}{5} \frac{L}{EI}$$

$$|\varepsilon_B| = \frac{3}{5} \frac{L^3}{EI}$$

$$|M_D^0| = \frac{qL^4}{EI} \left[ \frac{\sqrt{2}}{3} + 2 \right]$$

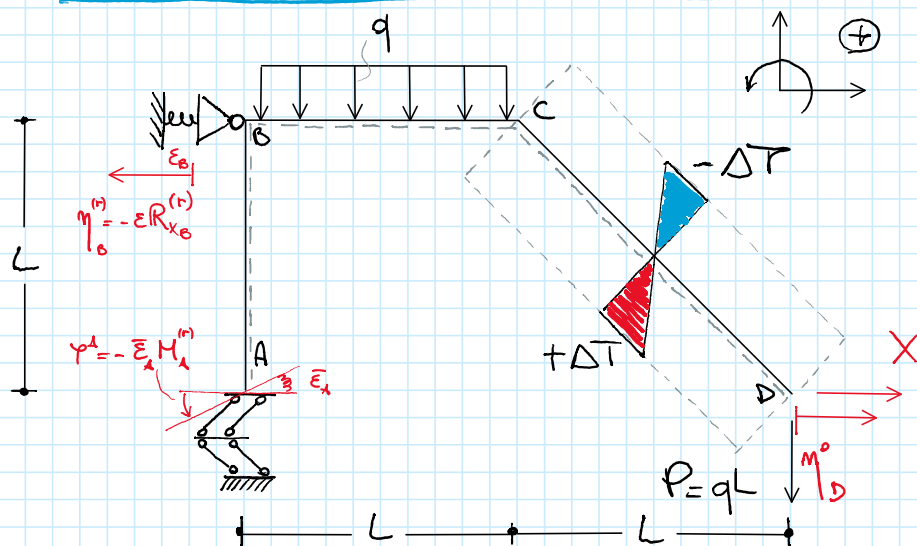
$$\left| \frac{\Delta T}{h} \right| = \frac{qL^2 \sqrt{2}}{EI} \left[ \frac{25}{6} + \frac{\sqrt{2}}{3} \right]$$

SOLUZIONI POSSIBILI:

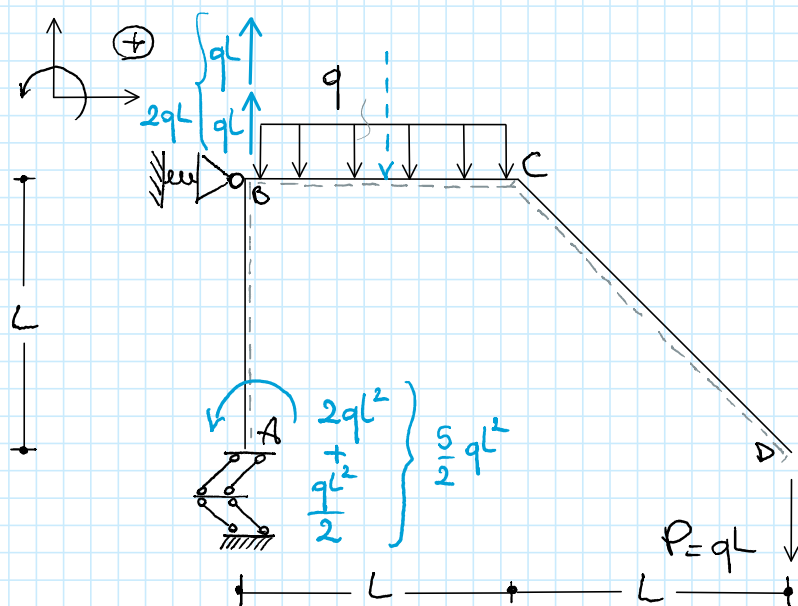


# SOLUZIONE # 1

## ● SISTEMA PRINCIPALE ISOSTATICO



## SCHEMA [0] SOLO CARICHI ESTERNI



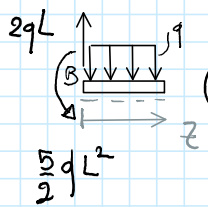
TRATTO AB  $0 \leq z \leq L$

$$\left( \begin{array}{c} \text{1} \\ \text{5/2 qL}^2 \end{array} \right) \left( \begin{array}{c} \text{1} \\ \text{0} \end{array} \right) \quad M^{(0)}(z) = -\frac{5}{2} qL^2 \quad \left\{ \begin{array}{l} M_A = -\frac{5}{2} qL^2 \\ M_B = -\frac{5}{2} qL^2 \end{array} \right.$$

TRATTO CD  $0 \leq z \leq L\sqrt{2}$

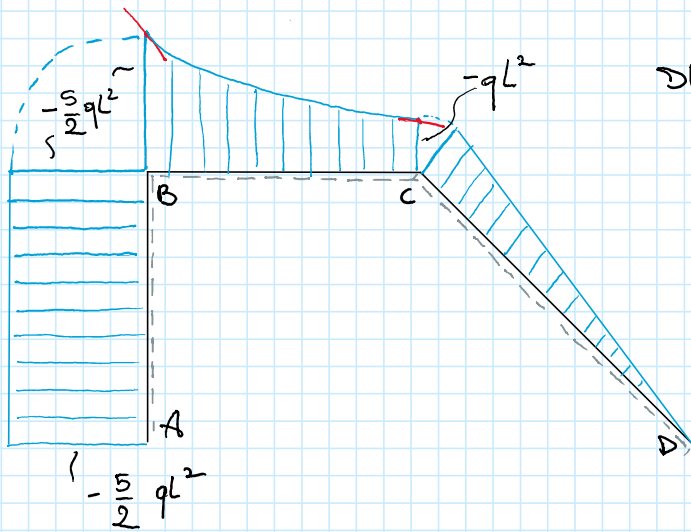
$$\left( \begin{array}{c} \text{0} \\ \text{1/2 qL} \frac{\sqrt{2}}{2} \end{array} \right) \left( \begin{array}{c} \text{0} \\ \text{1} \end{array} \right) \quad M^{(0)}(z) = -qL \frac{\sqrt{2}}{2} (L\sqrt{2} - z) \quad \left\{ \begin{array}{l} M_C = -qL^2 \\ M_D = 0 \end{array} \right.$$

TRATTO BC  $0 \leq z \leq L$

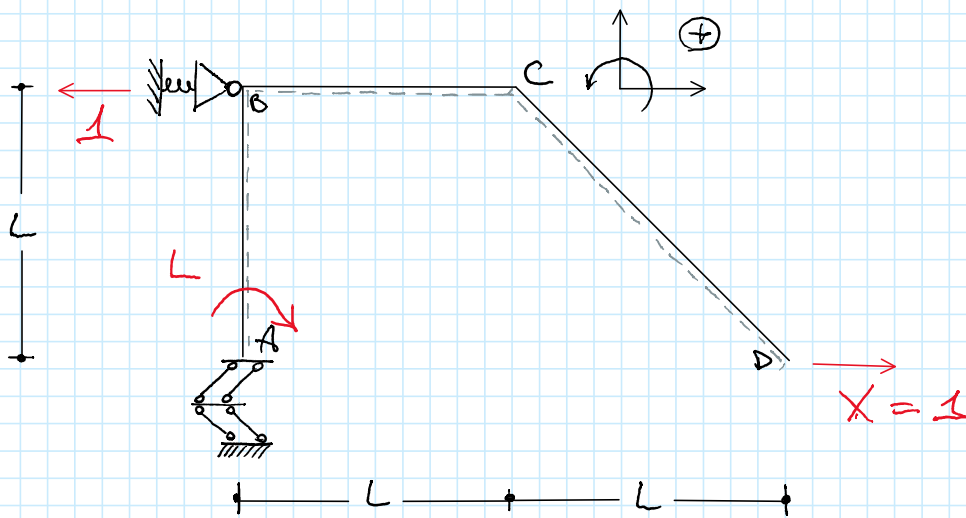


$$M''(z) = -\frac{5}{2}qL^2 + 2qLz - \frac{qz^2}{2}$$

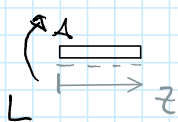
$$\begin{cases} M_B = -\frac{5}{2}qL^2 \\ M_C = -qL^2 \end{cases}$$



• SCHEMA [1] SOLO  $X=1$

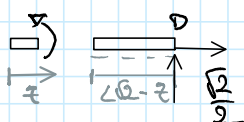


TRATTO AB  $0 \leq z \leq L$



$$M''(z) = L \quad \text{cost.}$$

TRATTO CD  $0 \leq z \leq L\sqrt{2}$



$$M''(z) = \frac{\sqrt{2}}{2} [L\sqrt{2} - z]$$

$$\begin{cases} M_C = L \\ M_D = 0 \end{cases}$$

TRATTO BC  $0 \leq z \leq L$

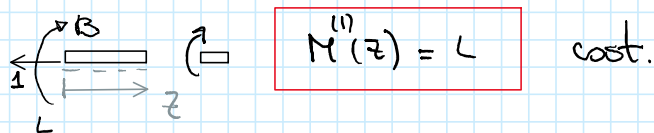
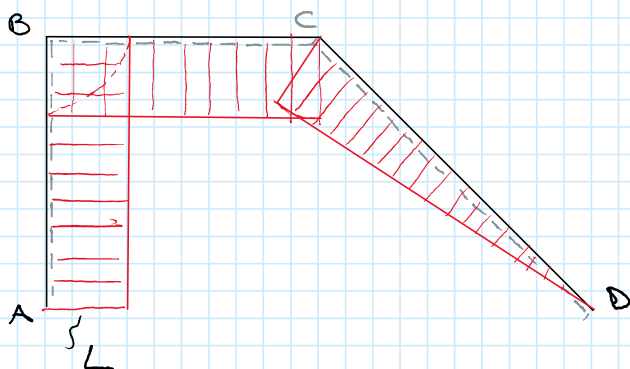


DIAGRAMMA  $M''(z)$



$$\begin{aligned}
 \underline{L_{ve}} &= 1 \cdot \eta_i^{(r)} + \sum_j R_j^{(r)} \cdot \eta_j^{(r)} = 1 \cdot \eta_0^{(r)} + M_A^{(r)} \cdot \eta_A^{(r)} + R_{XB}^{(r)} \cdot \eta_B^{(r)} = \\
 &= \eta_0^{(r)} - L \left[ -\bar{\varepsilon} M_A^{(r)} \right] - 1 \left[ -\varepsilon_B R_{XB}^{(r)} \right] = \\
 &\quad \underbrace{L_0 M_A^{(0)} + X_1 M_A^{(1)}}_{\frac{5}{2} q L^2} \underbrace{- L}_{-L} \quad \underbrace{L_0 R_{XB}^{(0)} + X R_{XB}^{(1)}}_{\emptyset} \underbrace{- 1}_{-1} \\
 &= \eta_0^{(r)} + \bar{\varepsilon} \left[ \frac{5}{2} q L^3 - X L^2 \right] - \varepsilon_B X
 \end{aligned}$$

$$\begin{aligned}
 \underline{L_{vi}} &= \int_{Str} M''^{(r)} \left( \frac{M''^{(r)}}{EI} \right) dStr + \int_{Str} M''^{(r)} \frac{\alpha \Delta T}{h} dStr = \\
 &= \frac{1}{EI} \int_{Str} M''^{(r)} M''^{(0)} dStr + \frac{X}{EI} \int_{Str} [M''^{(r)}]^2 dStr + \int_{Str} M''^{(r)} \frac{\alpha \Delta T}{h} dStr = \\
 &= \frac{1}{EI} \left\{ \int_{AB} M''^{(r)} M''^{(0)} dz + \int_{BC} M''^{(r)} M''^{(0)} dz + \int_{CD} M''^{(r)} M''^{(0)} dz \right\} + \\
 &\quad + \frac{X}{EI} \left\{ \int_{AB} [M''^{(r)}]^2 dz + \int_{BC} [M''^{(r)}]^2 dz + \int_{CD} [M''^{(r)}]^2 dz \right\} + \int_{Str} M''^{(r)} \frac{\alpha \Delta T}{h} dz =
 \end{aligned}$$



$$\begin{aligned}
&= \frac{1}{EI} \left\{ \int_0^L L \left[ -\frac{5}{2} q L^2 \right] dz + \int_0^L L \left[ -\frac{5}{2} q L^2 + 2qLz - q \frac{z^2}{2} \right] \right. \\
&\quad \left. + \int_0^{L\sqrt{2}} \left[ L\sqrt{2} - z \right] \left[ -qL\frac{\sqrt{2}}{2} (L\sqrt{2} - z) \right] dz \right\} + \frac{X}{EI} \left\{ \int_0^L L^2 dz + \right. \\
&\quad \left. + \int_0^L L^2 dz + \int_0^{L\sqrt{2}} \frac{2}{4} [L\sqrt{2} - z]^2 dz \right\} + \alpha \frac{\Delta T}{h} \int_0^{L\sqrt{2}} \frac{\sqrt{2}}{2} [L\sqrt{2} - z] dz = \\
&+ \frac{1}{EI} \left\{ -\frac{5}{2} q L^3 \left[ z \right]_0^L - \frac{5}{2} q L^3 \left[ z \right]_0^L + 2qL^2 \left[ \frac{z^2}{2} \right]_0^L - \frac{qL}{2} \left[ \frac{z^3}{3} \right]_0^L - qL^3 \left[ z \right]_0^{L\sqrt{2}} \right. \\
&\quad \left. + qL^2 \frac{\sqrt{2}}{2} \left[ \frac{z^2}{2} \right]_0^{L\sqrt{2}} + qL^2 \frac{\sqrt{2}}{2} \left[ \frac{z^2}{2} \right]_0^{L\sqrt{2}} - \frac{qL}{2} \left[ \frac{z^3}{3} \right]_0^{L\sqrt{2}} \right\} + \frac{X}{EI} \left\{ L^2 \left[ z \right]_0^L + \right. \\
&\quad \left. + L^2 \left[ z \right]_0^L + L^2 \left[ z \right]_0^{L\sqrt{2}} + \frac{1}{2} \left[ \frac{z^3}{3} \right]_0^{L\sqrt{2}} - L\sqrt{2} \left[ \frac{z^2}{2} \right]_0^{L\sqrt{2}} \right\} + \\
&\quad + \alpha \frac{\Delta T}{h} \left\{ L \left[ z \right]_0^{L\sqrt{2}} - \frac{\sqrt{2}}{2} \left[ \frac{z^2}{2} \right]_0^{L\sqrt{2}} \right\} = \\
&= \frac{1}{EI} \left\{ -5qL^4 + qL^4 - \frac{qL^4}{6} - qL^4\sqrt{2} + \frac{qL^4\sqrt{2}}{4} (2L^2) + qL^2\frac{\sqrt{2}}{4} 2L^2 - \frac{qL}{6} L^3 2\sqrt{2} \right\} + \\
&\quad + \frac{X}{EI} \left\{ 2L^3 + L^3\sqrt{2} + \frac{1}{6} L^3 \cdot 2\sqrt{2} - L\frac{\sqrt{2}}{2} \cdot 2L^2 \right\} + \alpha \frac{\Delta T}{h} \left\{ L^2\sqrt{2} - \frac{\sqrt{2}}{4} L^2 2 \right\} = \\
&= \frac{1}{EI} \left\{ -4qL^4 - \frac{1}{6} qL^4 - \frac{qL^4}{3} \sqrt{2} \right\} + \frac{X}{EI} \left\{ 2L^3 + \frac{L^3\sqrt{2}}{3} \right\} + \frac{\alpha \Delta T}{h} \frac{L^2\sqrt{2}}{2} = \\
&= \frac{1}{EI} \left\{ -\frac{25}{6} qL^4 - \frac{qL^4}{3} \sqrt{2} \right\} + \frac{X}{EI} \left\{ 2L^3 + \frac{L^3\sqrt{2}}{3} \right\} + \frac{\alpha \Delta T}{h} \frac{L^2\sqrt{2}}{2}
\end{aligned}$$

$L_{ve} = L_{vi}$  fornisce

$$\eta_b^0 + \bar{\varepsilon} \cdot \frac{5}{2} qL^3 - \bar{\varepsilon} L^2 X - \varepsilon_b X = -\frac{qL^4}{EI} \left[ +\frac{25}{6} + \frac{\sqrt{2}}{3} \right] + \frac{XL^3}{EI} \left[ 2 + \frac{\sqrt{2}}{3} \right] + \frac{\alpha \Delta T}{h} \frac{L^2 \sqrt{2}}{2}$$

$$X \left\{ \frac{L^3}{EI} \left( 2 + \frac{\sqrt{2}}{3} \right) + \bar{\varepsilon} L^2 + \varepsilon_b \right\} = \eta_b^0 + \bar{\varepsilon} \frac{5}{2} qL^3 + \frac{qL^4}{EI} \left[ \frac{25}{6} + \frac{\sqrt{2}}{3} \right] - \frac{\alpha \Delta T}{h} \frac{L^2 \sqrt{2}}{2}$$

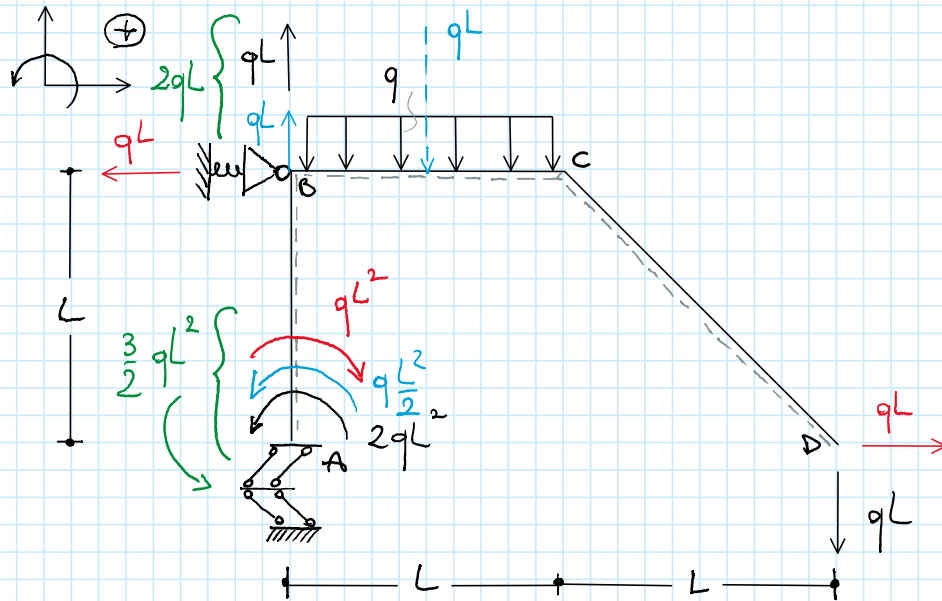
$\frac{2}{3} \frac{L}{EI}$      $\frac{3}{5} \frac{L^3}{EI}$      $\frac{qL^4}{EI} \left[ \frac{\sqrt{2}}{3} + 2 \right]$      $qL^2 \frac{\sqrt{2}}{EI} \left[ \frac{25}{6} + \frac{\sqrt{2}}{3} \right]$

$$X \left\{ \frac{L^3}{EI} \left( 2 + \frac{\sqrt{2}}{3} \right) + \frac{L^3}{EI} \right\} = \frac{qL^4}{EI} \left\{ \frac{\sqrt{2}}{3} + 2 \right\} + \frac{2}{5} \frac{L}{EI} \frac{5}{2} qL^3$$

$$X \left\{ \frac{L^3}{EI} \left( 2 + \frac{\sqrt{2}}{3} \right) + \frac{L^3}{EI} \right\} = qL \left\{ \frac{L^3}{EI} \left( 2 + \frac{\sqrt{2}}{3} \right) + \frac{L^3}{EI} \right\}$$

$$X = qL \text{ POSITIVA}$$

# SOLUZIONE SISTEMA PRINCIPALE ISOSTATICO



TRATTO AB  $0 \leq z \leq L$

Free body diagram of segment AB showing a constant counter-clockwise moment of  $\frac{3}{2}qL^2$ .

$$M^{(r)}(z) = -\frac{3}{2}qL^2 \quad \text{cost.}$$

TRATTO CD  $0 \leq z \leq L\sqrt{2}$

Free body diagram of segment CD showing zero internal moment.

$$M^{(r)}(z) = 0$$

TRATTO BC  $0 \leq z \leq L$

Free body diagram of segment BC showing a linearly varying moment.

$$M^{(r)}(z) = 2qLz - \frac{qz^2}{2} - \frac{3}{2}qL^2$$

$$\begin{cases} M_B = -\frac{3}{2}qL^2 \\ M_C = 0 \end{cases}$$

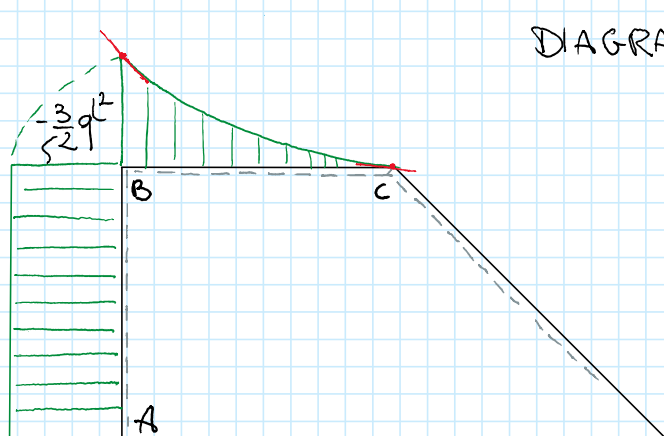
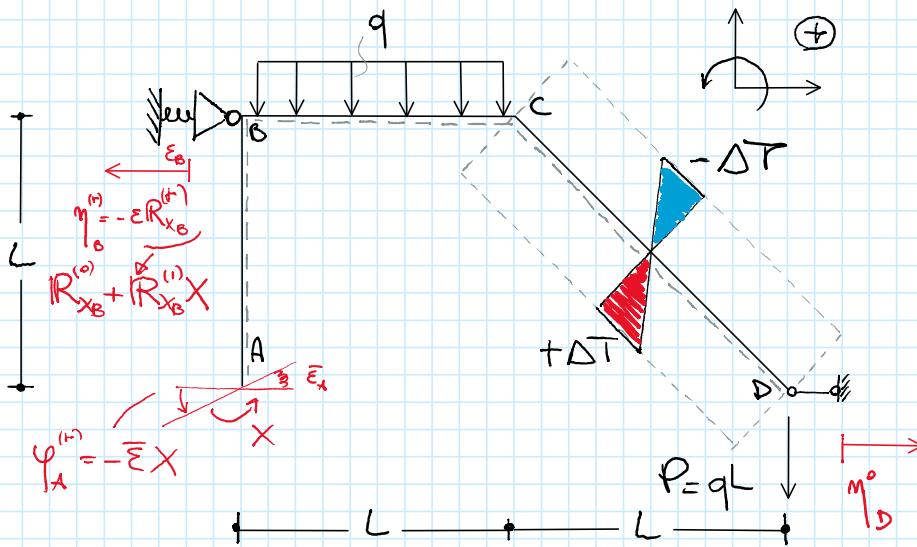


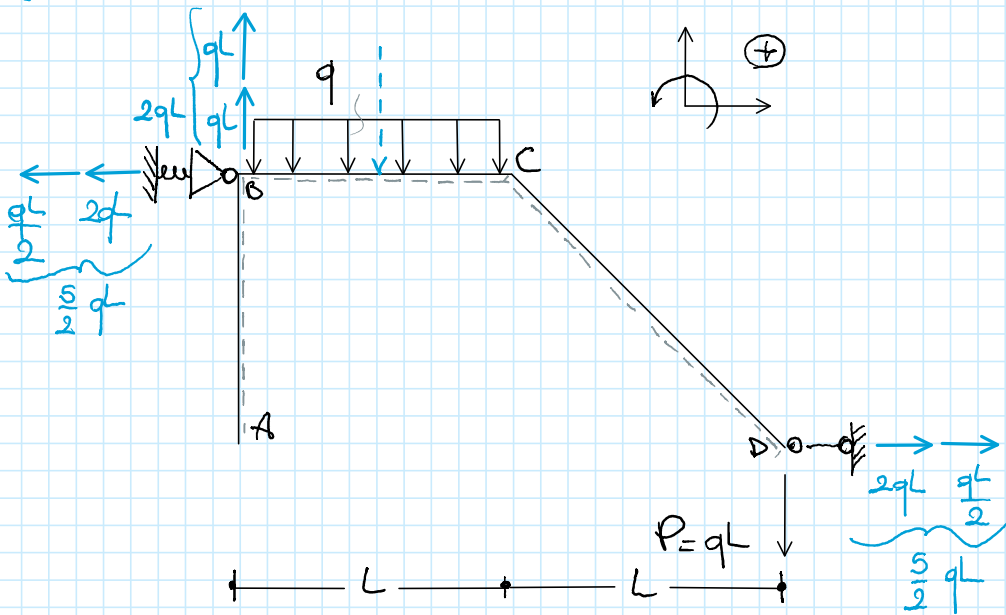
DIAGRAMMA FINALE  $M^{(r)}(z)$

## SOLUZIONE #2

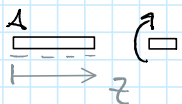
### SISTEMA PRINCIPALE ISOSTATICO



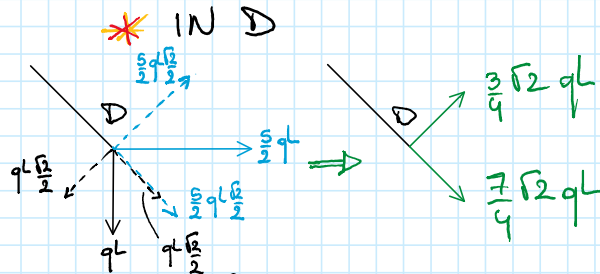
### SCHEMA [0] SOLO CARICHI ESTERNI



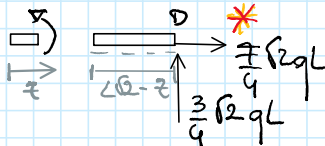
TRATTO AB  $0 \leq z \leq L$



$$M^{(1)}(z) = 0$$



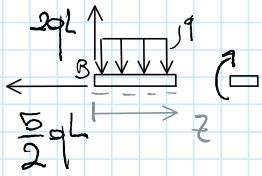
TRATTO CD  $0 \leq z \leq L\sqrt{2}$



$$M^{(1)}(z) = \frac{3\sqrt{2}}{4} qL (\sqrt{2}L - z)$$

$$\begin{cases} M_C = \frac{3}{2} qL^2 \\ M_D = 0 \end{cases}$$

TRATTO BC  $0 \leq z \leq L$



$$M^{(0)}(z) = 2qLz - q\frac{z^2}{2}$$

$$\begin{cases} M_B = 0 \\ M_C = \frac{3}{2} qL^2 \end{cases}$$

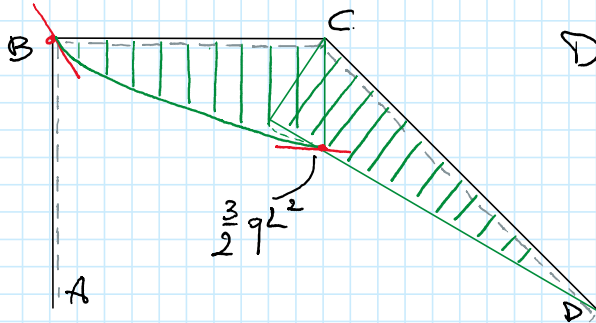
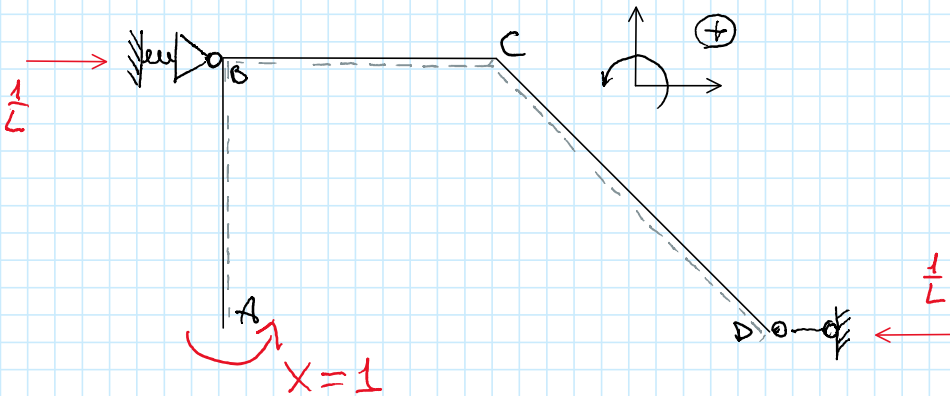


DIAGRAMMA  $M^{(0)}(z)$

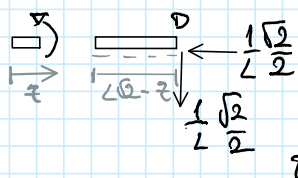
• SCHEMA [1] SOLO  $X=1$



TRATTO AB  $0 \leq z \leq L$

$$\left( \begin{array}{c} 1 \\ 1 \end{array} \right) \Rightarrow M^{(1)}(z) = -1 \quad \text{cost.}$$

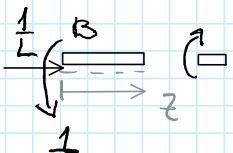
TRATTO CD  $0 \leq z \leq L\sqrt{2}$



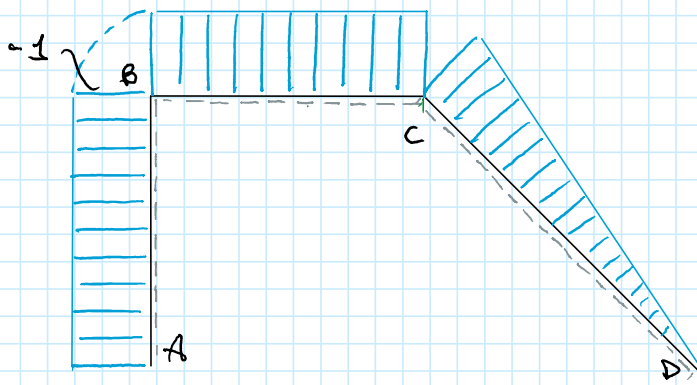
$$M^{(1)}(z) = -\frac{\sqrt{2}}{2L} (L\sqrt{2} - z)$$

$$\begin{cases} M_C = -1 \\ M_D = 0 \end{cases}$$

TRATTO BC  $0 \leq z \leq L$



$$M^{(1)}(z) = -1 \quad \text{cost.}$$



$$\begin{aligned} \delta u_e &= 1 \cdot (-\bar{\epsilon} X) + \frac{1}{L} \left( -\bar{\epsilon} \left( -\frac{5}{2} qL + \frac{1}{L} X \right) \right) + \left( -\frac{1}{L} M_D^0 \right) = \\ &= -\bar{\epsilon} X - \frac{\bar{\epsilon}}{L} \left[ \frac{X}{L} - \frac{5}{2} qL \right] - \frac{M_D^0}{L} \end{aligned}$$

$$\begin{aligned} \delta u_i &= \int_{str} M^{(1)} \frac{M^{(n)}}{\bar{\epsilon} I} dstr + \int_{str} M^{(1)} \alpha \frac{\Delta T}{h} dstr = \\ &= \frac{1}{\bar{\epsilon} I} \left\{ \int_{AB} M^{(1)} M^{(n)} d\bar{z} + \int_{BC} M^{(1)} M^{(n)} d\bar{z} + \int_{CD} M^{(1)} M^{(n)} d\bar{z} \right\} + \\ &\quad + \frac{X}{\bar{\epsilon} I} \left\{ \int_{AB} [M^{(1)}]^2 d\bar{z} + \int_{BC} [M^{(1)}]^2 d\bar{z} + \int_{CD} [M^{(1)}]^2 d\bar{z} \right\} + \int_{CD} M^{(1)} \alpha \frac{\Delta T}{h} d\bar{z} = \\ &= \frac{1}{\bar{\epsilon} I} \left\{ \int_0^L (-1) \cdot 0 d\bar{z} + \int_0^L (-1) \left( 2q\bar{z} - q \frac{\bar{z}^2}{2} \right) d\bar{z} + \int_0^{\sqrt{2}L} \left( -\frac{\sqrt{2}}{2L} (\sqrt{2}L - \bar{z}) \right) \left( \frac{3}{4} \sqrt{2} qL (\sqrt{2}L - \bar{z}) \right) d\bar{z} \right\} + \\ &\quad + \frac{X}{\bar{\epsilon} I} \left\{ \int_0^L (-1)^2 d\bar{z} + \int_0^L (-1)^2 d\bar{z} + \int_0^{\sqrt{2}L} \left[ -\frac{\sqrt{2}}{2L} (\sqrt{2}L - \bar{z}) \right]^2 d\bar{z} \right\} + \int_0^{\sqrt{2}L} \left( -\frac{\sqrt{2}}{2L} (\sqrt{2}L - \bar{z}) \right) \cdot \frac{\alpha \Delta T}{h} d\bar{z} = \\ &= \frac{1}{\bar{\epsilon} I} \left\{ \int_0^L -2qL\bar{z} + q \frac{\bar{z}^2}{2} + \int_0^{\sqrt{2}L} \left( -1 + \frac{\sqrt{2}}{2L} \bar{z} \right) \left( \frac{3}{2} qL^2 - \frac{3}{4} \sqrt{2} qL \bar{z} \right) d\bar{z} + \right. \\ &\quad \left. + \frac{X}{\bar{\epsilon} I} \left\{ \left[ \bar{z} \right]_0^L + \left[ \bar{z} \right]_0^L + \left[ \bar{z} \right]_0^{\sqrt{2}L} + \frac{1}{2L^2} \left[ \frac{\bar{z}^3}{3} \right]_0^{\sqrt{2}L} - \frac{\sqrt{2}}{L} \left[ \frac{\bar{z}^2}{2} \right]_0^{\sqrt{2}L} \right\} + \frac{\alpha \Delta T}{h} \left\{ -\left[ \bar{z} \right]_0^{\sqrt{2}L} + \frac{\sqrt{2}}{2L} \left[ \frac{\bar{z}^2}{2} \right]_0^{\sqrt{2}L} \right\} \right\} = \\ &= \frac{1}{\bar{\epsilon} I} \left\{ -2qL \left[ \frac{\bar{z}^2}{2} \right]_0^L + \frac{q}{2} \left[ \frac{\bar{z}^3}{3} \right]_0^L - \frac{3}{2} qL^2 \left[ \bar{z} \right]_0^{\sqrt{2}L} + \frac{3}{4} \sqrt{2} qL \left[ \frac{\bar{z}^2}{2} \right]_0^{\sqrt{2}L} + \frac{3}{4} \sqrt{2} qL \left[ \frac{\bar{z}^2}{2} \right]_0^{\sqrt{2}L} + \right. \end{aligned}$$

$$\begin{aligned}
& -\frac{3q}{4} \left[ \frac{x^3}{3} \right]_0^{\frac{\sqrt{2}L}{3}} \left. \right\} + \frac{X}{EI} \left\{ L + L + \sqrt{2}L + \frac{\sqrt{2}L}{3} - \sqrt{2}L \right\} + \frac{\alpha \Delta T}{h} \left\{ -\sqrt{2}L + \frac{\sqrt{2}}{2}L \right\} = \\
& = \frac{1}{EI} \left\{ -qL^3 + \frac{qL^3}{6} - \frac{3\sqrt{2}qL^3}{2} + \frac{3\sqrt{2}}{4}qL^3 + \frac{3\sqrt{2}}{4}qL^3 - \frac{\sqrt{2}}{2}qL^3 \right\} + \\
& + \frac{X}{EI} \left\{ 2L + \frac{\sqrt{2}L}{3} \right\} + \frac{\alpha \Delta T}{h} \left\{ -\frac{\sqrt{2}}{2}L \right\} = \\
& = \frac{1}{EI} \left[ -\frac{5}{6}qL^3 - \frac{\sqrt{2}}{2}qL^3 \right] + \frac{X}{EI} \left[ 2L + \frac{\sqrt{2}L}{3} \right] - \frac{\sqrt{2}L}{2} \frac{\alpha \Delta T}{h}
\end{aligned}$$

$\Delta u_e = \Delta u_i$  fornisce

$$-\bar{\epsilon}X - \frac{\epsilon}{L^2}X + \frac{5}{2}\epsilon q - \frac{\eta_D^0}{L} = \frac{2XL}{EI} - \frac{5}{6}\frac{qL^3}{EI} - \frac{\sqrt{2}}{2}\frac{qL^3}{EI} + \frac{X}{EI}\frac{\sqrt{2}L}{3} - \frac{\sqrt{2}L}{2}\frac{\alpha \Delta T}{h}$$

$$\begin{aligned}
& -\frac{2}{5}\frac{L}{EI}X - \frac{3}{5}\frac{L^3}{EI}\frac{X}{L^2} + \frac{5q}{2}\frac{3}{5}\frac{L^3}{EI} - \frac{qL^3}{EI}\left[\frac{\sqrt{2}}{3}+2\right] = \frac{2XL}{EI} - \frac{5}{6}\frac{qL^3}{EI} - \frac{\sqrt{2}}{2}\frac{qL^3}{EI} + \\
& + \frac{X}{EI}\frac{\sqrt{2}L}{3} - \frac{\sqrt{2}L}{2}\frac{qL^2}{EI}\left[\frac{25}{6}+\frac{\sqrt{2}}{3}\right]
\end{aligned}$$

$$-\frac{XL}{EI}\left(3+\frac{\sqrt{2}}{3}\right) = -\frac{3}{2}\frac{qL^3}{EI} + \frac{qL^3}{EI}\left(\frac{\sqrt{2}}{3}+2\right) - \frac{5}{6}\frac{qL^3}{EI} - \frac{\sqrt{2}}{2}\frac{qL^3}{EI} - \frac{qL^3}{EI}\left(\frac{25}{6}+\frac{\sqrt{2}}{3}\right)$$

$$-\frac{XL}{EI}\left(3+\frac{\sqrt{2}}{3}\right) = -\frac{9}{2}\frac{qL^3}{EI} - \frac{\sqrt{2}}{2}\frac{qL^3}{EI}$$

$$\frac{-9+12-5-25}{6} = \frac{-27}{6} = -\frac{9}{2}$$

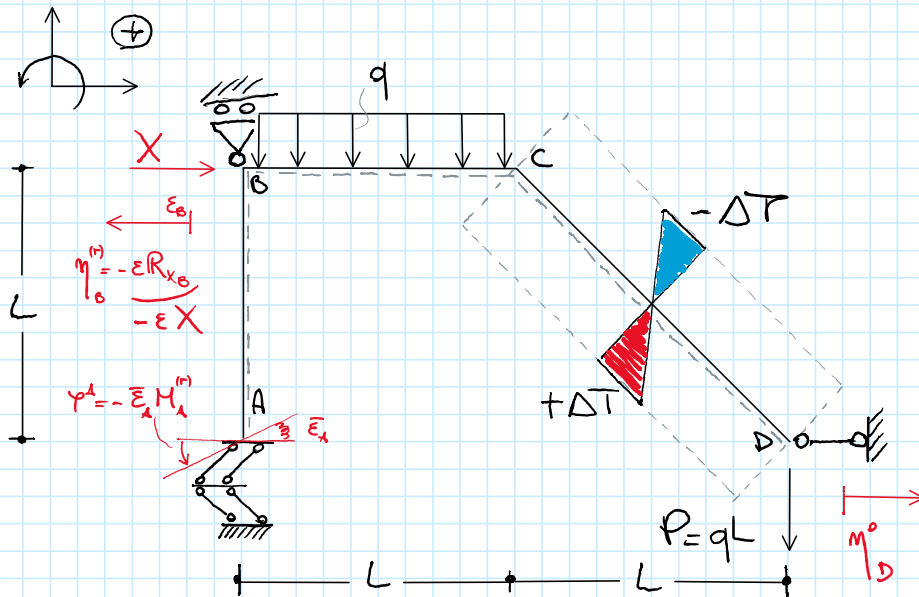
$$+X\left(3+\frac{\sqrt{2}}{3}\right) = \frac{qL^2}{2}(9+\sqrt{2})$$

$$+\frac{X}{3}(9+\sqrt{2}) = \frac{qL^2}{2}(9+\sqrt{2})$$

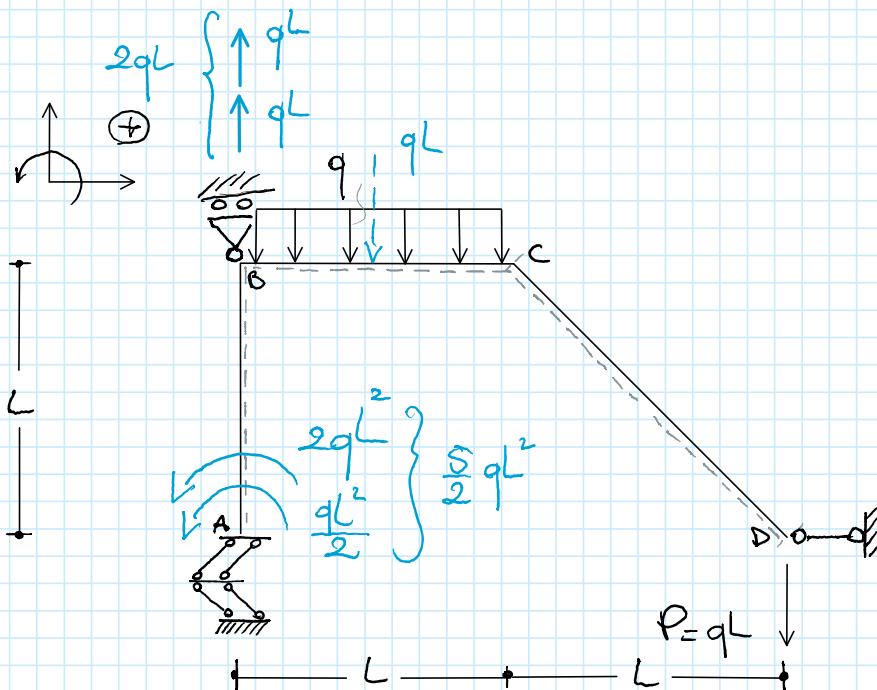
$$X = \frac{3}{2}qL^2$$

# SOLUZIONI # 3

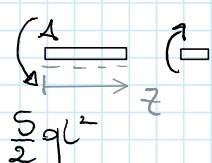
## SISTEMA PRINCIPALE ISOSTATICO



## SCHEMA [0] SOLO CARICHI ESTERNI



TRATTO AB  $0 \leq z \leq L$

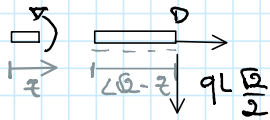


$$M^{(0)}(z) = -\frac{5}{2} qL^2$$

$$\begin{cases} M_A = -\frac{5}{2} qL^2 \\ M_B = -\frac{5}{2} qL^2 \end{cases}$$



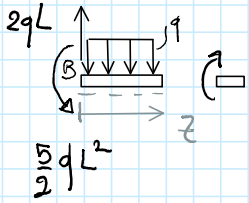
TRATTO CD  $0 \leq \bar{z} \leq L\sqrt{2}$



$$M^{(0)}(\bar{z}) = -qL\frac{\sqrt{2}}{2}(L\sqrt{2}-\bar{z})$$

$$\begin{cases} M_c = -qL^2 \\ M_D = 0 \end{cases}$$

TRATTO BC  $0 \leq \bar{z} \leq L$



$$M^{(0)}(\bar{z}) = -\frac{5}{2}qL^2 + 2qL\bar{z} - \frac{q\bar{z}^2}{2}$$

$$\begin{cases} M_B = -\frac{5}{2}qL^2 \\ M_C = -qL^2 \end{cases}$$

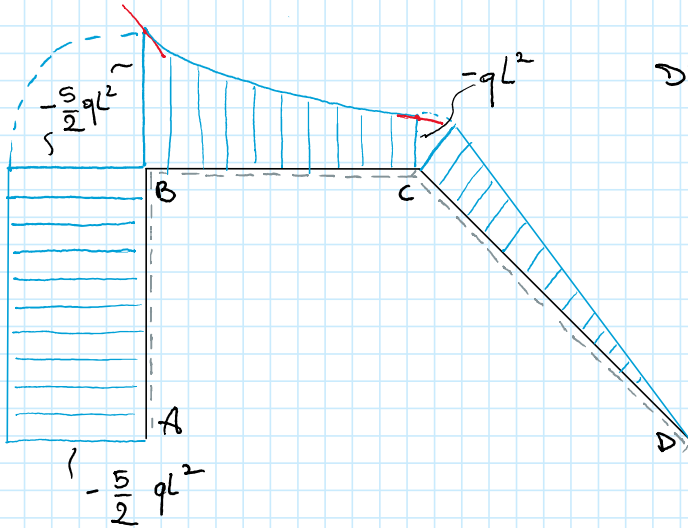
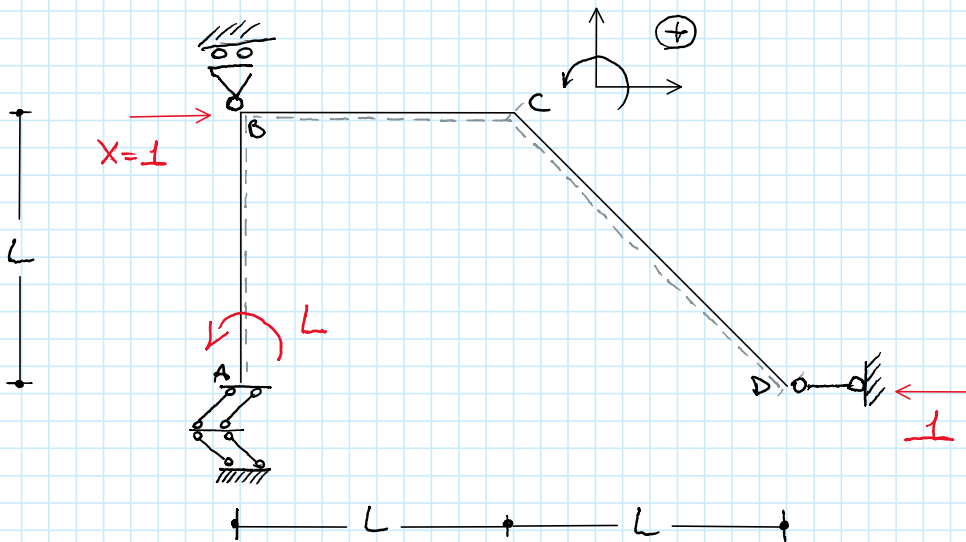
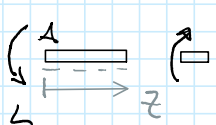


DIAGRAMMA  $M^{(0)}(\bar{z})$

• SCHEMA [1] SOLO  $X=1$

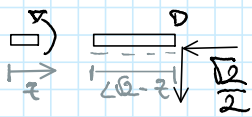


TRATTO AB  $0 \leq \bar{z} \leq L$



$$M^{(0)}(\bar{z}) = -L \quad \text{cost.}$$

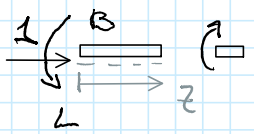
TRATTO CD  $0 \leq \bar{z} \leq L\sqrt{2}$



$$M^{(1)}(\bar{z}) = -\frac{\sqrt{2}}{2} [L\sqrt{2} - \bar{z}]$$

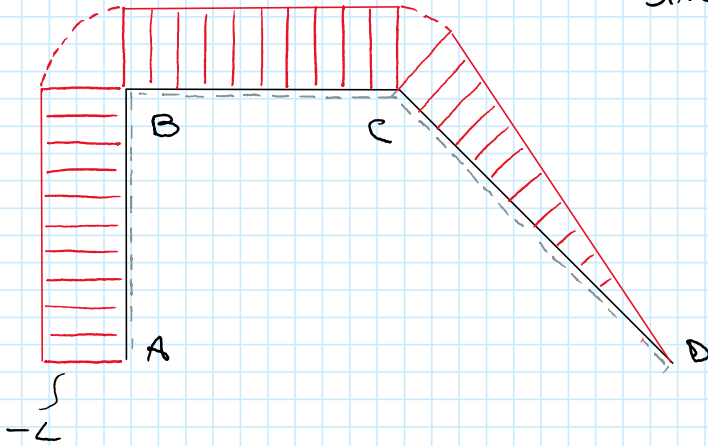
$$\begin{cases} M_c = -L \\ M_D = 0 \end{cases}$$

TRATTO BC  $0 \leq \bar{z} \leq L$



$$M^{(1)}(\bar{z}) = -L \quad \text{cost.}$$

DIAGRAMMA  $M^{(1)}(\bar{z})$



$$\begin{aligned} \Delta_{ve} &= 1 \cdot \eta_i^{(r)} + \sum_j R_j^{(1)} \cdot \eta_j^{(r)} = 1 \cdot \eta_B^{(r)} + M_A^{(1)} \cdot \eta_A^{(r)} + R_{XD}^{(1)} \cdot \eta_D^0 = \\ &= 1 \cdot -\varepsilon X + L \left[ -\varepsilon M_A^{(r)} \right] - 1 \cdot \eta_D^0 = \\ &= -\varepsilon X - \varepsilon L \left[ \frac{5}{2} L^2 + XL \right] - \eta_D^0 = \end{aligned}$$

$\frac{5}{2} L^2 \quad M_A^{(0)} + X \frac{M_A^{(1)}}{L}$

$$\begin{aligned} \Delta_{vi} &= \int_{Str} M^{(1)} \frac{M^{(r)}}{EI} dStr + \int_{Str} M^{(1)} \frac{\alpha \Delta T}{h} dStr = \\ &= \frac{1}{EI} \int_{Str} M^{(1)} M^{(0)} dStr + \frac{X}{EI} \int_{Str} [M^{(1)}]^2 dStr + \int_{Str} M^{(1)} \frac{\alpha \Delta T}{h} dStr = \\ &= \frac{1}{EI} \left\{ \int_{AB} M^{(1)} M^{(0)} d\bar{z} + \int_{BC} M^{(1)} M^{(0)} d\bar{z} + \int_{CD} M^{(1)} M^{(0)} d\bar{z} \right\} + \end{aligned}$$

$$\begin{aligned}
& + \frac{\chi}{EI} \left\{ \int_{AB} [M^{(1)}]^2 dz + \int_{BC} [M^{(1)}]^2 dz + \int_{CD} [M^{(1)}]^2 dz \right\} + \int_{\bar{O}} M^{(1)} \frac{\alpha \Delta T}{h} dz = \\
& = \frac{1}{EI} \left\{ \int_0^L -L \left[ -\frac{5}{2} q L^2 \right] dz + \int_0^L -L \left[ -\frac{5}{2} q L^2 + 2qLz - \frac{qz^2}{2} \right] + \right. \\
& + \int_0^{\sqrt{2}L} -\frac{\sqrt{2}}{2} \left[ L\sqrt{2} - z \right] \left[ -qL\frac{\sqrt{2}}{2} (L\sqrt{2} - z) \right] dz \left. \right\} + \frac{\chi}{EI} \left\{ \int_0^L L^2 dz + \right. \\
& + \int_0^L L^2 dz + \int_0^{\frac{\sqrt{2}L}{2}} \frac{2}{4} \left[ L\sqrt{2} - z \right]^2 dz \left. \right\} + \frac{\alpha \Delta T}{h} \int_0^{\sqrt{2}L} -\frac{\sqrt{2}}{2} \left[ L\sqrt{2} - z \right] dz = \\
& = \frac{1}{EI} \left\{ + \frac{5}{2} q L^3 \left[ z \right]_0^L + \frac{5}{2} q L^3 \left[ z \right]_0^L - 2qL^2 \left[ \frac{z^2}{2} \right]_0^L + \frac{qL}{2} \left[ \frac{z^3}{3} \right]_0^L + \right. \\
& + qL^3 \left[ z \right]_0^{\sqrt{2}L} - qL^2 \frac{\sqrt{2}}{2} \left[ \frac{z^2}{2} \right]_0^{\sqrt{2}L} - qL^2 \frac{\sqrt{2}}{2} \left[ \frac{z^2}{2} \right]_0^{\sqrt{2}L} + \frac{qL}{2} \left[ \frac{z^3}{3} \right]_0^{\sqrt{2}L} \left. \right\} + \frac{\chi}{EI} \left\{ L^2 \left[ z \right]_0^L + \right. \\
& + L^2 \left[ z \right]_0^L + L^2 \left[ z \right]_0^{\sqrt{2}L} + \frac{1}{2} \left[ \frac{z^3}{3} \right]_0^{\sqrt{2}L} - L\sqrt{2} \left[ \frac{z^2}{2} \right]_0^{\sqrt{2}L} \left. \right\} + \frac{\alpha \Delta T}{h} \left\{ -L \left[ z \right]_0^{\sqrt{2}L} + \frac{\sqrt{2}}{2} \left[ \frac{z^2}{2} \right]_0^{\sqrt{2}L} \right\} = \\
& = \frac{1}{EI} \left\{ + \frac{5}{2} q L^4 + \frac{5}{2} q L^4 - q L^4 + \frac{q L^4}{6} + \cancel{\sqrt{2} q L^4} - \cancel{\frac{\sqrt{2}}{2} q L^4} - \cancel{\frac{\sqrt{2}}{2} q L^4} + \right. \\
& + \frac{\sqrt{2}}{3} q L^4 \left. \right\} + \frac{\chi}{EI} \left\{ L^3 + L^3 + \cancel{\sqrt{2} L^3} + \frac{\sqrt{2}}{3} L^3 - \cancel{\sqrt{2} L^3} \right\} + \frac{\alpha \Delta T}{h} \left\{ -\sqrt{2} L^2 + \frac{\sqrt{2}}{2} L^2 \right\} = \\
& + \frac{1}{EI} \left\{ + \frac{25}{6} q L^4 + \frac{\sqrt{2}}{3} q L^4 \right\} + \frac{\chi}{EI} \left\{ 2L^3 + \frac{\sqrt{2}}{3} L^3 \right\} + \frac{\alpha \Delta T}{h} \left\{ -\frac{1}{2} \sqrt{2} L^2 \right\}
\end{aligned}$$

$\Delta v_e = \Delta v_i$  fornisce

$$\begin{aligned} -\varepsilon X - \bar{\varepsilon} L \left[ \frac{5}{2} q L^2 + X L \right] - \eta_0 &= + \frac{1}{\varepsilon I} \left[ + \frac{25}{6} q L^4 + \frac{\sqrt{2}}{3} q L^4 \right] + \\ + \frac{X}{\varepsilon I} \left[ 2 L^3 + \frac{\sqrt{2}}{3} L^3 \right] + \frac{\alpha \Delta T}{h} \left[ - \frac{1}{2} \sqrt{2} L^2 \right] \\ - \frac{3}{8} \frac{X L^3}{\varepsilon I} - \frac{2}{5} \frac{L^2}{\varepsilon I} \left[ \frac{5}{2} q L^2 + X L \right] - \frac{q L^4}{\varepsilon I} \left[ \frac{\sqrt{2}}{3} + 2 \right] &= \\ = + \frac{1}{\varepsilon I} \left[ + \frac{25}{6} q L^4 + \frac{\sqrt{2}}{3} q L^4 \right] + \frac{X}{\varepsilon I} \left[ 2 L^3 + \frac{\sqrt{2}}{3} L^3 \right] - \frac{q L^4}{\varepsilon I} \left[ \frac{25}{6} + \frac{\sqrt{2}}{3} \right] \\ \frac{X L^3}{\varepsilon I} \left[ - \frac{3}{8} - \frac{2}{5} - 2 - \frac{\sqrt{2}}{3} \right] &= \frac{q L^4}{\varepsilon I} \left[ + \frac{25}{6} + \frac{\sqrt{2}}{3} - \frac{25}{6} - \frac{\sqrt{2}}{3} + \frac{\sqrt{2}}{3} + 2 + 1 \right] \\ X \left[ - 3 - \frac{\sqrt{2}}{3} \right] &= q L \left[ \frac{\sqrt{2}}{3} + 3 \right] \end{aligned}$$

$$X = -qL \quad \text{NEGATIVO}$$